Persistent Currents for Interacting Bosons on a Ring with a Gauge Field

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Bose-Einstein condensates with cold atoms

Typical parameters

- Temperature: 10 - 100 nK
- Density: $10^{13} - 10^{14}$ cm$^{-3}$
- Number of atoms: $10^3 - 10^7$
- Size: 10 μm - 1 mm
- Lifetime: 10 s

Condensed species

- $^{87}$Rb
- Na
- $^7$Li
- H
- $^{85}$Rb
- $^4$He$^*$
- $^{41}$K
- Cs
- Yb
- Cr
- $^{39}$K

Anderson et al., Science 95
Davis et al. PRL 95
Persistent flow of BEC in torroidal trap with a weak link

Ryu et al. PRL 07
Ramanathan et al. PRL 11
Murray et al., PRA 13

(rotating state induced using optical fields; after expansion)

non-rotating state after expansion

weak link locally induces flow instability
Outline

1. Single particle on a rotating ring
   Defining concepts

2. Interacting bosons on a rotating ring
   Discovering the interplay between backscattering and interactions

3. Atomic SQUID (AQUID)
   Optimizing performance while playing with interaction and barrier strengths

4. Dynamics of AQUID
   Interacting bosons + barrier yields an open quantum system

Conclusions
I. Single particle on a rotating ring

Defining concepts
Particle on a uniform ring: angular momentum states

Hamiltonian and Schrödinger equation

\[ H = -\frac{\hbar^2}{2mR^2} \frac{\partial^2}{\partial \phi^2} \]

\[ H \psi(\phi) = E \psi(\phi) \]

\[ -\frac{\hbar^2}{2mR^2} \frac{\partial^2}{\partial \phi^2} \psi = E \psi \]

Eigenfunctions and eigenvalues

\[ \psi(\phi) = \psi_n(\phi) = \frac{1}{\sqrt{2\pi}} e^{in\phi} = \psi(\phi + 2\pi) \]

\[ E = E_n = \frac{\hbar^2 n^2}{2mR^2} \]

Wave function is strictly $2\pi$ periodic

Spectrum is discrete
Particle on a uniform rotating ring

Hamiltonian in rotating frame

\[ H = -\frac{\hbar^2}{2mR^2} \left( \frac{\partial}{\partial \phi} - i \frac{m\omega R^2}{\hbar} \right)^2 \]

\[ = -\frac{\hbar^2}{2mR^2} \left( \frac{\partial}{\partial \phi} - i \frac{\Phi}{\Phi_0} \right)^2 \]

Coriolis flux and flux quantum

\[ \Phi = \omega R^2 \quad \Phi_0 = \hbar/m \]

Eigenfunctions and eigenvalues

\[ \psi(\phi) = \psi_n(\phi) = \frac{1}{\sqrt{2\pi}} e^{in\phi} = \psi(\phi + 2\pi) \]

\[ E = E_n = \frac{\hbar^2}{2mR^2} (n - \Phi/\Phi_0)^2 \]

Wave function is strictly $2\pi$ periodic

Energy depends on $\Phi$: ring sustains persistent current $I \sim \partial E/\partial \Phi$
Periodicity is Coriolis flux quantum $\Phi_0$
Spectrum and persistent currents (rotating frame)

lowest energy states: well-defined angular momentum

\[ I_0 = \frac{\hbar}{2\pi m R^2} \]

persistent current is a saw-tooth as a function of Coriolis flux
Spectrum and persistent currents (laboratory frame)
Effect of an impurity

Impurity induces mixing of angular momentum eigenstates

\[ |n\rangle \quad |n - 1\rangle \]

Effective Hamiltonian near degeneracy point

\[ H_{\text{eff}} = \begin{pmatrix} E_{n-1} & U_{\text{eff}} \\ U_{\text{eff}} & E_n \end{pmatrix} \]

cf. band structure in solids
Mixing angular momentum states: superpositions

Coherent impurity scattering: formation of superpositions of angular momentum states.

Weight factors in superpositions are tunable with Coriolis flux

System is a tunable two-level system (qubit)
Persistent current as a function of Coriolis flux: rotating frame
Persistent current as a function of Coriolis flux: laboratory frame
II. Interacting bosons on a rotating ring

Discovering the interplay between backscattering and interactions
Rotating interacting 1D bosons on a ring with a barrier

Hamiltonian

\[ \mathcal{H} = \sum_{j=1}^{N} \frac{\hbar^2}{2M} \left( -i \frac{\partial}{\partial x_j} - \frac{2\pi}{L} \right)^2 + U_0 \delta(x_j) + \frac{g}{2} \sum_{j,l=1}^{N} \delta(x_l - x_j). \]

Aim: calculate persistent current

Various techniques:
- non-interacting bosons & Tonks-Girardeau bosons
- Gross-Pitaevskii equation (weak interactions)
- Luttinger liquid (strong interactions)
- DMRG (intermediate interactions)

Cominotti et al. PRL 14
Analytical solutions

Noninteracting bosons

Tonks-Girardeau bosons (infinite hard-core repulsion)

