Quantum correlations, Schrödinger cat states, and decoherence

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Strangeness of the quantum word...

PART I

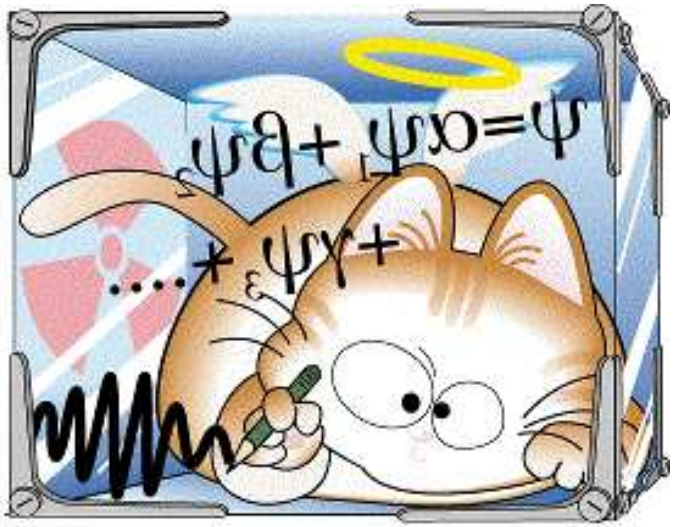
DECOHERENCE AND SCHRÖDINGER CAT STATES IN BOSE-JOSEPHSON JUNCTIONS

Schrödinger cat states

By the linearity principle of quantum mechanics, linear superpositions of macroscopically distinguishable states as

$$|\Psi_{\rm cat}\rangle = \frac{1}{\sqrt{2}}\big(|\uparrow\rangle|{\rm alive}\rangle + |\downarrow\rangle|{\rm dead}\rangle\big)$$

should exist in nature! *[Schrödinger '35]*



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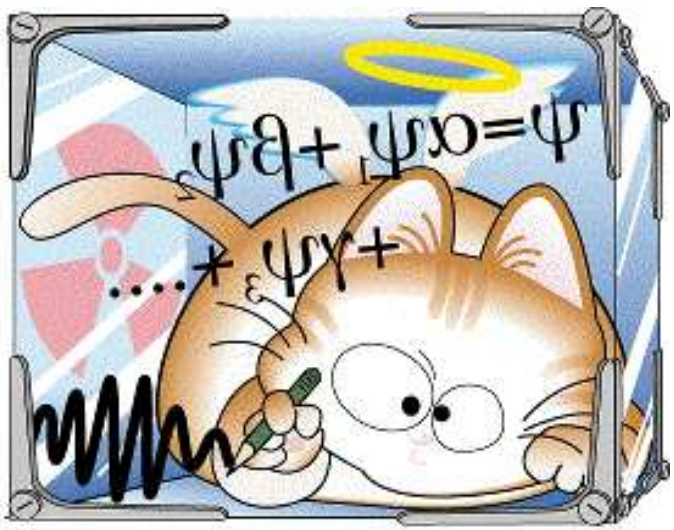

However, such "Schrödinger cat states" are very rapidly **transformed into statistical mixtures** by most system-environment couplings:

$$\rho_{\rm cat} = |\Psi_{\rm cat}\rangle\langle\Psi_{\rm cat}|$$

$$\longrightarrow \rho_{\rm mixture} = \tfrac{1}{2}\big(|\uparrow\rangle|{\rm alive}\rangle\langle\uparrow|\langle{\rm alive}| + |\downarrow\rangle|{\rm dead}\rangle\langle\downarrow|\langle{\rm dead}|\big)$$

Mesoscopic superpositions

Superpositions of states differing by 10-100 photons **have been observed in laboratories:**

- @ NIST

  [Monroe et al. Science 273 ('96)]

- @ LKB in Paris

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- @ Yale

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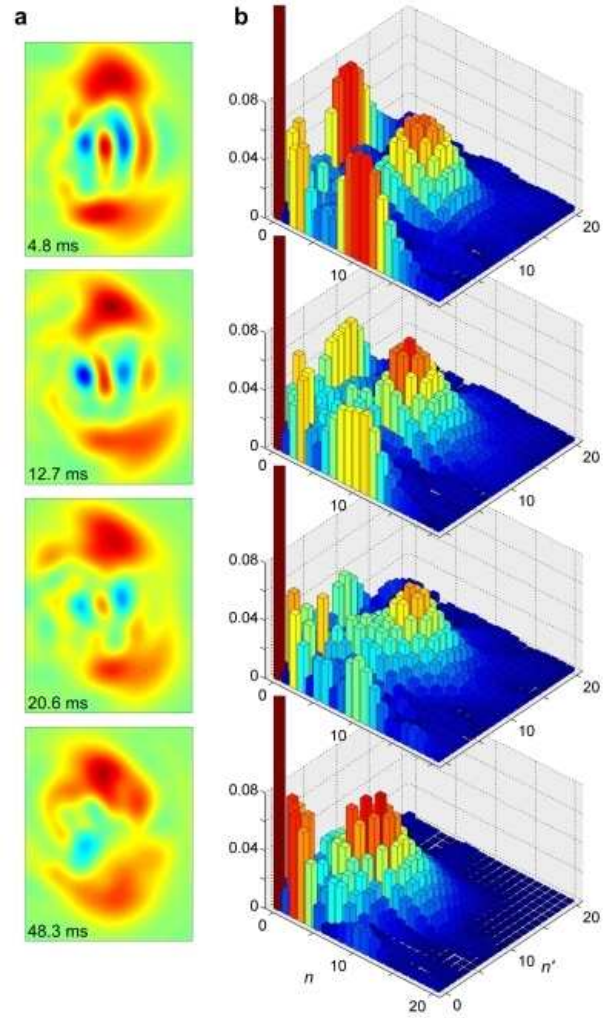

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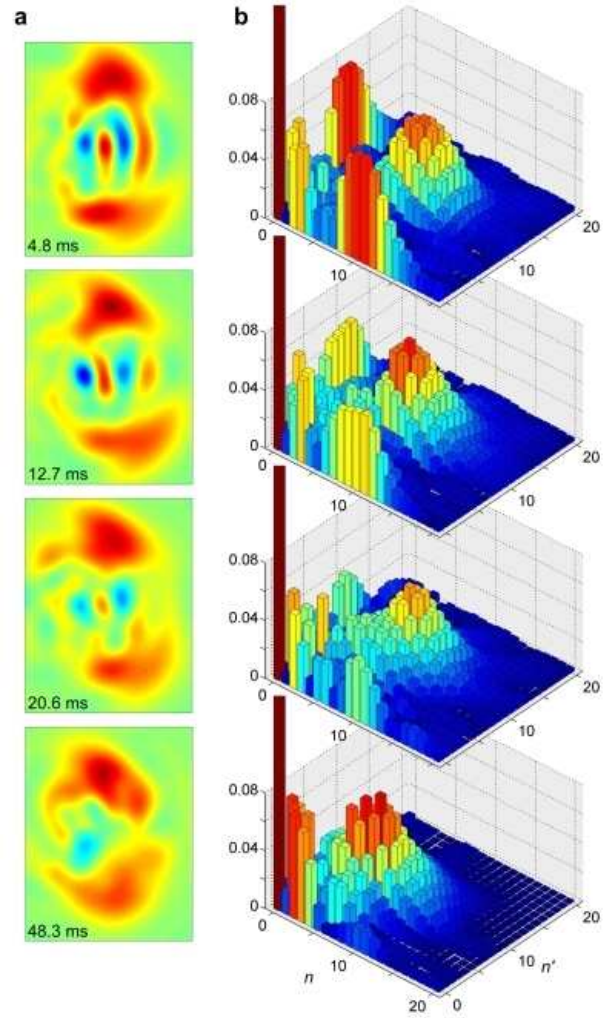
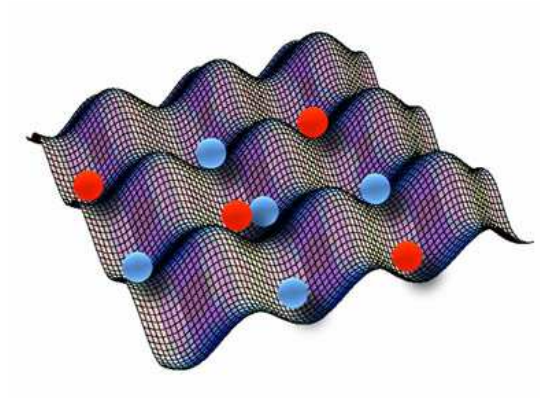


↪ **Challenge:** observe mesoscopic superpositions with matter waves in ultracold atomic gases.

Trapped ultracold atoms

→ **Tunable parameters :**

- strength and sign of interactions (via Feshbach resonances)
- trapping potential without or with disorder (speckle)
- artificial gauge field.



→ **Known sources of decoherence:** loss of atoms in the trap, experimental noises, ...

◇ **simulations** of ground state and dynamical properties of many-body systems, in particular from solid state physics

◇ **applications to** quantum information science: multipartite entanglement, high precision interferometry, flux qubits,...

