Entanglement evolution for quantum trajectories

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Outlines

- Evolution of entanglement in the presence of couplings with an environment
- Average concurrence for quantum trajectories
- Conclusions & Perspectives

Joint work with: Sylvain Vogelsberger (I.F. Grenoble)

Ref.: arXiv:1006.1317 [quant-ph]

Evolution of entanglement

Entanglement of formation E_ρ between 2 subsystems A & B in a **mixed state** ρ: by definition, E_ρ is an *infimum over all convex decompositions* $\rho = \sum_k p_k |\psi_k\rangle\langle\psi_k|$ (with $p_k \geq 0$),

$$E_\rho = \inf \sum_k p_k E_{\psi_k} \quad , \quad E_{\psi_k} = S_{\text{von Neuman}}\big(\operatorname*{tr}_A |\psi_k\rangle\langle\psi_k|\big)$$

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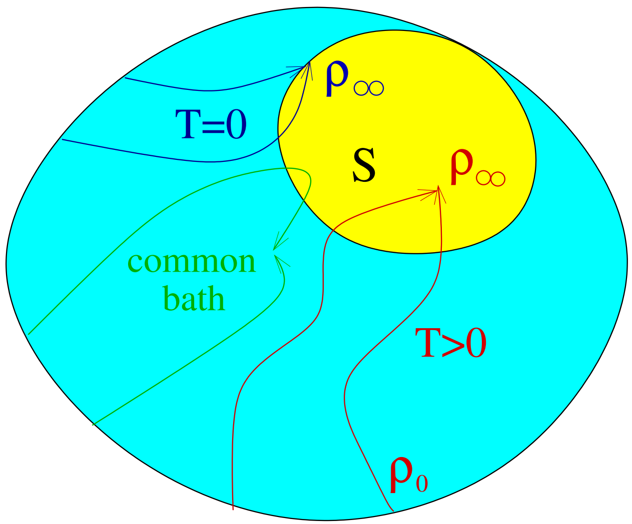
Q1: Can the A-B entanglement disappear completely?

Q2: Can one extract information from the environment (by measuring it) in order to "know" the optimal decomposition?

Entanglement sudden death

ENTANGLEMENT TYPICALLY DISAPPEARS BEFORE COHERENCES ARE LOST!



It can disappear after a finite time

- *always the case if the qubits relax to a Gibbs state ρ_∞ at positive temperature*
- *otherwise depends on the initial state.*

[Diosi '03], [Dodd & Halliwell PRA 69 ('04)], [Yu et Eberly PRL 93 ('04)]

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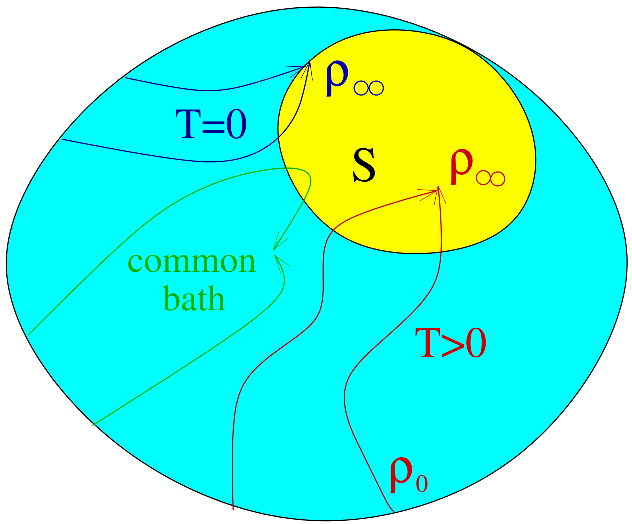

