Soutenance pour l'Habilitation à Diriger des Recherches :

Measurements, decoherence, and quantum correlations in composite quantum systems

Dominique Spehner

Institut Fourier
et Laboratoire de Physique et Modélisation des Milieux Condensés,
Grenoble

Institut Fourier, 10/04/2015

Outline

- Introduction
- Model for an ideal quantum measurement
- Geometric discord with Bures distance
- Summary of other results

Introduction: part I

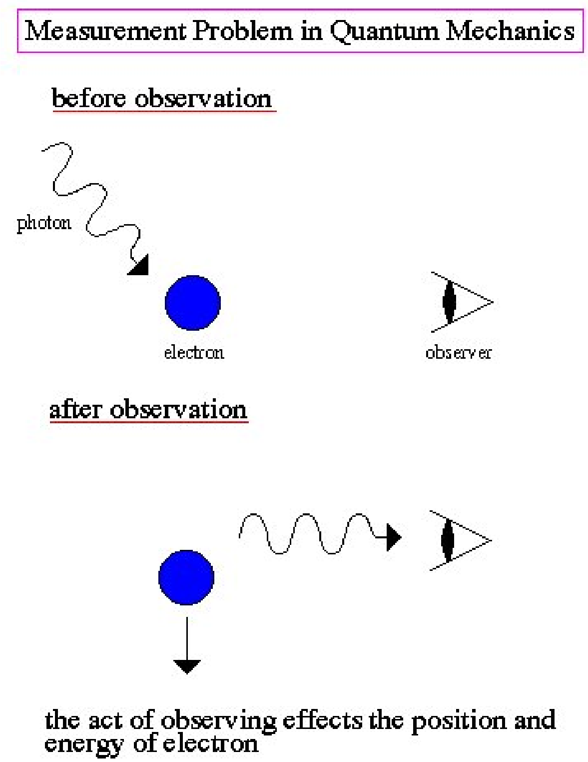
STRANGENESS OF THE QUANTUM WORLD...

AS NOTED BY ITS FOUNDING FATHERS

The quantum measurement problem

Reduction of the wavepacket:

- To obtain information on a quantum system, a measuring apparatus strongly perturbes its state.
- After the measurement of an observable S (= self-adjoint operator) giving the result "s_i", the system state is projected onto the eigenspace of S with eigenvalue s_i.



$\hookrightarrow$ Measurement = dynamical process seemingly independent from Schrödinger equation. *[Heisenberg '27, von Neumann '55,...]*

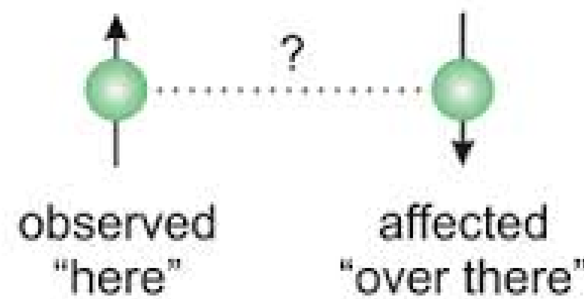
Entanglement

"Maximal knowledge on a total system does not necessarily include maximal knowledge on all its parts, even if those are completely separated from each other and for now cannot interact" (Schrödinger '35)

▻ Quantum mechanics is either nonlocal or incomplete

[Einstein, Podolsky, Rosen '35]

$$|\Psi_{\mathrm{EPR}}\rangle = \frac{1}{\sqrt{2}}\big(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle\big)$$



▻ Violation of the Bell inequalities: *[Bell '66, Clauser et al. '69]*

$$\langle ab\rangle + \langle a'b\rangle + \langle a'b'\rangle - \langle ab'\rangle \leqslant 2$$