Outlines of Part I

- Dynamics in Bose-Josephson junctions in the absence of tunelling
- Phase precision in atom interferometry
- Quantum Fisher information in the presence of decoherence
- Summary and perspectives of the results of part I

Joint work with:

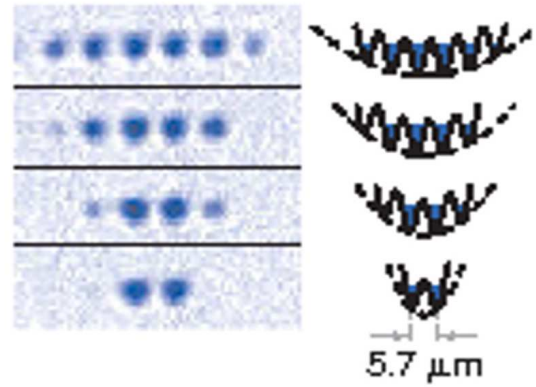
K. Pawlowski (CTP Warsaw),

A. Minguzzi (LPMMC, Univ. Grenoble Alpes)

G. Ferrini (Univ. Jussieu, Paris)

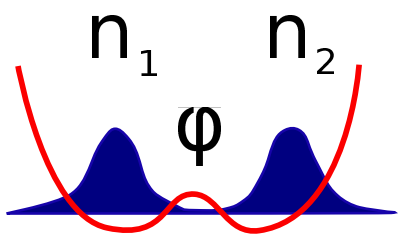
Bose-Josephson junctions

Bose-Einstein Condensate trapped in a double potential well



Fixed total # of atoms $N = n_1 + n_2$

$n_i = a_i^\dagger a_i$, $a_i^\dagger$ = creation operator of an atom in the well i



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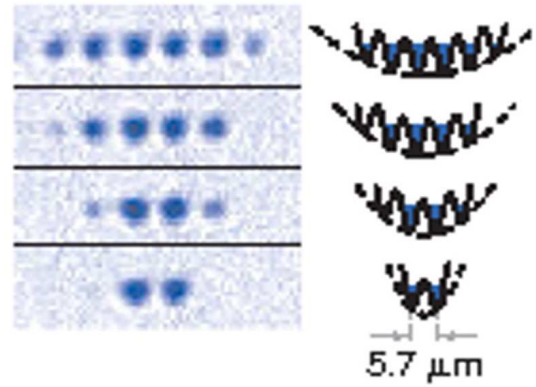

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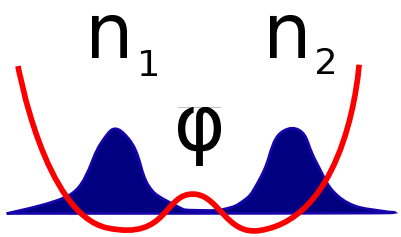

Two-mode Bose-Hubbard Hamiltonian

$$H_0 = \underbrace{E_1 n_1 + E_2 n_2}_{\text{one-atom site energies}} + \underbrace{K(a_1^\dagger a_2 + a_2^\dagger a_1)}_{\text{tunelling}} + \underbrace{\sum_{i=1,2} \frac{U_i}{2} n_i (n_i - 1)}_{\text{repulsive interactions}}$$

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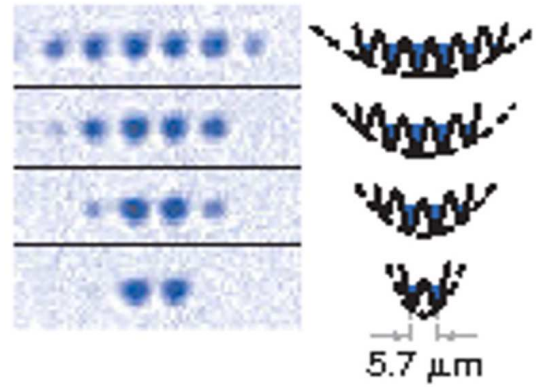

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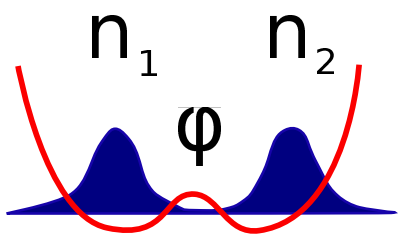

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Also: BEC trapped in a single well in two distincts hyperfine atomic states.

Bloch sphere representation

Schwinger transformation:

- ▷ $J_z = \frac{1}{2}\left(a_1^\dagger a_1 - a_2^\dagger a_2\right)$
- ▷ $J_x = \frac{1}{2}\left(a_1^\dagger a_2 + a_2^\dagger a_1\right)$
- ▷ $J_y = \frac{1}{2i}\left(a_1^\dagger a_2 - a_2^\dagger a_1\right)$

$[J_x, J_y] = 2iJ_z$, etc, $\vec{J}^2 = \frac{N}{2}(\frac{N}{2} + 1)$

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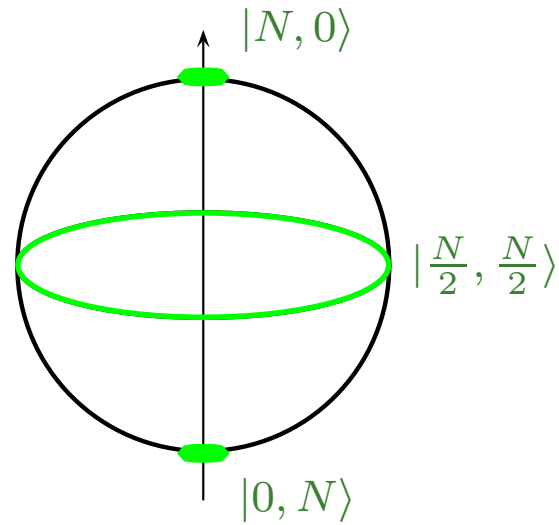

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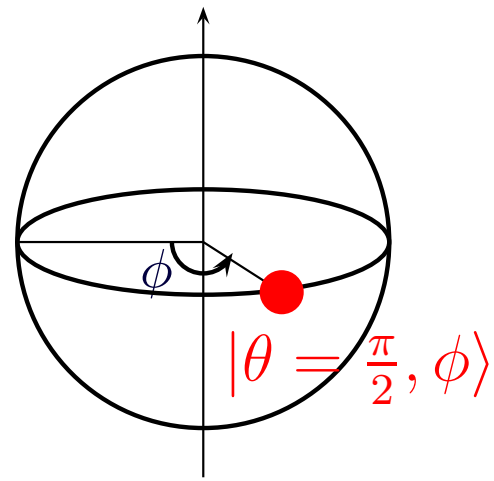

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$$\begin{aligned} |N;\theta,\phi\rangle &= \left(e^{-i\phi}\sin\tfrac{\theta}{2}\,a_1^\dagger + \cos\tfrac{\theta}{2}\,a_2^\dagger\right)^{\otimes N}|0,0\rangle \\ &= e^{-i\phi J_z}e^{-i\theta J_y}|0,N\rangle \end{aligned}$$

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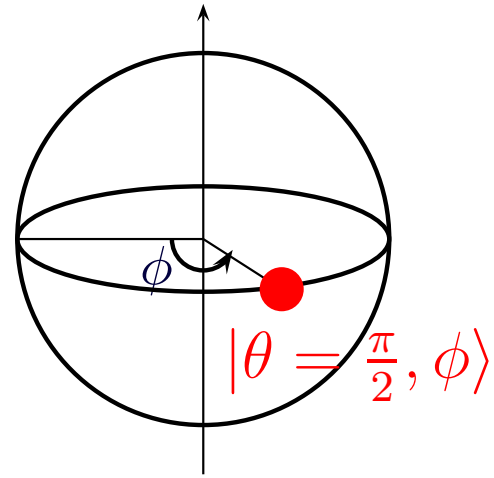

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Two-mode Bose-Hubbard Hamiltonian

$$H_0 = \lambda J_z + \underbrace{K J_x}_{\text{tunnel}} + \underbrace{\chi J_z^2}_{\text{inter.}} + c_N \quad , \quad \chi = \frac{U_1 + U_2 - 2U_{12}}{2}$$

Dynamics without tunelling

Initially, $|\psi(0)\rangle = |\theta = \frac{\pi}{2}, \phi = 0\rangle$ = ground state for $\chi \ll KN$

After a sudden quench to zero of the tunnel amplitude K, evolution under the Hamiltonian $H_0 = \chi J_z^2$

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$\longrightarrow$ observed experimentally

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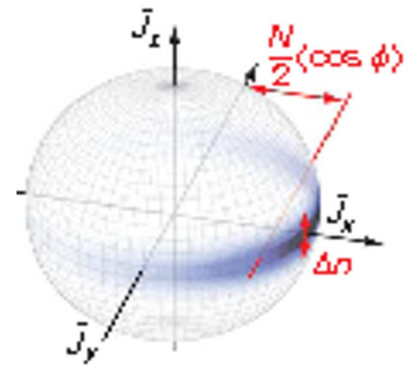

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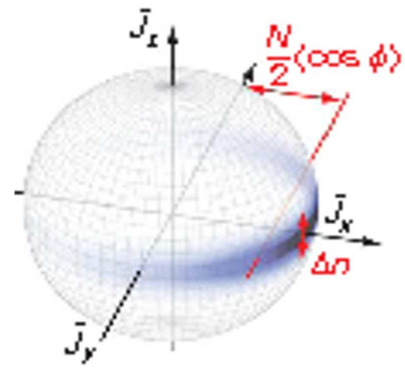

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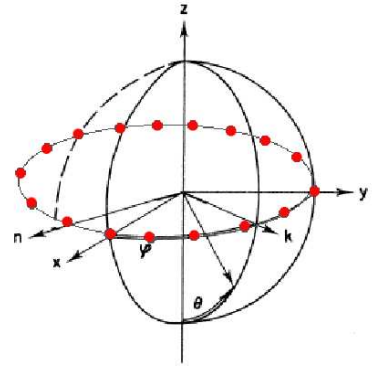
macroscopic superposition of coherent states *[Yurke & Stoler, PRL 57, '86]*



$$|\psi(t_q)\rangle = \frac{1}{\sqrt{q}} \sum_{k=0}^{q-1} c_k |N, \phi_k, \theta = \frac{\pi}{2}\rangle$$

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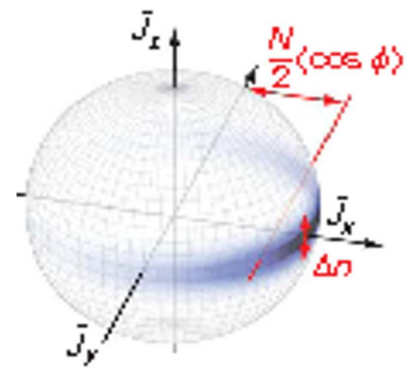

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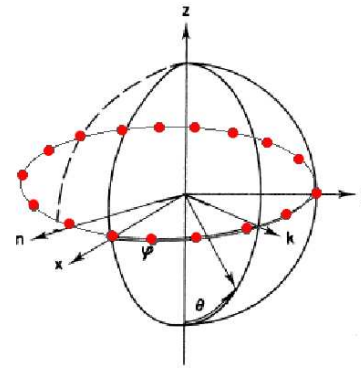

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◇ **At time** $t = T = 2\pi/\chi$: revival $|\psi(T)\rangle = |\psi(0)\rangle$.

