Problem 1. *Prove that the curve graph $C(S)$ for a surface $S = S_{g,n}$ with $3g-3+n \geq 2$ is connected. In fact, one can prove the following. Given two curves α and β, $d(\alpha,\beta) \leq i(\alpha,\beta)+1$.*

Problem 2. *Let S be a surface with a base point $\star$ with negative Euler characteristic. Recall the Birman short exact sequence $1 \to \pi_1(S,\star) \xrightarrow{push} \mathcal{MCG}(S,\star) \xrightarrow{Forget} \mathcal{MCG}(S) \to 1$. We will attempt to give a sketch of the proof of this.*

- *Consider the map $e : \mathrm{Homeo}^+(S) \to S$ defined by $e(f) = f(\star)$. Show this is a fiber bundle with fiber $\mathrm{Homeo}^+(S,\star)$.*
- *Apply the long-exact sequence of homotopy groups to this fiber bundle to obtain:*

$$\ldots \to \pi_1(\mathrm{Homeo}^+(S)) \to \pi_1(S,\star) \to \pi_0(\mathrm{Homeo}^+(S,\star)) \to \pi_0(\mathrm{Homeo}^+(S)) \to \pi_0(S,\star).$$

  Now derive Birman's exact sequence using the following fact:

  Theorem 3 (Hamstrom 60s)**.** *If S has negative Euler characteristic, then $\mathrm{Homeo}_0(S)$, the connected component of the identity of $\mathrm{Homeo}^+(S)$, is contractible.*

- *Of course, you should convince yourself that the connecting homomorphisms given by the long-exact sequence are indeed the maps in Birman's sequence.*
- *Also, show Hamstrom's theorem isn't true for the torus T^2.*

Problem 4. *Recall that the mapping class group $\mathcal{MCG}(T^2)$ of the torus T^2 can be identified $\mathrm{SL}_2(\mathbb{Z})$ via the action on $H_1(T^2)$. Let $\mathbb{H}^2 = \{z \in \mathbb{C} : \mathrm{im}(z) > 0\}$ be the upper half-space. Recall the action of $\mathrm{SL}_2(\mathbb{Z})$ act on $\mathbb{H}^2$ by linear fractional transformations, i.e. $\begin{pmatrix} a & b \\ c & d \end{pmatrix}(z) = \frac{az+b}{cz+d}$, are isometries of $\mathbb{H}^2$. Verify the classification of elements of $\mathrm{SL}_2(\mathbb{Z})$ given by the following table.*

Isometry type	*Nielsen-Thurston Type*	*Trace*	*Example*
Elliptic	*Periodic*	$\lvert \mathrm{trace}(A) \rvert < 2$	$A = \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix}$
Parabolic	*Reducible*	$\lvert \mathrm{trace}(A) \rvert = 2$	$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$
Hyperbolic	*Anosov*	$\lvert \mathrm{trace}(A) \rvert > 2$	$A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$

Problem 5. *Classify the finite-order elements of $\mathrm{SL}_2(\mathbb{Z})$.*

Problem 6. *Try out Thurstons example: Model $S_{0,4}$ as the plane with three punctures p_1, p_2, p_3 in a row on the x–axis (the fourth puncture is the point at infinity). Let σ_1 be the counterclockwise half-twist switching p_1 and p_2, and let σ_2 be the counterclockwise half-twist switching p_2 and p_3. Let $f = \sigma_1^{-1}\sigma_2$. Pick an essential simple closed curve c, say the one that goes around p_1 and p_2 and draw $f^k(c)$ for several positive integers k.*

Here's how to interpret your picture: roughly speaking, the pieces of your curve are somehow approaching the leaves of an unstable singular foliation for f. This picture led Thurston to the idea of train tracks, one of three constructions commonly used to characterize the structure of pseudo-Anosov mapping classes.