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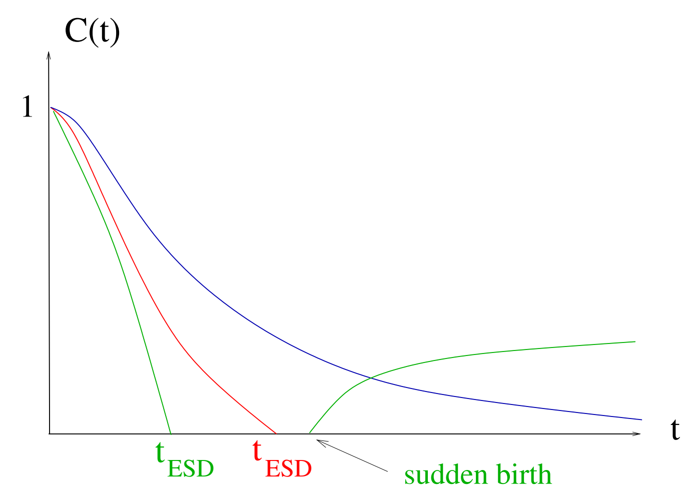
If the two qubits are coupled to a **common bath**, entanglement can also suddently reappear

$\rightsquigarrow$ *due to effective (bath-mediated) qubit interaction creating entanglement*

[Ficek & Tanás PRA 74 ('06)], [Hernandez & Orszag PRA 78 ('08)], [Mazzola et al. PRA ('09)]



Quantum trajectories

As a result of continuous measurements on the environment, the bipartite system remains in a pure state $|\psi(t)\rangle$ at all times $t > 0$

$$t \in \mathbb{R}_+ \mapsto |\psi(t)\rangle \quad \text{quantum trajectory}$$

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Averaging over the measurements, one gets the density matrix:

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In general this decomposition is NOT THE OPTIMAL one,

$$\overline{E_{\psi(t)}} \geq E_{\rho(t)}$$

[Nha & Carmichael PRL 98 ('04)].

But for specific models, one can find measurement schemes with $\overline{C_{\psi(t)}} = C_{\rho(t)}\ \forall\, t \geq 0$ with $C =$ Wootters concurrence for 2 qubits

[Carvalho et al. PRL 98 ('07), Viviescas et al. ('10)].

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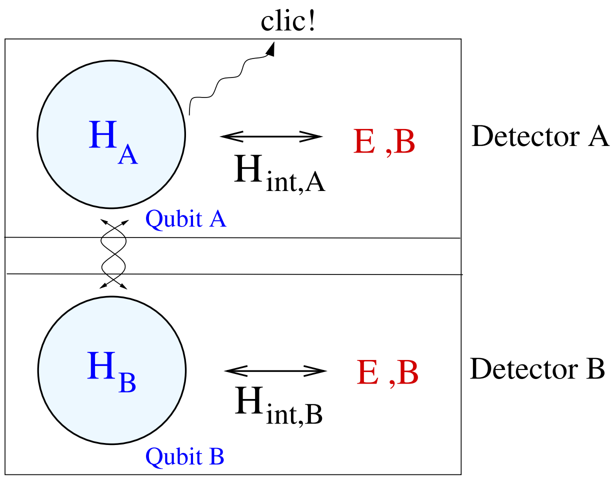
Photon counting



Two 2-level atoms (qubits) initially in state $|\psi\rangle = \sum_{s,s'=0,1} c_{ss'}|s\rangle|s'\rangle$ are coupled to independent modes of the electromagnetic field initially in the vacuum.

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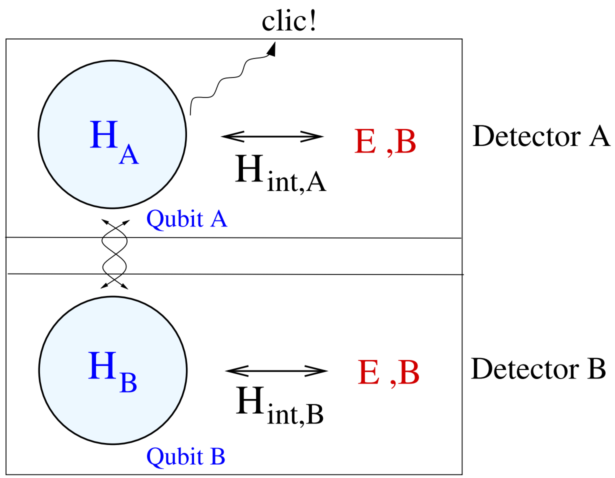