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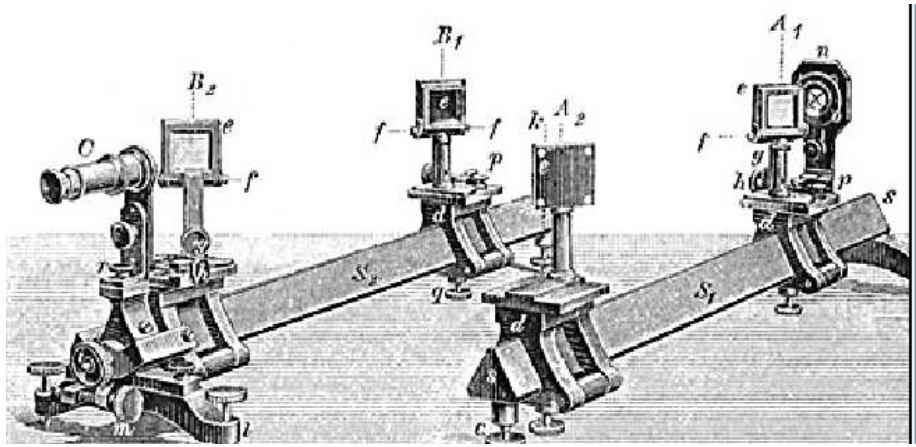
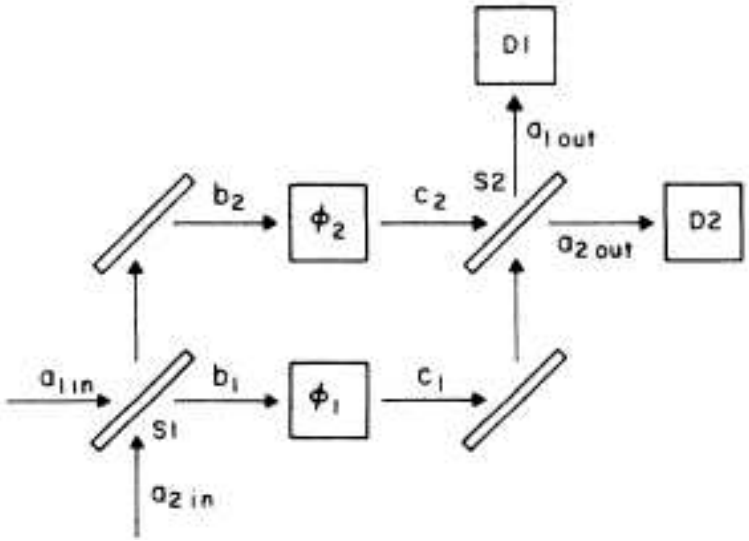
- ✓ Dynamics in Bose-Josephson junctions in the absence of tunelling
- Phase precision in atom interferometry

Mach-Zehnder interferometers



Goal: estimate an unknown phase shift

$$\phi = \phi_2 - \phi_1$$



Two (photon) **modes** $i = 1, 2$ (corresponding to the two arms of the interferometer)

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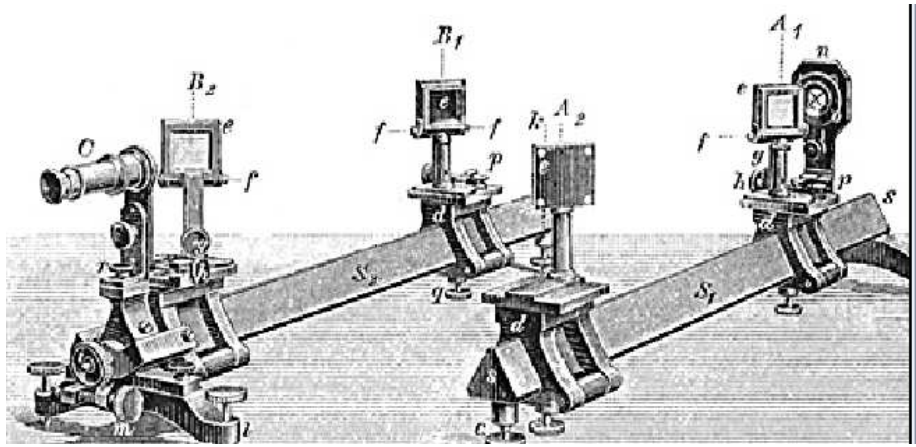

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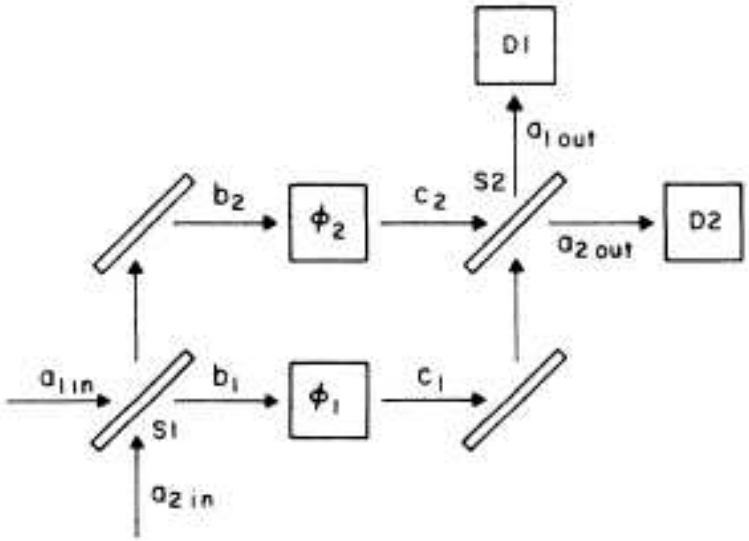

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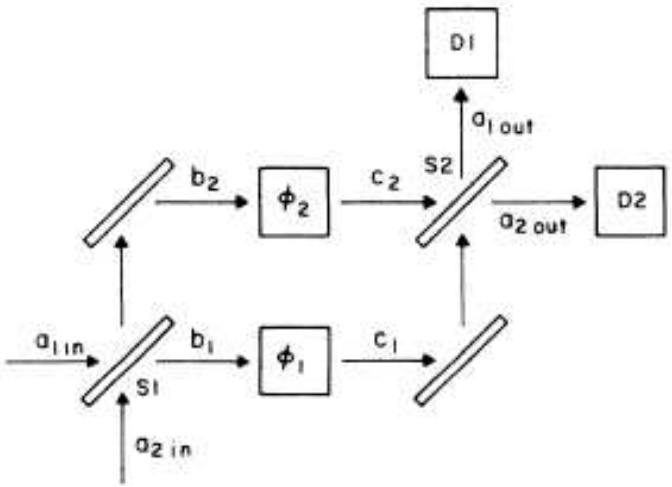
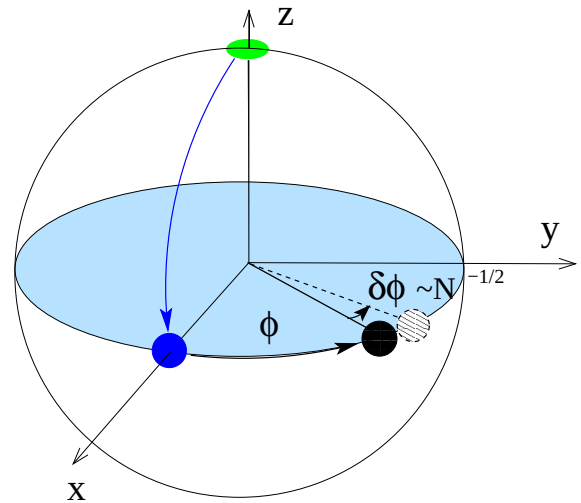
The phase precision $\delta\phi$ depends on the *input state*, the *type of measurement on the outcome state*, and the *statistical estimator* used to obtain ϕ from the meas. results

Rotations on the Bloch sphere





Schwinger transformation *[Yurke, McCall, Klauder PRA ('86)]*

- $\begin{cases} b_1 = 2^{-\frac{1}{2}}(a_{1\text{in}} - a_{2\text{in}}) \\ b_2 = 2^{-\frac{1}{2}}(a_{1\text{in}} + a_{2\text{in}}) \end{cases}$ 50% beam splitter $\Leftrightarrow \; |\psi\rangle \to e^{i\frac{\pi}{2}J_y}|\psi\rangle$
- $c_i = e^{-i\phi_i} b_i$, $i = 1, 2$ phase shifts $\Leftrightarrow \; |\psi\rangle \to e^{-i\phi J_z}|\psi\rangle$