Single-particle wave function on a ring with a barrier

Persistent current amplitude larger for interacting bosons
Macroscopic persistent current as a function of Coriolis flux:
Mean-field theory of condensate on a rotating ring

Cominotti et al. PRL 14

Macroscopic wave function

\[ \Phi(x) = |\Phi(x)|e^{i\phi(x)} \]

Superfluid density

Superfluid phase

Gross-Pitaevskii energy

\[ E_{GP} = \int dx \left\{ \Phi^*(\hbar^2/2M)[-i\partial_x - (2\pi/L)\Omega]^2\Phi] + U_0\delta(x)|\Phi|^2 + g|\Phi|^4 \right\} \]

Kinetic energy in rotating frame

Barrier repulsive interaction

Configuration that minimizes energy

@ \( \Omega = 0.4 \)

Characteristic scale: healing length

\[ \xi = \hbar/\sqrt{2Mgn_0} \]

Persistent current amplitude increases with interaction strength

\( \lambda = 1.9 \)

\( \lambda = 19.1 \)
Luttinger liquid approach

Hamiltonian of the liquid

\[ \mathcal{H}_0 = \frac{\hbar}{2}\int_0^L dx \left[ K \left( \partial_x \phi(x) - \frac{2\pi}{L} \right)^2 \right. \]

\[ + \frac{1}{K} \left( \partial_x \theta(x) \right)^2 \]

\[ v_s K = \hbar \pi n_s / m \]

Mode expansion for fluctuating fields

\[ \theta(x) = \theta_0 + \frac{1}{2} \sum_{q \neq 0} \left| \frac{2\pi K}{qL} \right|^{1/2} [e^{iqx} b_q + e^{-iqx} b_q^\dagger] , \]

\[ \phi(x) = \phi_0 + \frac{2\pi x}{L} (J - \Omega) + \frac{1}{2} \sum_{q \neq 0} \left| \frac{2\pi}{qLK} \right|^{1/2} \text{sgn}(q) [e^{iqx} b_q + e^{-iqx} b_q^\dagger] , \]

Commutation relations

\[ [b_q, b_{q'}^\dagger] = \delta_{q,q'} \]

[\text{bosonic low-energy excitations (phonons)}]

\[ [J, e^{-2i\theta_0}] = e^{-2i\theta_0} \]

[\text{raising angular momentum (phase-slip)}]
Changing angular momentum: phase-slips

\[ E = J_2 - J_1 - J = 0, 1/2, 3/2, 5/2 \]

\[ \phi \]

\[ 0 \quad x \quad L \]

\[ 2\pi \]

\[ 0 \quad x \quad L \]
Luttinger liquid approach

Cominotti et al. PRL 14

Hamiltonian of the liquid

\[ \mathcal{H}_0 = \frac{\hbar v_s}{2\pi} \int_0^L dx \left[ K \left( \partial_x \phi(x) - \frac{2\pi}{L} \Omega \right)^2 + \frac{1}{K} (\partial_x \theta(x))^2 \right] \]

\[ v_s K = \hbar \pi n_s / m \]

fluctuating phase

fluctuating density

kinetic energy

interaction term

Hamiltonian of the barrier

\[ \mathcal{H}_b = \int dx \Psi^\dagger(x) U_0 \delta(x) \Psi(x) = U_0 \rho(0) \]

\[ \sim 2U_0 n_0 \cos(2\theta(0)) = n_0 U_0 \sum_J |J - 1\rangle \langle J| e^{2i\delta(0)} + |J\rangle \langle J + 1| e^{-2i\delta(0)} \]

Barrier induces transitions between angular momentum states (phase-slips)

Role of interactions: barrier renormalization

\[ \langle e^{\pm 2i\delta(0)} \rangle = e^{-2\langle \delta\theta^2(0) \rangle} = \left( \frac{\alpha}{L} \right)^K \]

\[ U_0 \rightarrow U_{\text{eff}} = U_0 (\alpha / L)^K \]

Persistent current amplitude decreases with interaction strength
Optimal persistent current

Cominotti et al. PRL 14

Current amplitude as a function of interaction strength for various barrier strengths

NI: all bosons in the lowest band

TG: Bosons occupy Fermi sphere

GP: interactions induce healing

LL: interactions induce quantum phase fluctuations

MPS: competition between healing and quantum fluctuations
III. Atomic SQUID (AQUID)

Optimizing performance while playing with interaction and barrier strengths
Macroscopic persistent-current qubit

\[ H_J = E_0(J - \Omega)^2 + n_0 U_{\text{eff}} \sum_J |J + 1\rangle \langle J| + \text{H.c.} \]

Weight factors in macroscopic superpositions are tunable with Coriolis flux

System is a tunable two-level system (qubit)
Qubit energy states: role of interaction and barrier strengths

Aghamalyan, Cominotti et al. NJP 15

Spectrum as a function of $\Omega$ @ $U/\Lambda = 20$ & 0.4

Level splitting & anharmonicity as a function of interaction and barrier strengths @ $\Omega = 0.5$

healing
quantum fluctuations
Qubit energy states: mesoscopic finite-size scaling

Aghamalyan, Cominotti et al. NJP 15

Level splitting and anharmonicity as a function of system size

Interactions « protect » qubit splitting & anharmonicity against scaling
Ground-state momentum distribution