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- If D_i detects a photon between t and $t + \mathrm{d}t$, the qubits suffer a quantum jump [occurs with proba. $\gamma_i \|\sigma_-^i|\psi(t)\rangle\|^2 \mathrm{d}t$]

$$|\psi(t)\rangle \longrightarrow \sigma_-^i|\psi(t)\rangle = |0\rangle_i \otimes |\phi(t)\rangle \quad \rightsquigarrow \text{separable.}$$

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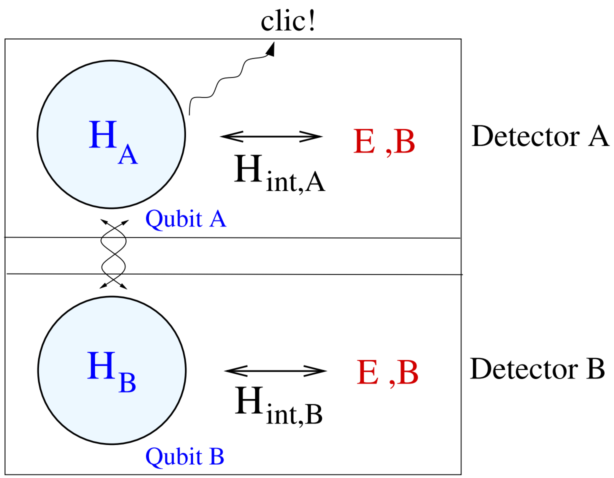

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- If no click occurs between t_0 and t [proba. $\|e^{-itH_{\text{eff}}}|\psi(t_0)\rangle\|^2$]

$$|\psi(t)\rangle = \frac{e^{-i(t-t_0)H_{\text{eff}}}|\psi(t_0)\rangle}{\|e^{-itH_{\text{eff}}}|\psi(t_0)\rangle\|} \;,\quad H_{\text{eff}} = H_0 - \frac{i}{2}\sum_{i=A,B} \gamma_i\,\sigma_+^i\sigma_-^i .$$

Photon counting (2)

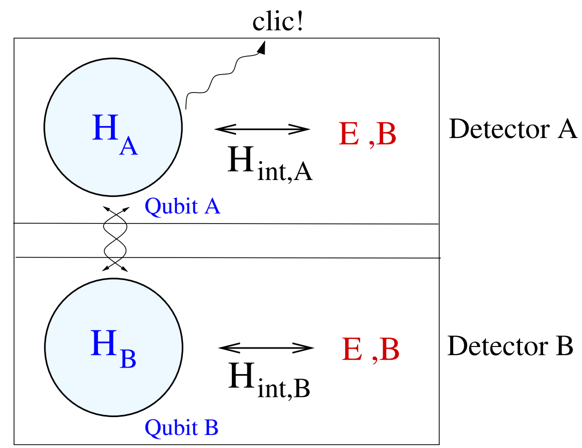
Concurrence: *[Wootters PRL 80 ('98)].*

$$C_{\psi(t)} = |\langle\psi(t)|\sigma_y \otimes \sigma_y T|\psi(t)\rangle|$$

$T =$ complex conjugation op.

$\sigma_y =$ Pauli matrix

$\hookrightarrow E_{\psi(t)} = f(C_{\psi(t)})$, f convex $\nearrow$



- Trajectories with 1 or more jumps between 0 and t have a concurrence $C_{\psi(t)} = 0$ (since $|\psi(t)\rangle$ separable after 1 jump).
- If no jump occurs between 0 and t, one finds for $H_0 = 0$:
  $C_{\text{no jump}}(t) = \mathcal{N}_t^{-2}\, C_0\, e^{-(\gamma_A+\gamma_B)t}$ with $\mathcal{N}_t = \|e^{-itH_{\text{eff}}}|\psi\rangle\|$.