Output state of the interferometer:

$$|\psi_{\text{out}}(\phi)\rangle = e^{-i\frac{\pi}{2}J_y} e^{-i\phi J_z} e^{i\frac{\pi}{2}J_y} |\psi_{\text{in}}\rangle = e^{-i\phi J_x}|\psi_{\text{in}}\rangle$$

Useful states for interferometry

Pb: *find the input states which give the highest phase sensitivity, i.e. the smallest error* $\delta\phi^2 = \left\langle \left(\frac{\phi_{\rm est}}{\partial\langle\phi_{\rm est}\rangle/\partial\phi} - \phi \right)^2 \right\rangle$

Optimizing over all statistical estimators $\phi_{\rm est}$ and all kinds of measurement on the output state, one finds

$$\delta\phi \geq (\delta\phi)_{\rm best} = \frac{1}{\sqrt{\mathcal{N}_m F_Q}}$$

Quantum Crámer-Rao bound

[Braunstein & Caves PRL ('94)]

F_Q = quantum Fisher information , $\mathcal{N}_m$ = # measurements

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(*Standard Quantum Limit*)

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Cat state $|\psi_{\rm in}\rangle = |\psi_{\rm cat}\rangle$

$$(\delta\phi)_{\rm best} = \frac{1}{\sqrt{\mathcal{N}_m}}\frac{1}{N}$$

(*Heisenberg limit*)

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= $4\langle(\Delta J_x)^2\rangle$ for pure states.

Coherent state $|\psi_{\rm in}\rangle = |\theta, \phi\rangle$

$$(\delta\phi)_{\rm best} = \frac{1}{\sqrt{\mathcal{N}_m}} \frac{1}{\sqrt{N}}$$

(*Standard Quantum Limit*)

Cat state $|\psi_{\rm in}\rangle = |\psi_{\rm cat}\rangle$

$$(\delta\phi)_{\rm best} = \frac{1}{\sqrt{\mathcal{N}_m}} \frac{1}{N}$$

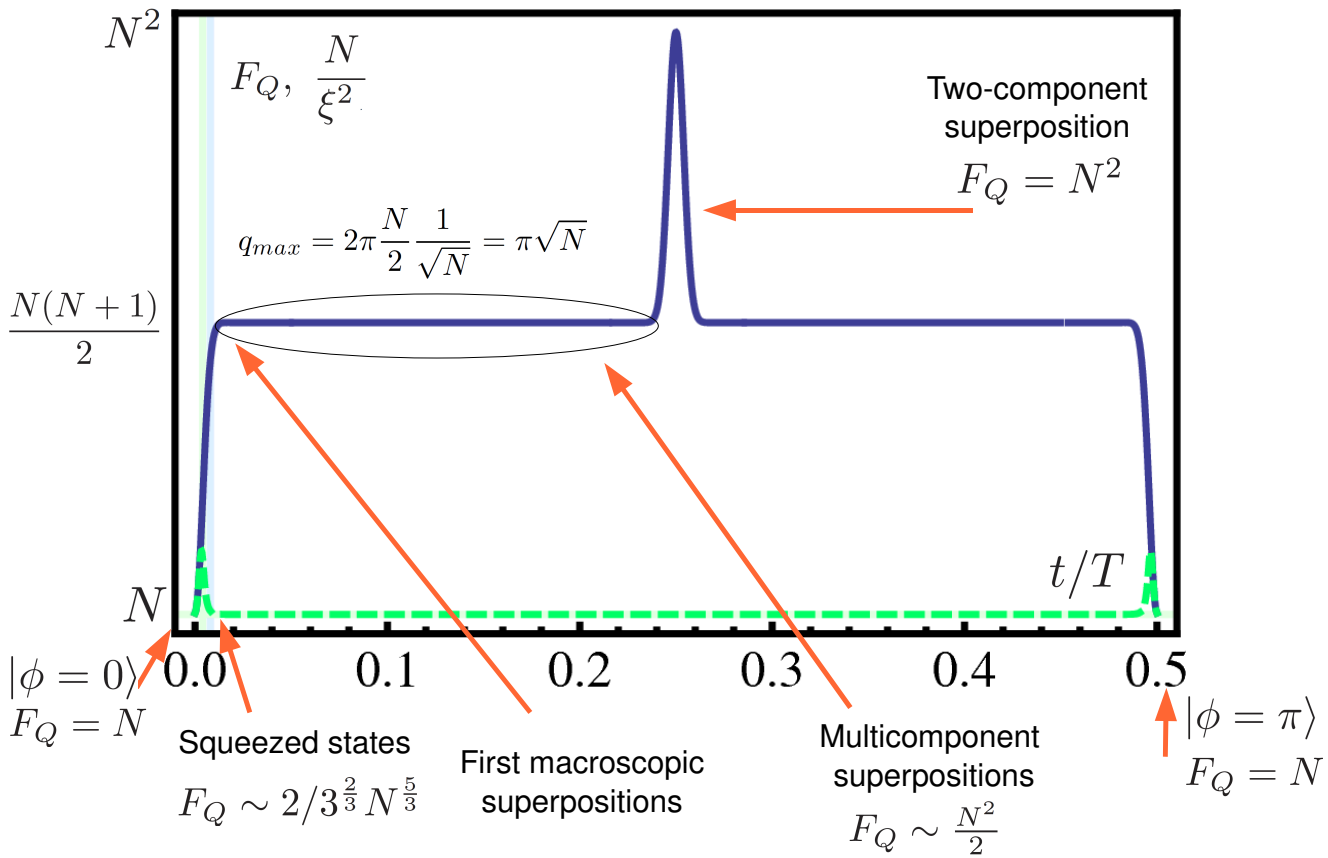
(*Heisenberg limit*)

$\hookrightarrow$ By using **highly entangled states**, one **increases the sensitivity** by a factor $\sqrt{N}$

[Giovannetti et al., PRL 96 ('06)]

Fisher information vs time in a lossless BJJ

Fisher information during the quenched dynamics



[Pezzé & Smerzi PRL ('09)], [Ferrini, Spehner, Minguzzi & Hekking ('11)]

Best phase sensitivity $(\delta\phi)_{\text{best}} = 1/\sqrt{\mathcal{N}_m F_Q}$

$(\delta\phi)_{\text{best}} < (\delta\phi)_{SQL}$ observed experimentally in the squeezing regime *[Gross et al., Nature 464 ('10)], [Riedel et al., Nature 464 ('10)]*

Outlines

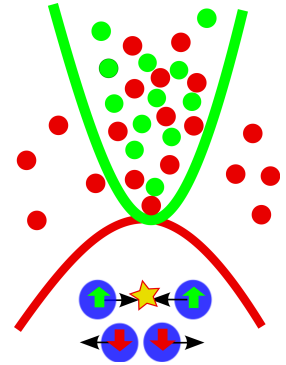
- ✓ Dynamics in Bose-Josephson junctions in the absence of tunelling
- ✓ Phase precision in atom interferometry
- • Quantum Fisher information in the presence of decoherence

Main sources of decoherence in BJJs

★ **Atom losses:**

→ collisions with background gas (1-body loss)

→ scattering of 2 or 3 atoms from the BEC (2 and 3-body losses)



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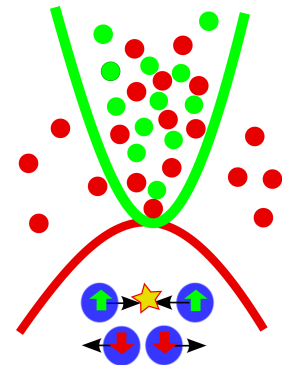

★ Random **fluctuations of the trapping potential** or of the magnetic field

→ fluctuations of the one-site energy ≠ (phase noise),

$$H(t) = \underbrace{\lambda(t) J_z}_{\text{random noise}} + \chi J_z^2$$

[Ferrini, Spehner, Minguzzi, Hekking, PRA 84 ('11)]
[Ferrini, Spehner, Minguzzi & Hekking PRA 82 ('10)]

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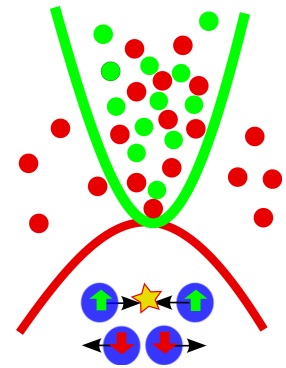

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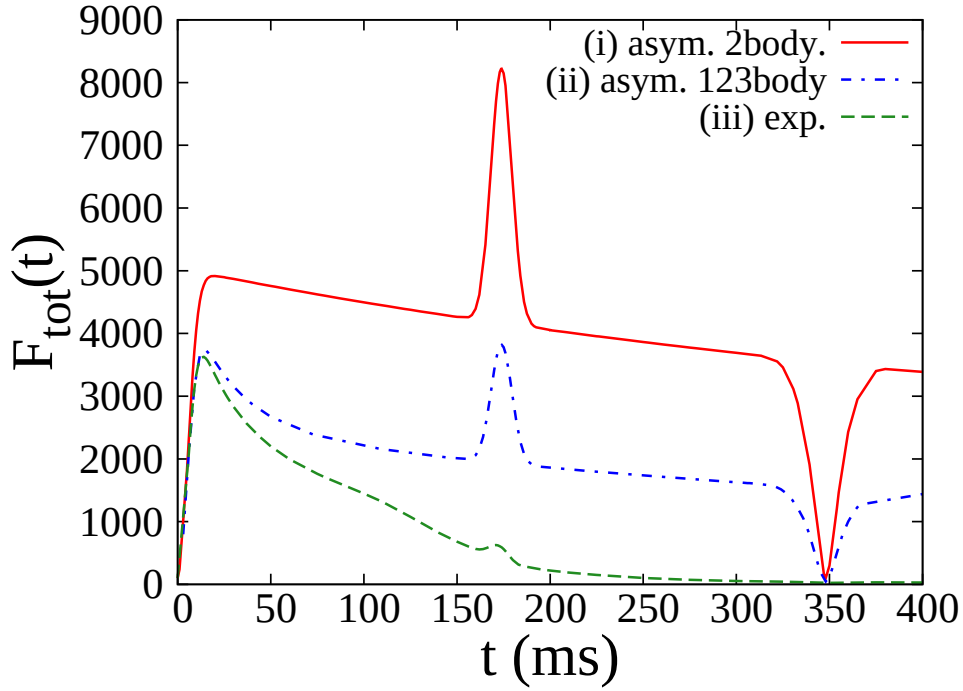
Question: do the cat states remain useful for interferometry in the presence of decoherence ?