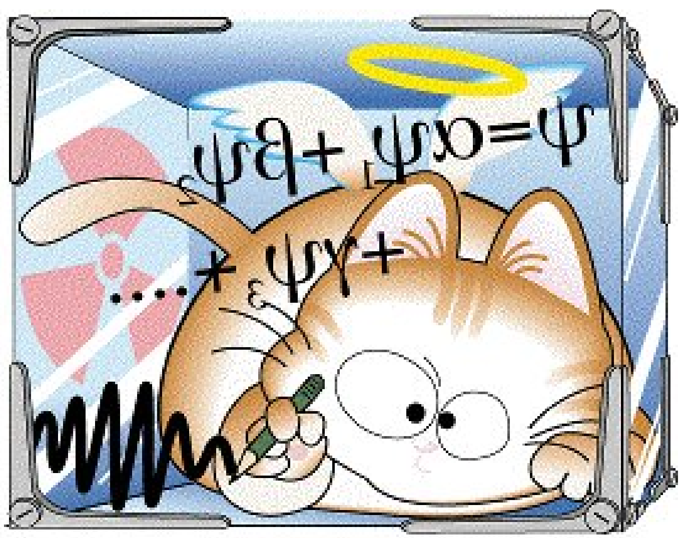
observed experimentally! *[Aspect, Dalibard, Roger '82,...]*

Schrödinger cat states

By the linearity principle (=THE principle of quantum mechanics!), linear superpositions of macroscopically distinguishable states as

$$|\Psi_{\text{cat}}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle|\text{alive}\rangle + |\downarrow\rangle|\text{dead}\rangle)$$

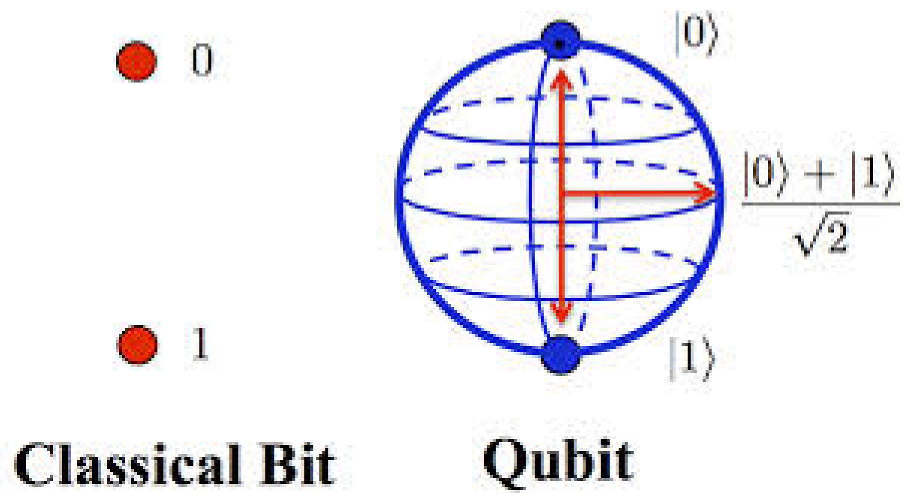
should exist in nature! *[Schrödinger '35]*



Introduction: part II

STRANGENESS OF THE QUANTUM WORLD...

AS WE LOOK AT IT IN 2015



Theory of open quantum systems

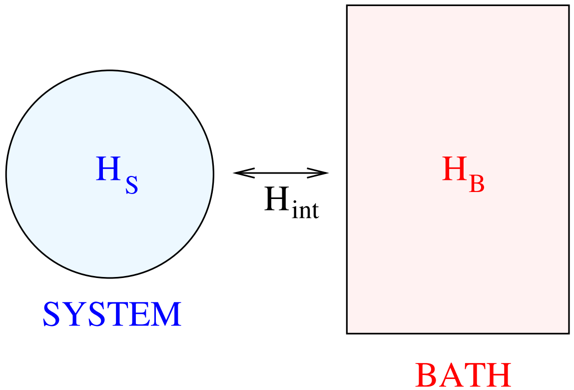
Isolated quantum systems

$\hookrightarrow$ **reversible** unitary dynamics in the Hilbert space $\mathcal{H}$, governed by Schrödinger's equation $\mathrm{i}\partial_t|\Psi(t)\rangle = H|\Psi(t)\rangle$

Open quantum systems

$=$ systems coupled to their environment.

$\hookrightarrow$ **irreversible** dynamics for density matrices $\rho(t)$, characterized by two timescales:



1) relaxation time τ_{R} (e.g. convergence to equilibrium)

2) decoherence time τ_{dec}: superpositions are transformed into statistical mixtures.