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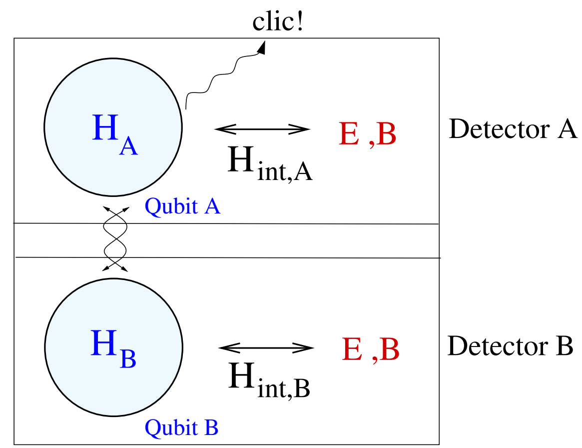

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Average concurrence over all trajectories:

$\overline{C_{\psi(t)}} =$ proba (no jump in $[0,t]$) $\times\, C_{\text{no jump}}(t)$

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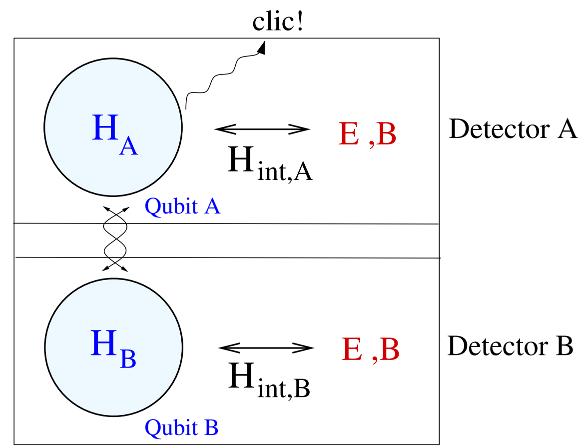

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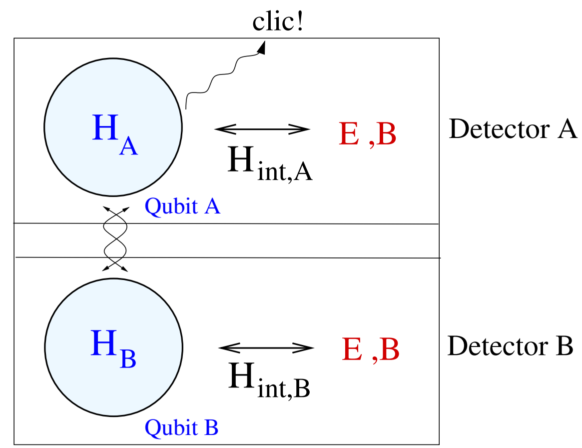

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$\hookrightarrow \overline{C_{\psi(t)}}$ vanishes asymptotically $\Rightarrow$ **sudden death of entanglement never occurs for quantum trajectories!**

General quantum jump dynamics

Consider 2 noninteracting qubits coupled to *independent baths* monitored by means of *local measurements*

$\Rightarrow$ the jump operators $J = J^A \otimes 1$ or $1_A \otimes J_B$ are *local*.

- *The no-jump trajectories have a non-vanishing concurrence* $C_{\rm nj}(t) > 0$ *at all finite times* (if $C_0 > 0$).

  Proof: assume the contrary, i.e. $|\psi_{\mathsf{nj}}(t)\rangle$ separable, then $|\psi(0)\rangle \propto e^{itH_{\rm eff}}|\psi_{\mathsf{nj}}(t)\rangle$ would be separable since $e^{itH_{\rm eff}}$ is a tensor product of two local operators acting on each qubits.