Fisher information with atom losses



[Spehner, Pawlowski, Ferrini, Minguzzi, EPJ B 87 ('14)]

Initially $N = 100$ atoms

Tuning of the interaction energies so that:

$U_2 - U_{12} = 0$

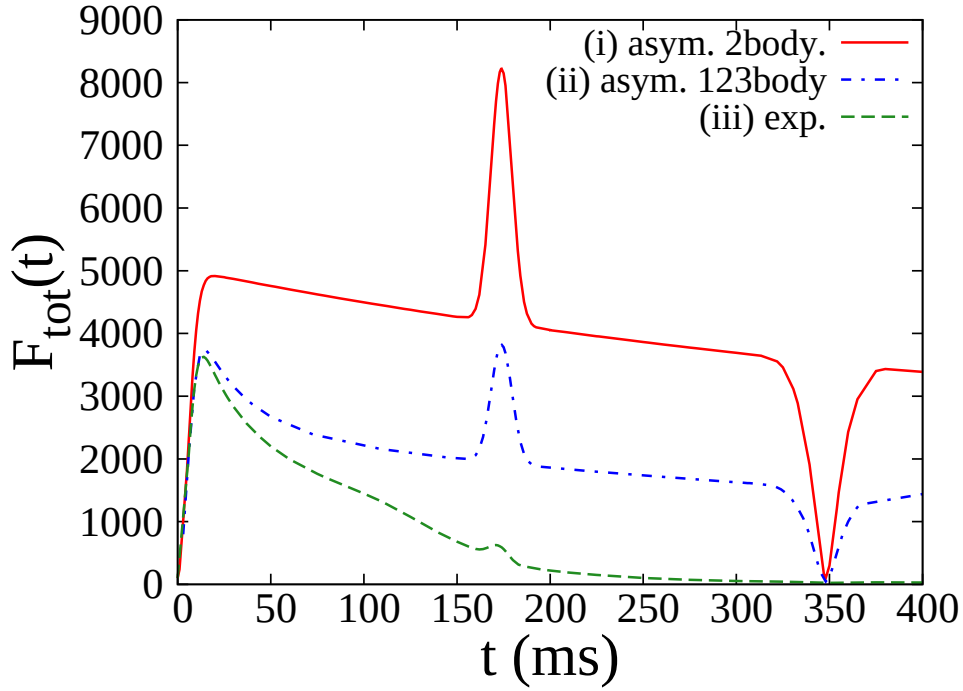
$U_1 - U_{12} = 18.056$ Hz

(i): *2-body losses in 2nd mode only*, $\Gamma_{2,0} = 0$ and $\Gamma_{0,2} = 0.0127$ Hz

(ii): *1-,2-, and 3-body losses in 2nd mode only*, $\Gamma_{0,1} = 0.4$ Hz, $\Gamma_{0,2} = 0.0127$ Hz, and $\Gamma_{0,3} = 1.08 \times 10^{-6}$ Hz

(iii): *asymmetric 2-body & symmetric 1- and 3-body losses*, $\Gamma_{0,1} = \Gamma_{1,0} = 0.2$ Hz, $\Gamma_{2,0} = 0$, $\Gamma_{0,2} = 0.0127$ Hz, and $\Gamma_{0,3} = \Gamma_{3,0} = 0.54 \times 10^{-6}$ Hz

Fisher information with atom losses



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$\hookrightarrow$ The impact of atom losses on quantum correlations useful for interferometry **depends strongly on the asymmetry between the loss rates** in the two modes

Outlines of Part I

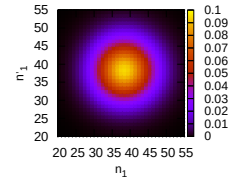
- ✓ Dynamics in Bose-Josephson junctions in the absence of tunelling
- ✓ Phase precision in atom interferometry
- ✓ Quantum Fisher information in the presence of decoherence
- • Summary and perspectives of the results of part I

Summary of the results

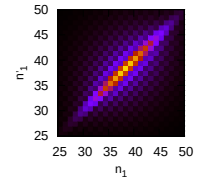
[Spehner, Pawlowski, Ferrini, Minguzzi, EPJ B 87 ('14)]
[Pawlowski, Spehner, Minguzzi, Ferrini, PRA 88 ('13)]

✓ The **conditional state** given **a single 2-body loss event** between $t = 0$ at the cat-formation time t_q is, for "weak" losses:

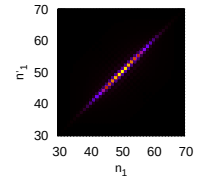
↗ almost a perfect cat state with $N - 2$ atoms if no loss in 1st mode and $U_2 = U_{12}$



→ "half" a cat state for equal loss rates in the two modes and $U_2 = U_{12} \neq U_1$



↘ completely decohered for equal loss rates in the two modes and $U_1 = U_2 \neq U_{12}$



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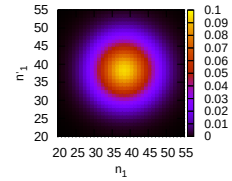

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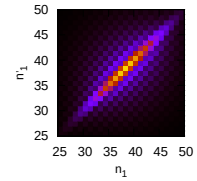

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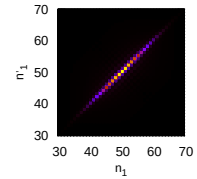


✓ For asymmetric loss rates, even if 20% of atoms are lost in the lossy mode, one still has quantum correlations useful for interferometry by tuning the energy $U_2 = U_{12}$ while keeping constant the effective interaction $\chi = (U_1 + U_2 - 2U_{12})/2$.

Perspectives

$\rightsquigarrow$ Find optimal **experimental conditions** to observe a cat state in a BJJ *(project in collaboration with the experimental group of P. Treutlein in Basel).*

- $\diamond$ Tuning of interaction energies U_i by Fesbach resonances is not possible
- $\diamond$ The atoms in the two hyperfine states see different trappping potentials

★ 2-body losses are negligible in the lower state

★ By reducing the # atoms in the upper state to $\simeq 10$, 2-and 3-body atom losses can be strongly reduced in this upper internal state.

Design time-dependent trapping protentials s.t. $U_2 \simeq U_{12}$
$\Rightarrow$ suppress the effect of 1-body losses in the upper state.

PART II

GEOMETRIC APPROACH TO QUANTUM CORRELATIONS

Outlines of Part II

- Quantum discord
- Geometric discord with Bures distance
- Geometric discord with Hellinger distance
- Summary and perspectives of the results of part II

Joint work with:

M.Orszag (PUC Santiago),

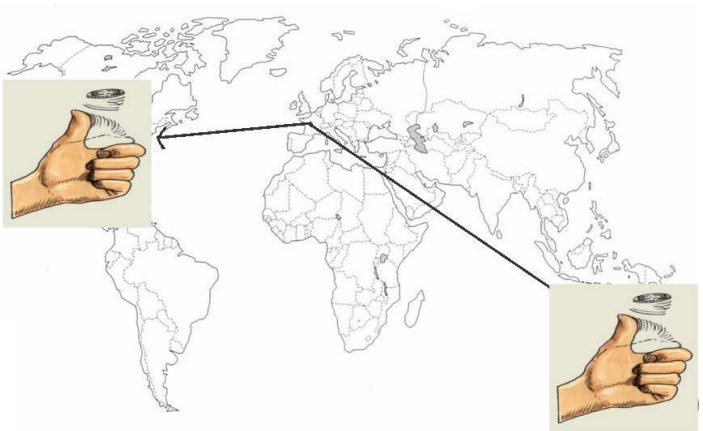
F. Illuminati (Univ. degli Study di Salerno, Italy)

W. Roga (Univ. of Strathclydeb, Glasgow, UK)

Quantum vs classical correlations

◇ **Central question in Quantum Information theory:** identify (and try to protect) the Quantum Correlations responsible for the efficiency of quantum algorithms.

classical correlations



quantum correlations

◇ For mixed states, two (at least) kinds of QCs

↗ **entanglement** *[Schrödinger ('36)]*

↘ **nonclassicality** (quantum discord) *[Ollivier & Zurek ('01)]* *[Henderson & Vedral ('01)]*

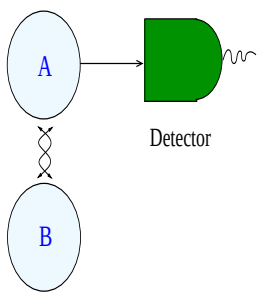
Quantum discord

- **Total correlations** between two parties A and B:

$$I_{A:B}(\rho) = S(\rho_A) + S(\rho_B) - S(\rho) \text{ mutual information}$$

$S(\rho)$ = von Neumann entropy of ρ.