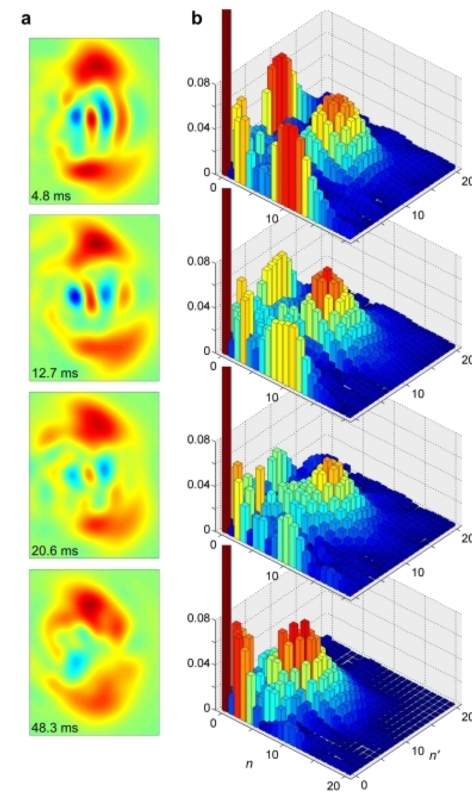
Schrödinger cat states and decoherence

For Schrödinger cat states, typically $\tau_{\rm dec} \ll \tau_{\rm R}$

$\hookrightarrow$ after a short time $\tau_{\rm dec}$, decoherence transforms the state as

$$|\Psi_{\rm cat}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle|\text{alive}\rangle + |\downarrow\rangle|\text{dead}\rangle\right)$$

$$\begin{array}{ll} \nearrow & |\uparrow\rangle|\text{alive}\rangle \ \text{ with proba } 1/2 \\ \searrow & |\downarrow\rangle|\text{dead}\rangle \ \text{ with proba } 1/2\,. \end{array}$$



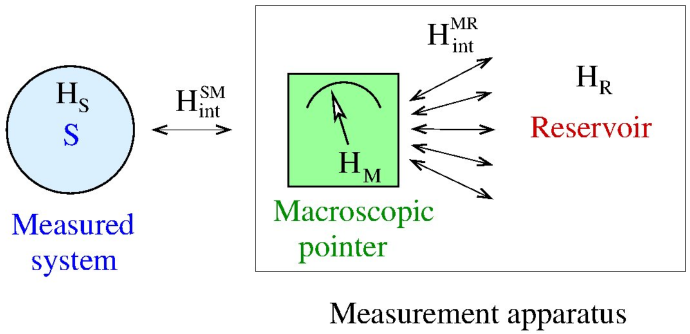
Superpositions of states differing by 10-100 photons **have been observed in laboratories:**

- at NIST *[Monroe et al. Science 273, '96]*
- at LKB in Paris *[Deléglise et al. Nature 455, '08]*
- at Yale *[Vlastakis et al., Science 342 '13]*

$\rightsquigarrow$ result on decoherence for superpositions in ultracold atoms.

Solution to the measurement problem

A measuring apparatus consists of a macroscopic pointer M coupled to large reservoir R.



- The system-pointer interaction builds up Schrödinger cat states

$$|\Psi_{\mathsf{SM}}\rangle = \sum_i c_i |i\rangle \otimes |\mu_i\rangle$$

$|i\rangle =$ eigenstate of the measured observable S.

- Decoherence due to the M-R coupling transforms this state into

$$\rho_{\mathsf{SM}} = \sum_i |c_i|^2 |i\rangle\langle i| \otimes |\mu_i\rangle\langle\mu_i|$$

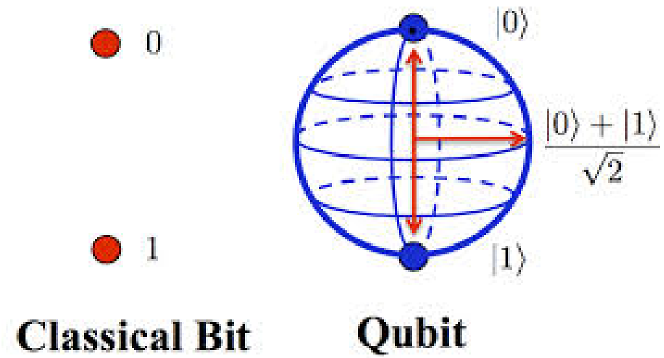
$\Rightarrow$ derivation of the reduction of the wavepacket from the Schrödinger equation for the system $+$ apparatus

[Zurek '91, Allahverdyan, Balian, Nieuwenhuizen '13]

$\rightsquigarrow$ study an explicit model.