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- *The average concurrence over all trajectories is*

$$\overline{C_{\psi(t)}} = C_0\, e^{-\kappa t}$$

  where $\kappa \geq 0$ *depends on the measurement scheme only (but not on the initial state).*

Note: $\overline{E_{\psi(t)}} \geq f(\overline{C_{\psi(t)}})$ by convexity of f.

Quantum state diffusion

- The result $\overline{C_{\psi(t)}} = C_0\, e^{-\kappa t}$ is not only true for quantum jump dynamics but also for quantum state diffusion, e.g. for trajectories given by the stochastic Schrödinger equation

$$|\mathrm{d}\psi\rangle = \Big[(-iH_0 - K)\mathrm{d}t + \sum_{J \text{ local}} \gamma_J \Big(\Re\langle J\rangle_\psi\, J - \frac{1}{2}(\Re\langle J\rangle_\psi)^2\Big)\mathrm{d}t + \sum \sqrt{\gamma_J}\big(J - \Re\langle J\rangle_\psi\big)\mathrm{d}w\Big]|\psi\rangle$$

  which describes **homodyne detection**.

- The disentanglement rates κ are different for photon-counting, homodyne, and heterodyne detections:

$$\kappa_{\mathrm{QJ}} = \frac{1}{2}\sum_J \gamma_J \Big(\mathrm{tr}(J^\dagger J) - 2|\det(J)|\Big)$$

$$\kappa_{\mathrm{ho}} = \frac{1}{2}\sum_J \gamma_J \Big(\mathrm{tr}(J^\dagger J) - 2\Re\det(J) - (\Im\,\mathrm{tr}(J))^2\Big)$$

$$\kappa_{\mathrm{het}} = \frac{1}{2}\sum_J \gamma_J \Big(\mathrm{tr}(J^\dagger J) - \frac{1}{2}|\,\mathrm{tr}(J)|\Big).$$

  Adjusting the laser phases $J \to e^{-i\theta} J$ yields $\kappa_{\mathrm{ho}} \le \kappa_{\mathrm{QJ}}, \kappa_{\mathrm{het}}$.

Discussion

It is **not possible** to have $\overline{C_{\psi(t)}} = C_{\rho(t)}$ if one measures locally the independent environments of the qubits (since $C_{\rho(t)}$ may vanish at a finite time t_{ESD}, whereas $\overline{C_{\psi(t)}} > 0 \ \ \forall\, t$).

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- $\ast$ This raises the question: **is ESD observable?**

  [Almeida et al., Science 316 ('07)]. $\longrightarrow$ simulation of master eq.

  [Viviescas et al., arXiv:1006.1452]. $\longrightarrow$ YES with some nonlocal measurements $\Rightarrow$ require additional quantum channels...

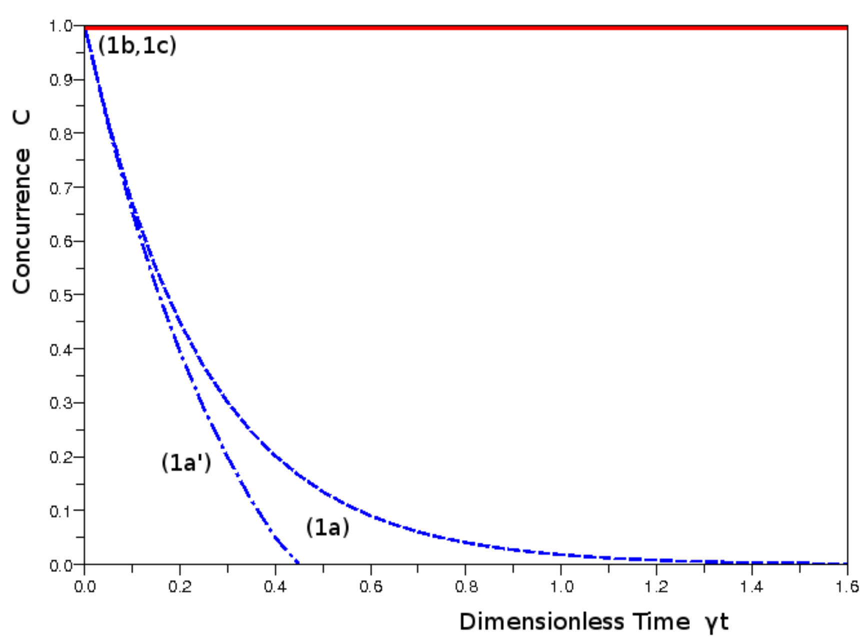
- $\ast$ For A-B entanglement, "ignoring" the environment state is not the same as measuring it without reading the results.