- **Quantum discord** = *mutual information not accessible by local measurements on subsystem A*



$$\mathcal{D}(\rho) = \min_{\{\Pi_i^A\}} \left\{ I_{A:B}(\rho) - I_{A:B}\Big(\sum_i \Pi_i^A \otimes 1 \, \rho \, \Pi_i^A \otimes 1 \Big) \right\}$$

Π_i^A = rank-one orthogonal projectors for A

[Ollivier & Zurek PRL ('01)]

[Henderson & Vedral JPA ('01)]

- The A**-classical** (=classical-quantum) states are

$$\sigma_{A\text{-cl}} = \sum_i q_i |\alpha_i\rangle\langle\alpha_i| \otimes \rho_{B|i} \quad \Leftrightarrow \quad \mathcal{D}(\sigma_{A\text{-cl}}) = 0$$

with $\{|\alpha_i\rangle\}$ = orthonormal basis for A.

Quantum discord as a resource

- Certain mixed separable states are neither A- nor B-classical, and thus have QCs different from entanglement.

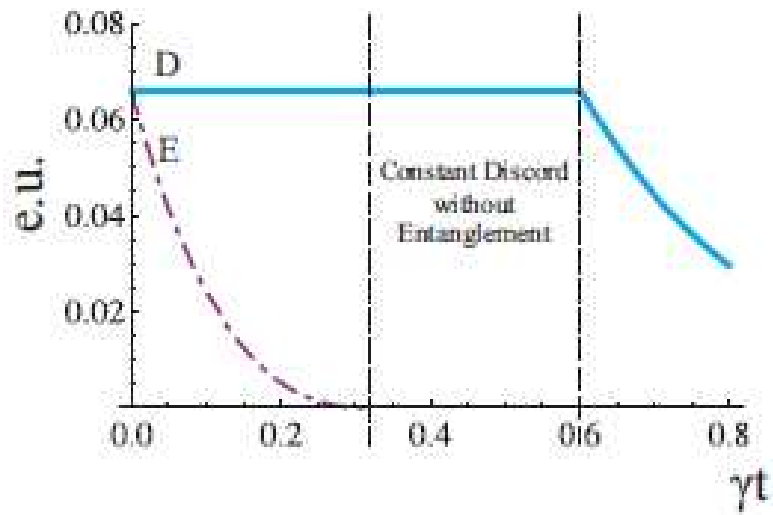
  $\hookrightarrow$ *such states may be useful as resources for quantum computation or quantum communication* (e.g. for the Knill & Laflamme DQC1 algorithm)

  [Datta, Shaji & Caves PRL 100 ('08)]

  $\hookrightarrow$ *they are presumably less fragile than entangled states.*

Time evolution of the entanglement and discord for 2 qubits subject to pure dephasing

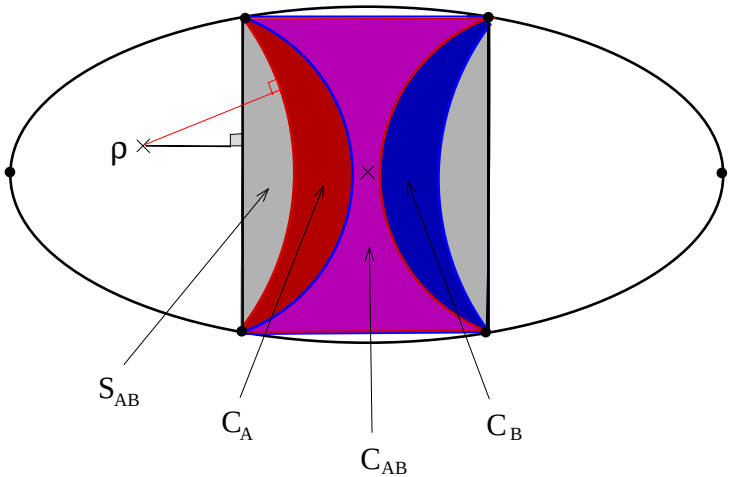
[Mazzola, Piilo & Maniscalco PRL ('10)]



Outlines of Part II

- ✓ Quantum discord
- Geometric discord with Bures distance

Geometric view of QCs



$C_A = \{A\text{-classical states}\}$,

$C_B = \{B\text{-classical states}\}$

$\subset S_{AB} = \{\text{separable states}\}$

Geometric discord: *[Dakic, Vedral & Bruckner PRL 105 ('01)]*

$$\mathcal{D}_G(\rho) = \min_{\sigma_{A\text{-cl}} \in C_A} d(\rho, \sigma_{A\text{-cl}})^2$$

with d = distance on the set of quantum states.

Similarly, **geometric measure of entanglement**

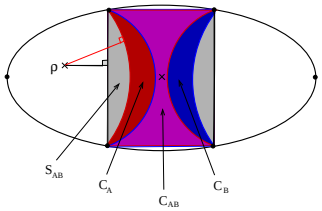
[Vedral et al PRL 78 ('97)] [Vedral & Plenio PRA 57 ('98)]

$$E_G(\rho) = \min_{\sigma_{\text{sep}} \in S_{AB}} d(\rho, \sigma_{\text{sep}})^2$$

Advantages of the geometric approach

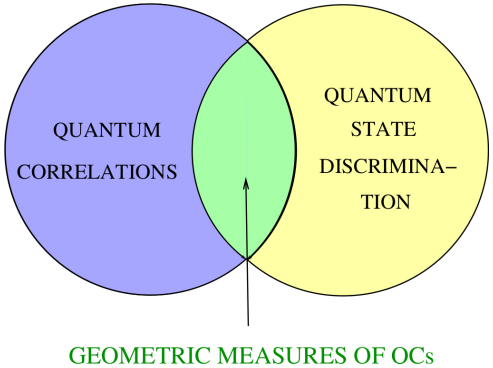
↪ 

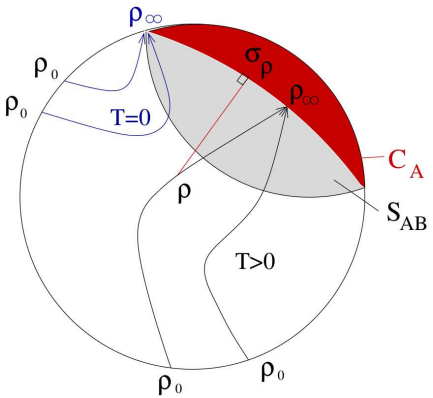
$\mathcal{D}_G$ is typically easier to compute than the entropic discord $\mathcal{D}$.

↪ 

Geometric entanglement is always smaller or equal to the geometric discord, $E_G(\rho) \leq \mathcal{D}_G(\rho)$ (not true for $\mathcal{D}$ and E_{EoF}).

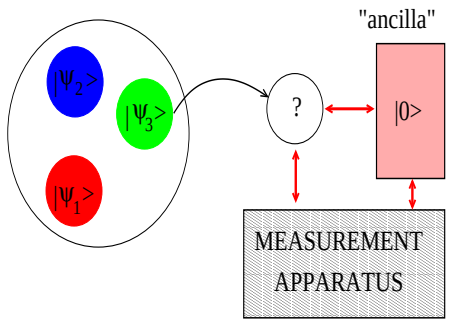
↪ Operational interpretation of $\mathcal{D}_G$ related to the distinguishability of quantum states



↪ 

Useful geometrical information on ρ given by the closest A-classical state(s) σ_ρ to ρ.

Quantum State Discrimination (QSD)



- A receiver gets a state ρ_i randomly chosen with probability η_i among a known set of states $\{\rho_1, \cdots, \rho_n\}$.
- To determine the state he has in hands, he performs a measurement on it.

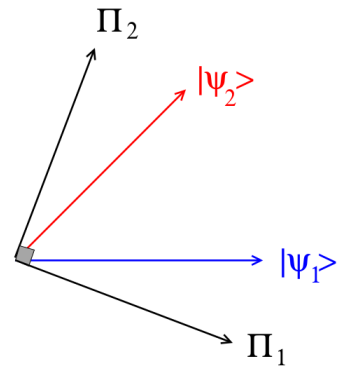
$\hookrightarrow$ **Applications :** quantum communication, cryptography,...

◇ If the ρ_i are $\perp$, one can discriminate them unambiguously.

◇ Otherwise one succeeds with probability

$$p_S = \sum_i \eta_i \, \mathrm{tr}(M_i \rho_i)$$

M_i = non-negative operators describing the generalized measurement, $\sum_i M_i = 1$.



Open pb (for $n > 2$): *find the optimal measurement $\{M_i^{\mathrm{opt}}\}$ and highest success probability p_S^{opt}.*

Link between the geometric discord & QSD

★ **Bures distance:** $d_B(\rho,\sigma) = \left(2 - 2\sqrt{F(\rho,\sigma)}\right)^{\frac{1}{2}}$ with the fidelity $F(\rho,\sigma) = \|\sqrt{\rho}\sqrt{\sigma}\|_1^2$ *[Bures ('69), Uhlmann ('76)]*

If $\rho(t) = e^{-iHt}\rho e^{iHt}$, then the infinitesimal distance $ds^2 = d_B(\rho,\rho+d\rho)^2 \;\propto\; F_Q(\rho)$ **Quantum Fisher information.**

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★ The geometric discord with Bures distance is given by solving a QSD problem, *[Spehner & Orszag NJP 15 ('14)]*

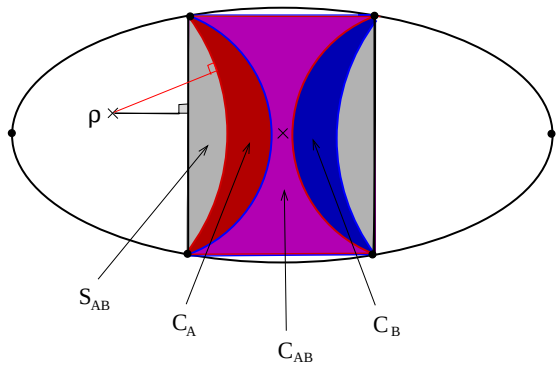
$$\mathcal{D}_A(\rho) = 2 - 2\max_{\{|\alpha_i\rangle\}} \sqrt{p_S^{\rm opt}(|\alpha_i\rangle)}$$

$p_S^{\rm opt}(|\alpha_i\rangle) =$ optimal success probability in discriminating the states ρ_i with proba η_i, with

$$\rho_i = \eta_i^{-1}\sqrt{\rho}\,|\alpha_i\rangle\langle\alpha_i| \otimes 1\,\sqrt{\rho}\;,\;\; \eta_i = \langle\alpha_i|\,\mathrm{tr}_B(\rho)|\alpha_i\rangle$$

where $\{|\alpha_i\rangle\} =$ orthonormal basis for subsystem A.