Entanglement & theory of quantum information

- A quantum computer works with qubits, i.e. two-level quantum systems in linear combinations of $|0\rangle$ and $|1\rangle$.



- **Entanglement** can be viewed as a **resource** to accomplish certain information-processing tasks with higher efficiencies than can be done classically (e.g. factorizing into prime numbers, improving sensitivity of interferometers,...) *[Bennett et al. '96]*

- Other kinds of "quantum correlations" differing from entanglement could explain the quantum efficiencies, like those captured by the **quantum discord**.

[Ollivier, Zurek '01, Henderson, Vedral '01]

$\rightsquigarrow$ geometric approach to nonclassicality and quantum discord.

Outline

✓ Introduction

- Model for an ideal quantum measurement

ARTICLES:

- D. Spehner, F. Haake, *Quantum measurements without macroscopic superpositions*, Phys. Rev. A 77 (2008), 052114
- D. Spehner, F. Haake, *Decoherence bypass of macroscopic superpositions in quantum measurement*, J. Phys. A: Math. Theor. 41 (2008), 072002
- D. Spehner, F. Haake, *Quantum measurements without Schrödinger cat states*, J. Phys.: Conf. Series 84 (2007), 012018 (conference proceedings)

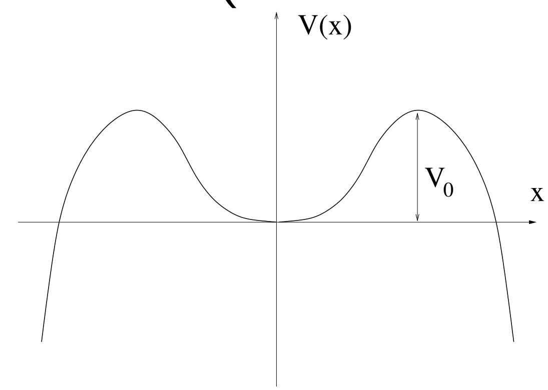
The measuring apparatus

- Consider a pointer M with a single degree of freedom (Hilbert space $L^2(\mathbb{R})$) and Hamiltonian

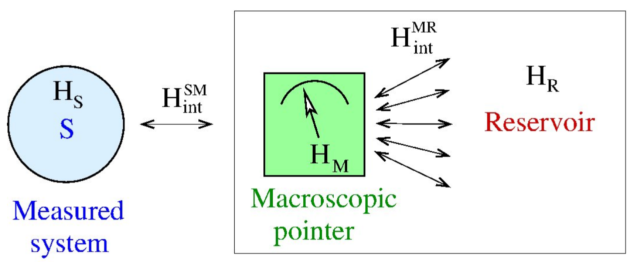
$$H_{\mathsf{M}} = \frac{P^2}{2m} + V(X)$$

$X =$ position op., $P =$ momentum op.

The potential V has a local minimum at $x = 0$.



1) M interacts with the system S between times $t = 0$ and $t = t_{\text{int}}$



Measurement apparatus

$$\boxed{H^{\mathsf{SM}}_{\text{int}} = gS \otimes P}$$

$S =$ measured observable

2) M is coupled to a reservoir R with $N \gg 1$ degrees of freedom

$$H^{\mathsf{MR}}_{\text{int}} = X \otimes B \quad \text{with} \quad B = N^{-\frac{1}{2}} \sum_{\nu=1}^{N} B_\nu \text{ acting on } \otimes_{\nu=1}^{N} \mathcal{H}_\nu .$$

Initial state

- The system S is initially in a pure state $|\psi\rangle = \sum_i c_i |i\rangle$ with $\{|i\rangle\}$ $=$ eigenbasis of S (assume S has discrete nondegen. spectrum).
- Initially there are no system-apparatus correlations, i.e.

$$\rho_{\mathsf{SMR}}(0) = |\psi\rangle\langle\psi| \otimes \rho_{\mathsf{MR}}(0)\,.$$