[Mascararenhas et al., arXiv:1006.1233].

Entanglement protection

One may use the continuous monitoring by the measurements to protect the qubits against disentanglement.

- For ex., for pure phase dephasing ($J^i = \mathbf{u}_i \cdot \sigma^i$, $i = A, B$), $\kappa_{\rm QJ} = \kappa_{\rm ho} = \kappa_{\rm het} = 0$ so that $\overline{C_{\psi(t)}} = C_0 = \text{const.}$



Bell initial state

$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + e^{-i\varphi}|\downarrow\downarrow\rangle)$

$C_0 = 1 \Rightarrow C_{\psi(t)} = 1$ for all quantum trajectories and all times

$\hookrightarrow$ perfect entanglement protection!

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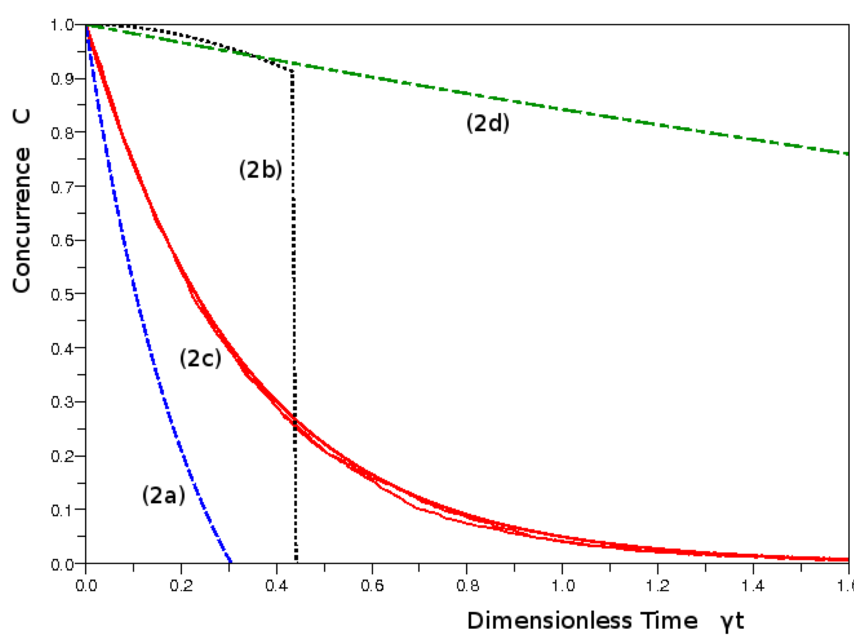
- For two baths at temperatures $T_i > 0$, the smallest rate is $\kappa_{\rm QJ} = \sum_{i=A,B} \gamma_+^i (e^{\frac{\omega_0}{2kT_i}} - 1)^2$ (jump op. $J \propto \sqrt{\gamma_-^i}\sigma_-^i + \sqrt{\gamma_+^i}\sigma_+^i$)