Closest A-classical states & QSD



The closest A-classical state(s) σ_ρ to ρ is (are) given in terms of the optimal von Neumann measurement for discriminating the states ρ_i,

[Spehner & Orszag NJP 15 ('13)]

$$\rho_{A\text{-cl}} \propto \sum_i |\alpha_i^{\rm opt}\rangle\langle\alpha_i^{\rm opt}| \otimes \langle\alpha_i^{\rm opt}|\sqrt{\rho}\,\Pi_i^{\rm opt}\sqrt{\rho}\,|\alpha_i^{\rm opt}\rangle$$

$\{\Pi_i^{\rm opt}\}$ = optimal von Neumann measurement for discriminating the states ρ_i (orthogonal projectors with rank $\dim \mathcal{H}_B$)

$\{|\alpha_i^{\rm opt}\rangle\}$ = orthonormal basis for A maximizing $p_S^{\rm opt}$.

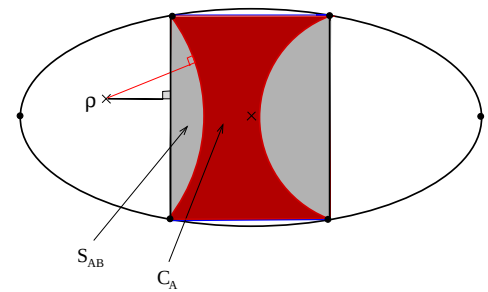
Hellinger geometric discord

[Roga, Spehner & Illuminati ('15)]

◇ **Hellinger distance:** $d_{\mathrm{Hel}}(\rho,\sigma) = \left(2 - 2\,\mathrm{tr}\,\sqrt{\rho}\sqrt{\sigma}\right)^{1/2}$

◇ The geometric discord for the Hellinger distance is a reliable measure of QCs like in the Bures case. For pure states,

$$\mathcal{D}_G^{\mathrm{Hel}}(|\Psi\rangle) = 2 - 2K^{-1/2}$$



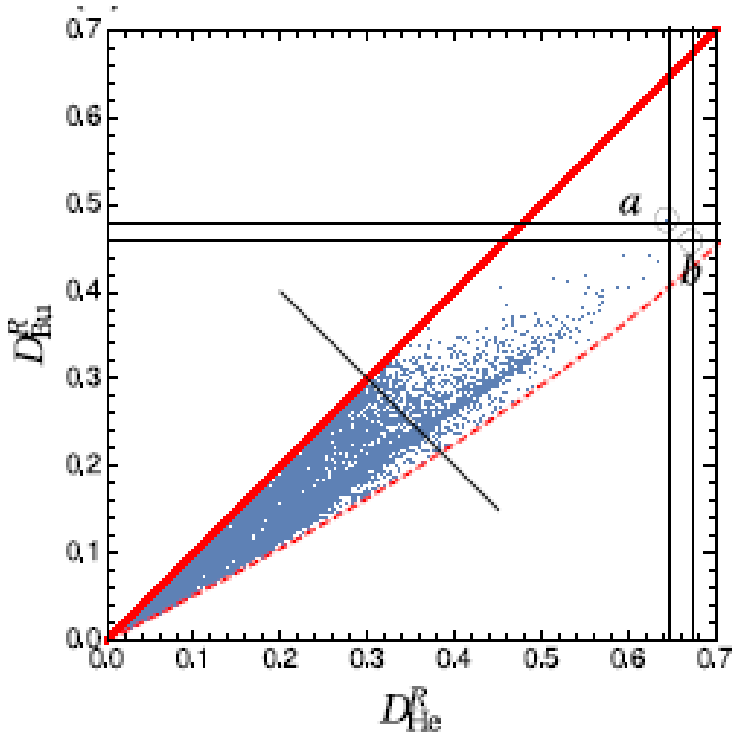
$K = (\sum \mu_i^2)^{-1}$ Schmidt number.

◇ $\mathcal{D}_G^{\mathrm{Hel}}(\rho)$ is simply related to the Hilbert-Schmidt geometric discord of the square root of ρ,

$$\mathcal{D}_G^{\mathrm{Hel}}(\rho) = 2 - 2\big(1 - \mathcal{D}_G^{\mathrm{HS}}(\sqrt{\rho})\big)^{1/2}$$

$\mathcal{D}_G^{\mathrm{Hel}}$ is (almost) as easy to compute as the Hilbert-Schmidt discord $\mathcal{D}_G^{\mathrm{HS}}$, but unlike $\mathcal{D}_G^{\mathrm{HS}}$ it is a proper measure of QCs!

Comparison of the two discords



Distribution of the discords of response with Hellinger and Bures distances for randomly generated two-qubit states

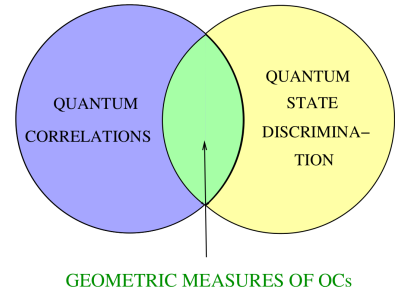
$\hookrightarrow$ The 2 measures of QCs lead to **different orders** on the set of quantum states.

Outlines of Part II

- ✓ Quantum discord
- ✓ Geometric discord
- • Summary of the results of part II

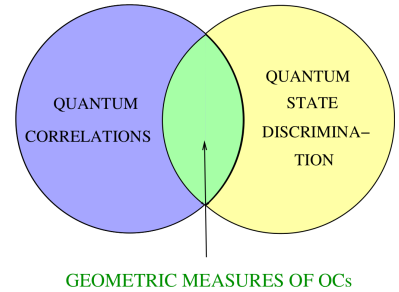
Summary & perspectives of part II

$\rightsquigarrow$ The geometric discord for the Bures distance is related to a quantum state discrimination task.

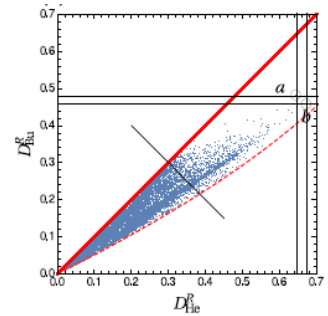


Summary & perspectives of part II

- ⇝ The geometric discord for the Bures distance is related to a quantum state discrimination task.



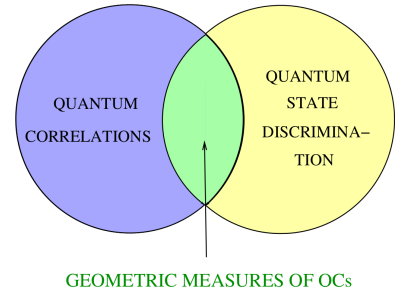
- ⇝ The geometric discord for the Hellinger distance provides the 1st instance of a *fully computable* and *physically reliable* measure of QCs.



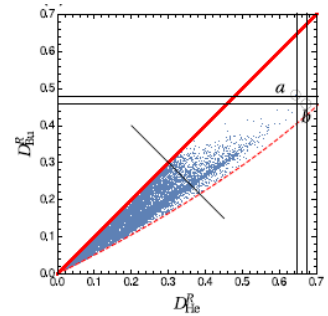
- ⇝ These 2 discords are not equivalent but general bounds between them exist.

Summary & perspectives of part II

- ⇝ The geometric discord for the Bures distance is related to a quantum state discrimination task.



- ⇝ The geometric discord for the Hellinger distance provides the 1st instance of a *fully computable* and *physically reliable* measure of QCs.



- ⇝ These 2 discords are not equivalent but general bounds between them exist.

PERPECTIVES

★ Evolution of QCs and closest classical states in concrete dissipative or dephasing channels.

★ Closest classical states of correlated ground states near a quantum phase transition.

THAT'S ALL

THANK YOU FOR YOU ATTENTION!

Acknowledgments to :

- A. Minguzzi (Univ. Grenoble Alpes, France)
- K. Pawlowski (CTP Warsaw, Poland)
- G. Ferrini (Univ. Jussieu, Paris, France)
- M. Orszag (PUC Santiago, Chile)
- F. Illuminati (Salerno, Italy)
- W. Roga (Glasgow, UK)

Complementary material (Part I)

EFFECTIVE PHASE NOISE DUE TO RANDOM LOSS TIMES

Quantum trajectories

- The BEC wavefunction is transformed by an atom loss as

$$|\psi(t)\rangle \longrightarrow \frac{M_m|\psi(t)\rangle}{\|M_m|\psi(t)\rangle\|}, \;\; M_m \text{ = jump operator}$$

  e.g. $M_m = a_1^2$, a_2^2 or $a_1 a_2$ for two-body losses.