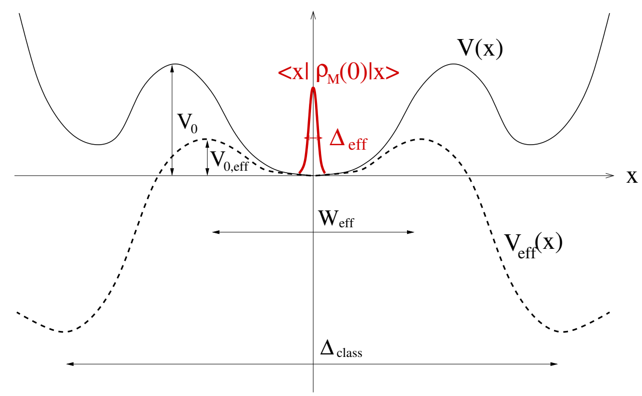
- Pointer and reservoir are initially in a metastable equilibrium

$$\rho_{\mathsf{MR}}(0) = Z^{-1} e^{-\beta(H_{\mathsf{M}} + H_{\mathsf{R}} + H_{\text{int}}^{\mathsf{MR}})}$$

β = inverse temperature

$\Rightarrow \langle x|\rho_{\mathsf{M}}(0)|x\rangle \propto e^{-\beta V_{\text{eff}}(x)} \simeq e^{-(x/\Delta_{\text{eff}})^2}$

$V_{\text{eff}}(x) = V(x) - \gamma_0 x^2$ with $\gamma_0 \geqslant 0$.



- Separation of times and lengthscales

$$\boxed{t_{\text{int}}, \beta \ll T_{\mathsf{M}}, T_{\mathsf{S}} \quad , \quad \Delta_{\text{eff}} \ll W_{\text{eff}} \ll \Delta_{\text{clas}}}$$

Dynamics

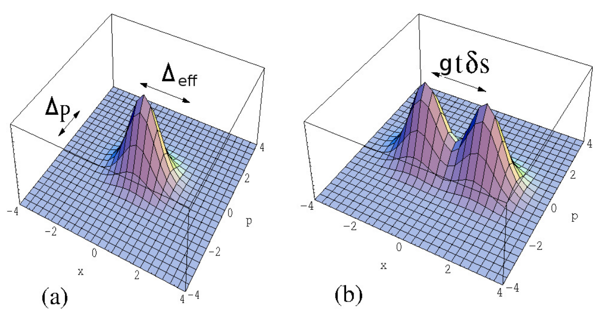
Density matrix of the system and pointer at time $t \leqslant t_{\text{int}}$

$$\rho_{\mathsf{SM}}(t) = \text{tr}_{\mathsf{R}}(e^{-\text{i}tH}\rho_{\mathsf{SMR}}(0)e^{\text{i}tH}) \ , \ H = \underbrace{H_{\mathsf{S}} + H_{\mathsf{M}}}_{\text{neglected}} + H_{\mathsf{R}} + H_{\text{int}}^{\mathsf{SM}} + H_{\text{int}}^{\mathsf{MR}}$$

$\rightsquigarrow$ decoherence and entanglement act simultaneously.

MAIN RESULT

$$\boxed{\langle i, x | \rho_{\mathsf{SM}}(t) | j, y \rangle = c_i \bar{c}_j \underbrace{\langle x - g s_i t | \rho_{\mathsf{M}}(0) | y - g s_j t \rangle}_{\text{shifted initial pointer state}} \underbrace{\exp\{-D_t(x, s_i; y, s_j)\}}_{\text{decoherence factor} \leqslant 1}}$$



(a) (b)

◇ At the **entanglement time**

$$\tau_{\text{ent}} = \Delta_{\text{eff}}/(g\delta s) \ , \ \delta s = |s_i - s_j| \ ,$$

one can resolve the peaks in pointer positions tied up with distinct eigenvalues s_i.

◇ **Decoherence time:** $e^{-D_t^{\text{peak}}} \simeq \exp\left\{ - \left(\frac{t}{\tau_{\text{dec}}}\right)^{\gamma} \right\}$ with $\gamma = 2, 3, 4$

$\tau_{\text{dec}} = c\beta[\tau_{\text{ent}}/(\langle B^2 \rangle^{\frac{1}{2}} \beta^2 \Delta_{\text{eff}})]^{\frac{2}{\gamma}}$ (short time and Markov regimes).