**Definition**

\[ n(k) = \int dx \int dx' \langle \hat{\psi}^\dagger(x) \hat{\psi}(x') \rangle e^{ik(x-x')} \]

**Response at** \( \Omega = \frac{1}{2} + \delta \) (just away from perfect balance)

Superposition state involving many angular momentum states (Fermi gas)

Superposition state involving two angular momentum states

State with one angular momentum quantum
IV. Dynamics of AQUID

*Interacting bosons + barrier yields an open quantum system*
Phonon modes on a uniform ring

Hamiltonian of harmonic fluid

\[
\mathcal{H}_0 = \frac{\hbar v_s}{2\pi} \int_0^L dx \left[ K \left( \partial_x \phi(x) - \frac{2\pi}{L} \Omega \right)^2 + \frac{1}{K} \left( \partial_x \theta(x) \right)^2 \right]
\]

Mode expansion: propagating phonon modes

\[
\theta(x) = \theta_0 + \frac{1}{2} \sum_{q \neq 0} \left| \frac{2\pi K}{qL} \right|^{1/2} \left[ e^{iqx} b_q + e^{-iqx} b_q^\dagger \right],
\]

\[
\phi(x) = \phi_0 + \frac{2\pi x}{L} (J - \Omega) + \frac{1}{2} \sum_{q \neq 0} \left| \frac{2\pi}{qLK} \right|^{1/2} \text{sgn}(q) \left[ e^{iqx} b_q + e^{-iqx} b_q^\dagger \right],
\]

Harmonic fluid equivalent to harmonic oscillator bath

\[
\mathcal{H}_0 = E_0 (J - \Omega)^2 + \sum_{q \neq 0} \hbar \omega_q b_q^\dagger b_q = E_0 (J - \Omega)^2 + \frac{1}{2} \sum_{q \neq 0} (P_q^2 + \omega_q^2 Q_q^2)
\]

linear spectrum \( \omega_q \sim v_s |q| \)
Effect of the barrier

Angular momentum tunneling (phase-slips) excites phonon modes

\[ \mathcal{H}_b \approx n_0 U_0 \sum_J |J - 1 \rangle \langle J| e^{2i\delta \theta(0)} + |J \rangle \langle J + 1| e^{-2i\delta \theta(0)} \]

coupling to density fluctuations

\[ \delta \theta(0) = \sum_{q \neq 0} \lambda_q Q_q \]

particle-bath coupling

Caldeira-Leggett Hamiltonian

\[ \mathcal{H} = E_0 (J - \Omega)^2 - 2U_0 n_0 \cos(2\theta_0) + \frac{1}{2} \sum_{q \neq 0} \{[P_q - \mu_q (J - \Omega)]^2 + \omega_q^2 Q_q^2 \} \]

quantum particle oscillator bath
Tunable persistent current qubit with dissipation: spin-boson model

\[ \frac{1}{2} \left( 1 + \sigma_z \right) \]

\[ \frac{1}{2} \sigma_x \]

\[ \mathcal{H} = E_0 (J - \Omega(t))^2 - 2U_0(t)n_0 \cos(2\theta_0) + \frac{1}{2} \sum_{q \neq 0} \left\{ [P_q - \mu_q (J - \Omega(t))]^2 + \omega_q^2 Q_q^2 \right\} \]

\[ -\frac{1}{2} B_z \sigma_z - \frac{1}{2} B_x \sigma_x \]

\[ n_0 U_{\text{eff}}(t) \]

\[ \beta |J - 1\rangle - \alpha |J\rangle \]

\[ \alpha |J - 1\rangle + \beta |J\rangle \]

bath coupling linearly to \( \sigma_z \)

Spin-boson model

Polo et al. in progress
Tunable nonlinear oscillator: quantum Langevin problem

Hamiltonian (no dissipation, no drive)

\[ H = E_0 (J - \Omega)^2 - 2U_0 n_0 \cos(2\theta_0) \]

Classical phase portrait

Equation of motion

\[ \ddot{\theta}_0 = -a \sin 2\theta_0 \]

running state (dual to self-trapping)

separatrix

oscillating state

\[ \theta_0 \]
Tunable nonlinear oscillator: quantum Langevin problem

Quantum Langevin equation for driven system

\[ \ddot{\theta}_0 = -a(t) \sin 2\theta_0 - b\dot{\theta} + c\dot{\Omega}(t) + \xi(t) \]

Ergül et al., PRB13
Conclusions

Spectrum of condensate on rotating loop periodic with Coriolis flux

*Ring sustains persistent currents*

Response depends on barrier and interaction strengths

*From sawtooth to sinusoidal behaviour*

Coherent phase-slips induce superposition of angular momentum states

*Ring can be used as a qubit*

Dynamics governed by Caldeira-Legget model

*From spin-boson physics to Langevin equation*