- A loss event occurs in the time interval $[t, t+dt]$ with proba

$$dp_m = \Gamma_m \|M_m|\psi(t)\rangle\|^2 dt, \;\; \Gamma_m \text{ = jump rate}$$

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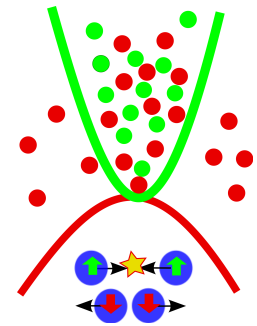
- Unormalized wavefunction at time t after J loss events at consecutive times $s_1, s_2, \cdots s_J$

$$\begin{aligned} |\widetilde{\psi}(t)\rangle &= e^{-i(t-s_J)H_{\text{eff}}} M_{m_J} \cdots \\ &\quad \cdots e^{-i(s_2-s_1)H_{\text{eff}}} M_{m_1} e^{-is_1 H_{\text{eff}}} |\psi(0)\rangle \end{aligned}$$



$H_{\text{eff}} = H_0 - i \sum_m \Gamma_m M_m^\dagger M_m / 2$ non self-adjoint Hamiltonian

[Dalibard, Castin & Møllmer ('92), Carmichael ('92),...]

Master equation

Density matrix of the BEC = average of $|\psi(t)\rangle\langle\psi(t)|$ over the random loss times s_j and types m_j and # loss events J,

$$\rho(t) = \overline{|\psi(t)\rangle\langle\psi(t)|}$$

$\Rightarrow \rho(t)$ satisfies the Lindblad master equation

[Anglin PRL 78 ('97), Jack PRL 89 ('02)]

$$\frac{d\rho}{dt} = -i[H_0, \rho] + \sum_m \Gamma_m \Big(M_m \rho M_m^\dagger - \frac{1}{2}\{M_m^\dagger M_m \, , \, \rho\} \Big)$$

with the jump operators $M_m = a_1^{m_1} a_2^{m_2}$

$$m = \begin{cases} \text{(1,0), (0,1)} & \text{for 1-body losses} \\ \text{(2,0),(1,1),(0,2)} & \text{for 2-body losses} \\ \text{(3,0),(2,1),(1,2),(0,3)} & \text{for 3-body losses} \end{cases}$$

Interplay between losses & interactions

- ★ To simplify formulas, we restrict ourselves to 2-body losses (1- and 3-body losses are similar).
- ★ A single loss event at time s transforms a coherent state

$$|N,\phi,\theta\rangle = \sum_{n_1+n_2=N} \binom{N}{n_1}^{1/2} \frac{(\tan(\theta/2))^{n_1} e^{-in_1\phi}}{[1+\tan^2(\theta/2)]^{N/2}} |n_1,n_2\rangle,$$

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$$\left|N,0,\frac{\pi}{2}\right\rangle \to e^{-itH_{\rm eff}}\left|N-2,\phi_1(s)=2(U_1-U_{12})s\,,\,\theta_1(s)\right\rangle$$

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Reason: dynamical phases accumulated by Fock states $|n_1,n_2\rangle$, e.g. for a loss of 2 atoms in mode 1:

$$\underbrace{sE(n_1,n_2)}_{\text{evol. in } [0,s]} + \underbrace{(t-s)E(n_1-2,n_2)}_{\text{evol. in } [s,t]} = tE(n_1-2,n_2) + \phi_1 n_1 + c$$

Conditional state after a single loss event

[Spehner, Pawlowski, Ferrini, Minguzzi, EPJ B 87 ('14)]
[Pawlowski, Spehner, Minguzzi, Ferrini, PRA 88 ('13)]

The wavefunction after a single 2-body loss event in channel m is

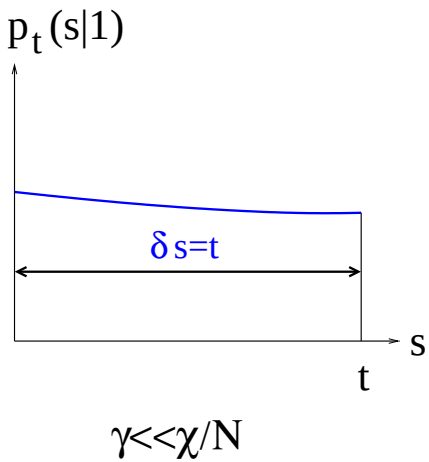
$$|\psi_1(t)\rangle \propto e^{-itH_{\rm eff}}|N-2, \phi_m(s), \theta_m(s)\rangle$$

with **phases depending on the random jump time** s,

$$\phi_m(s) = \begin{cases} 2s(U_1 - U_{12}) \\ -2s(U_2 - U_{12}) \end{cases} \qquad \tan(\tfrac{\theta_m(s)}{2}) = \begin{cases} \exp\{-s(2\Gamma_{2,0} - \Gamma_{1,1})\} & \text{if } m = (2,0) \\ \exp\{s(2\Gamma_{0,2} - \Gamma_{1,1})\} & \text{if } m = (0,2). \end{cases}$$

$\hookrightarrow$ Atom losses & interactions lead to an **effective phase noise** *see also [Sinatra, Dornstetter, Castin, Front Phys. 7 ('12)]*

Small loss rates regime



When $\Gamma_m \ll \chi/N$, at the time $t_q = \frac{\pi}{\chi q}$ of formation of the q-component cat state, the random phases have fluctuations

$$\delta\theta_m \ll 1/\sqrt{N}, \;\; \delta\phi_{0,2} + \delta\phi_{2,0} = \frac{4\pi}{q}$$

$$\delta\phi_{2,0} = \frac{2(U_2 - U_{12})}{U_1 + U_2 - 2U_{12}} \frac{2\pi}{q}$$

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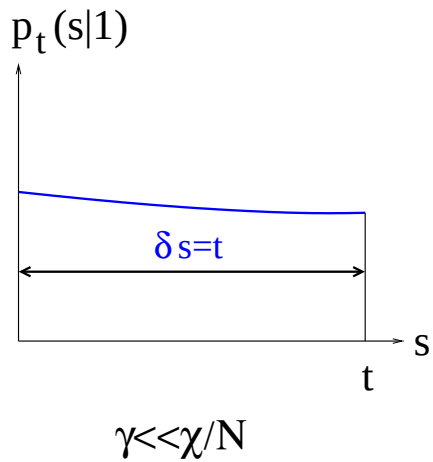

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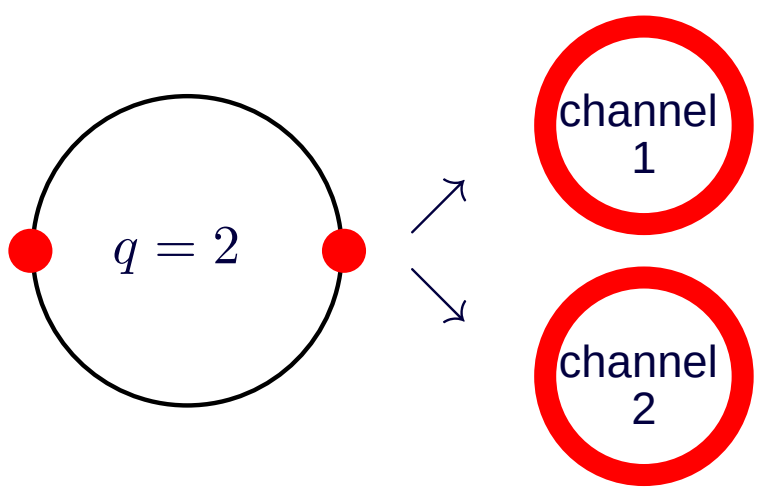

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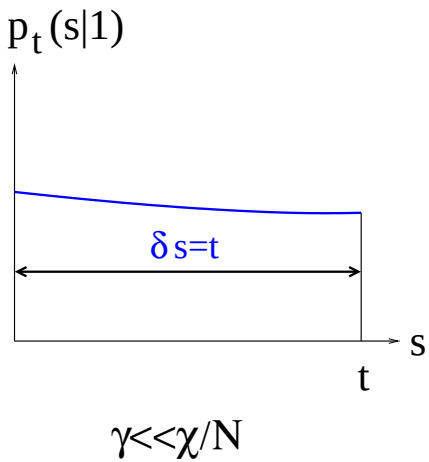

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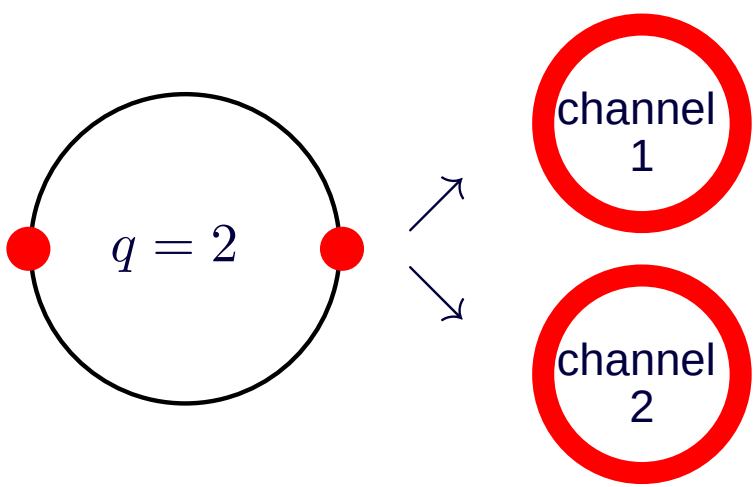

Asymm. energies $U_1 > U_2 = U_{12}$

$$\delta\phi_{2,0} = 4\pi/q, \; \delta\phi_{0,2} = 0$$