Reduction of the wavepacket

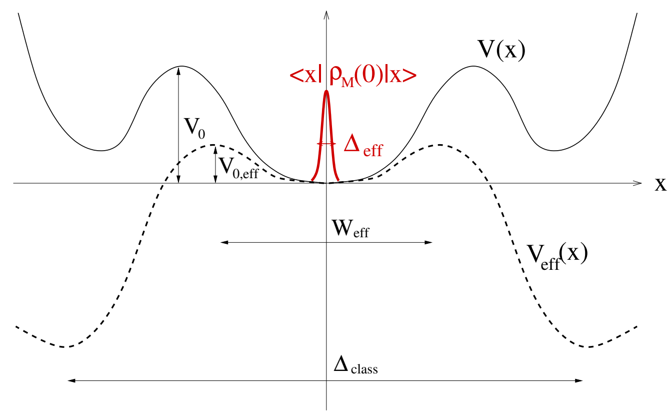
- At time $t \gg \tau_{\rm dec}$, all matrix elements for $i \neq j$ vanish,

$$\boxed{\rho_{\sf SM}(t) \;\simeq\; \sum_i |c_i|^2 \quad \underbrace{|i\rangle\langle i|}_{\text{system in an eigenstate of } S} \quad \otimes \quad \underbrace{\rho_{{\sf M}|i(t)}}_{\text{pointer with position } x \simeq g s_i t}}$$

- Take system-pointer **interaction time**

$$t_{\rm int} \approx W_{\rm eff}/(g\delta s)$$

$\hookrightarrow$ evolution for $t \geqslant t_{\rm int}$ in the unstable pointer potential $V_{\rm eff}$ provides an amplification mechanism



$\Rightarrow$ pointer positions at time $t_{\rm mes}$ separated by macroscopic distances $g\delta s\, t_{\rm mes} \approx \Delta_{\rm clas}$

- The different timescales of the measurement are related as
  $\tau_{\rm ent}, \tau_{\rm dec} \ll t_{\rm int} \ll t_{\rm mes}$ (in most cases $\tau_{\rm ent} \leqslant \tau_{\rm dec}$).

Outline

✓ Introduction

✓ Model for an ideal quantum measurement

- Geometric discord with Bures distance

ARTICLES:

- D. Spehner, *Quantum correlations and Distinguishability of quantum states*, J. Math. Phys. 55 (2014), 075211 (review article)

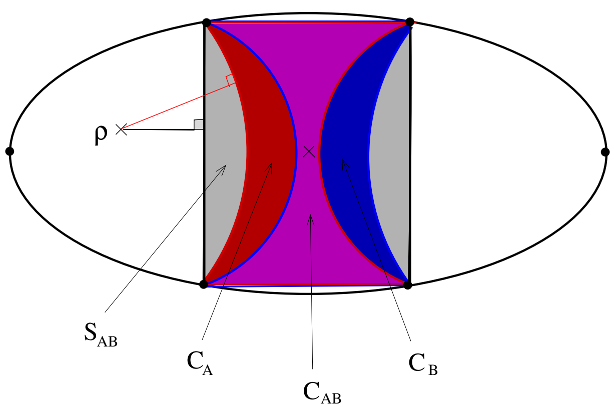
- D. Spehner, M. Orszag, *Geometric quantum discord with Bures distance: the qubit case*, J. Phys. A: Math. Theor. 47 (2014), 035302

- D. Spehner, M. Orszag, *Geometric quantum discord with Bures distance*, New J. of Phys. 15 (2013), 103001

Geometric measures of quantum correlations

Bipartite system AB with finite dim. Hilbert space $\mathcal{H}_{\mathsf{AB}} = \mathcal{H}_{\mathsf{A}} \otimes \mathcal{H}_{\mathsf{B}}$

The set of all (pure or mixed) states ρ of AB (nonnegative trace-one op. on $\mathcal{H}_{\mathsf{AB}}$) is equipped with a contractive Riemannian distance d.