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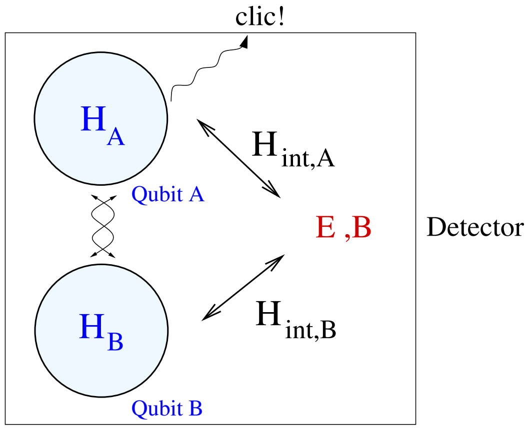
$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle - i|\downarrow\downarrow\rangle)$$

$$\overline{C_{\psi(t)}} = e^{-\kappa t}$$

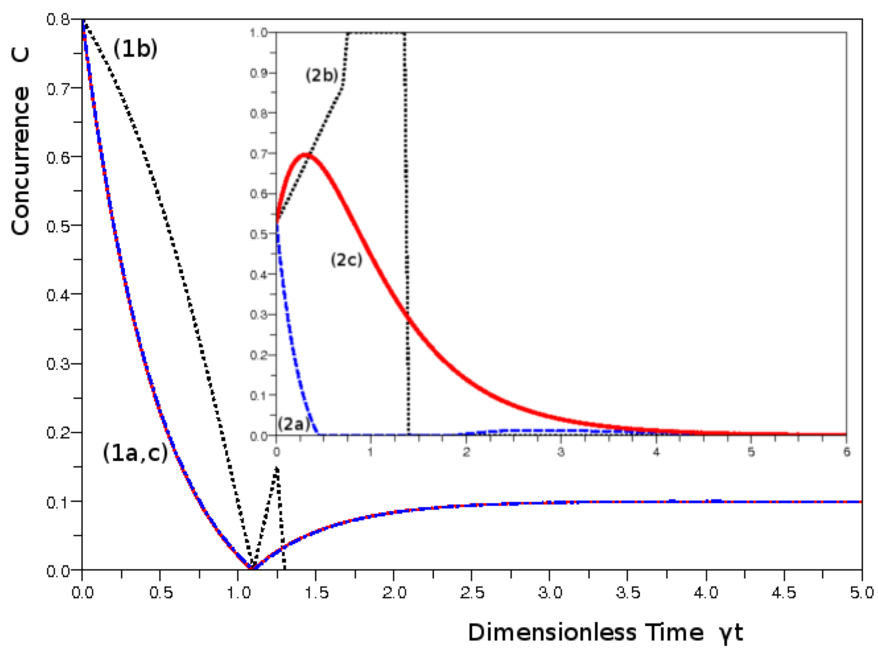
$\hookrightarrow$ perfect entanglement protection only possible at infinite temperature!

Qubits coupled to a common bath



Two 2-level atoms (qubits) initially in state $|\psi\rangle = \sum_{s,s'=0,1} c_{ss'}|s\rangle|s'\rangle$ are coupled to the **same** modes of the electromagnetic field initially in the vacuum.

$$\overline{C_{\psi(t)}} = \frac{1}{2}\left|c_-^2 - c_+^2 e^{-2\gamma t} + 4c_{11}c_{00}\, e^{-\gamma t}\right| + 2|c_{11}|^2\gamma t\, e^{-2\gamma t}$$



with $c_\pm = c_{11} \pm c_{00}$.

- If $c_{11} = 0$ then $\overline{C_{\psi(t)}} = C_{\rho(t)}$.
- If $c_{11} > 0$ then $\overline{C_{\psi(t)}}$ increases at small times.

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- The mean concurrence $\overline{C(t)}$ of two qubits coupled to **independent baths** monitored by continuous **local measurements** decays exponentially with a rate depending on the measurement scheme only.

  $\hookrightarrow$ in order that $\overline{C(t)}$ coincides at all times with $C_{\rho(t)}$ for the density matrix having an entanglement sudden death, one has to measure joint observables of the two baths.

- Measuring the baths helps to protect entanglement, sometimes perfectly!

- For two qubits coupled to a **common bath**, the time behavior of the mean concurrence depends strongly on the initial state. One may have $\overline{C(t)} = C_{\rho(t)}$.

Open problems: non-Markov unravelings, multipartite systems,...