(i) Geometric discord

$$D_A(\rho) = \min_{\sigma \in \mathcal{C}_{\mathsf{A}}} d(\rho,\sigma)^2$$

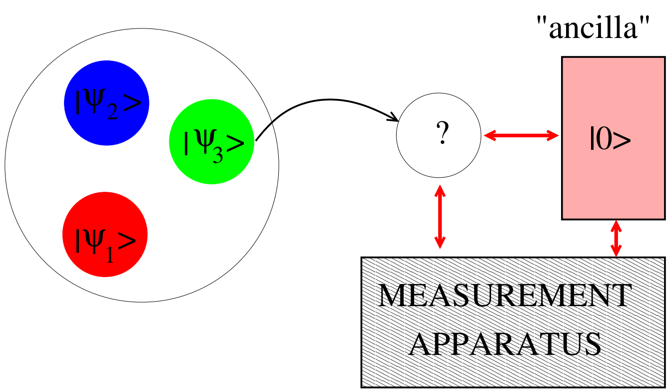
(ii) Geometric entanglement

$$E(\rho) = \min_{\sigma \in \mathcal{S}_{\mathsf{AB}}} d(\rho,\sigma)^2 \,.$$

(i) $\sigma \in \mathcal{C}_{\mathsf{A}}$ *iff* $\sigma = \sum_i p_i |\alpha_i\rangle\langle\alpha_i| \otimes \sigma_{\mathsf{B}|i}$
with $p_i \geqslant 0$, $\{|\alpha_i\rangle\}$ ONB of $\mathcal{H}_{\mathsf{A}}$, and $\rho_{\mathsf{B}|i}$ arbitrary states of B.
$\sigma \in \mathcal{C}_{\mathsf{A}}$ is called an A-**classical (or zero-discord) state**.

(ii) $\sigma \in \mathcal{S}_{\mathsf{AB}}$ *iff* $\sigma = \sum_i p_i |\Psi_i\rangle\langle\Psi_i|$ with $p_i \geqslant 0$ and $|\Psi_i\rangle = |\psi_i\rangle \otimes |\phi_i\rangle$
product states for all i $\longrightarrow$ $\mathcal{S}_{\mathsf{AB}}$ is the convex hull of $\mathcal{C}_{\mathsf{A}}$.
$\sigma \in \mathcal{S}_{\mathsf{AB}}$ is called **separable**, $\rho \notin \mathcal{S}_{\mathsf{AB}}$ is called **entangled**.

Quantum State Discrimination



- A receiver gets a state ρ_i randomly chosen with probability η_i among a known set of states $\{\rho_1, \cdots, \rho_n\}$.
- To determine the state he has in hands, he performs a measurement on it.

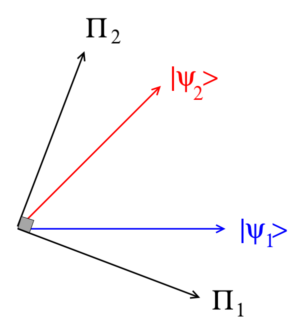
$\hookrightarrow$ **Applications :** quantum communication, cryptography,...

◇ If the ρ_i are $\perp$, one can discriminate them unambiguously.

◇ Otherwise one succeeds with probability

$$p_S = \sum_i \eta_i \,\mathrm{tr}(M_i \rho_i)$$

M_i = non-negative operators describing the generalized measurement, $\sum_i M_i = 1$.



Open pb (for $n > 2$): *find the optimal measurement $\{M_i^{\mathrm{opt}}\}$ and highest success probability p_S^{opt}*.

Link between the geometric discord and QSD

★ **Bures distance:** $d_B(\rho,\sigma) = \big(2 - 2\sqrt{F(\rho,\sigma)}\big)^{\frac{1}{2}}$ with the fidelity

$$F(\rho,\sigma) = \|\sqrt{\rho}\sqrt{\sigma}\|_1^2 = F(\sigma,\rho)$$ *[Bures '69, Uhlmann '76]*

Other contractive distances: trace or Kubo-Mori distances *[Petz '96]*

★ The geometric discord with Bures distance is given by solving a quantum state discrimination problem,

$$\boxed{D_A(\rho) = 2 - 2\max_{\{|\alpha_i\rangle\}}\sqrt{p_S^{\mathrm{opt}}(|\alpha_i\rangle)}}$$

$p_S^{\mathrm{opt}}(|\alpha_i\rangle) =$ optimal success proba. in discriminating the states

$$\boxed{\rho_i = \eta_i^{-1}\sqrt{\rho}\,|\alpha_i\rangle\langle\alpha_i|\otimes 1\,\sqrt{\rho}}$$

with proba $\eta_i = \langle\alpha_i|\,\mathrm{tr}_{\mathsf{B}}(\rho)|\alpha_i\rangle$, where $\{|\alpha_i\rangle\} =$ ONB of $\mathcal{H}_{\mathsf{A}}$.

Outline

- ✓ Introduction
- ✓ Model for an ideal quantum measurement
- ✓ Geometric discord with Bures distance
- Summary of other results

ARTICLES:

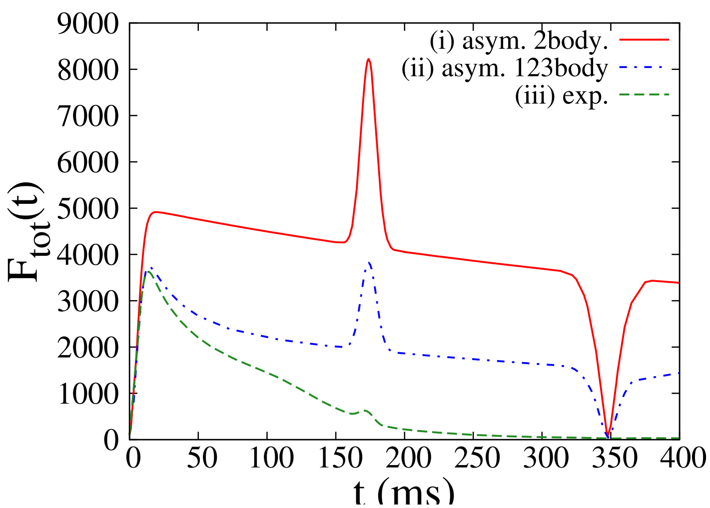
- D. Spehner, K. Pawlowski, G. Ferrini, A. Minguzzi, Eur. Phys. J. B 87, 157 (2014)
- K. Pawlowski, D. Spehner, A. Minguzzi, G. Ferrini, Phys. Rev. A 88, 013606 (2013)
- G. Ferrini, D. Spehner, A. Minguzzi, F.W.J. Hekking, Phys. Rev. A 84, 043628 (2011)
- G. Ferrini, D. Spehner, A. Minguzzi, F.W.J. Hekking, Phys. Rev. A 82, 033621 (2010)
- S. Vogelsberger, D. Spehner, Phys. Rev. A 82, 052327 (2010)

Decoherence in Bose-Josephson junctions

- Bose-Josephson junctions are formed by trapping ultracold bosonic atoms in two modes, e.g. BEC in two internal states.
- Due to interatomic interactions, the unitary dynamics generates superpositions of coherent states (Schrödinger cat states).

Pb: impact of decoherence due to atom losses on the superposition?

MAIN RESULTS:



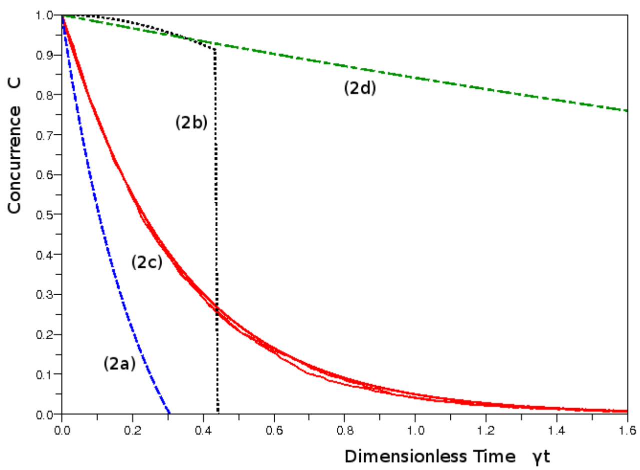
1. The amount of quantum correlations at time t depends strongly on the asymmetry between the loss rates and interaction energies in the two modes.
2. The cat states can be protected by tuning the interaction energies.

Entanglement evolution for quantum trajectories

- Two qubits initially entangled coupled to independent baths $\hookrightarrow$ decay of entanglement with time.
- Typically, entanglement in the density matrix disappears after a finite time. *[Diósi '03, Dodd, Halliwell '04, Yu, Eberly '04]*

Pb: what happens if one performs local continuous measurements on the baths?

MAIN RESULT:



(PhD of S. Vogelsberger)

Mean concurrence decays exponentially

$$\overline{C(\Psi(t))} = C_0 e^{-\kappa t}$$

with a rate κ depending on the measurement scheme only.