Problem 1. *Apply the proof of the Dehn-Licorish Theorem to find an explicit set of generators for $\mathcal{PMCG}(S_2)$ starting with the two Dehn twists that generate $\mathcal{PMCG}(S_{1,1})$.*

Problem 2. *Verify the lantern relation using the Alexander method.*

Problem 3. *Use the lantern relation, the change of coordinates principle, and the Dehn–Lickorish theorem to prove that $\mathcal{MCG}(S_g)$ has trivial abelianization for $g \geq 3$.*

Problem 4. *Assuming that $\mathcal{MCG}(S_g)$ is finitely presentable, deduce that $\mathcal{MCG}(S_{g,n,b})$ is finitely presentable.*

Problem 5. *Show that the map $\mathcal{MCG}(S_g) \to \mathrm{Out}(\pi_1(S_g))$ is well defined. Hint: consider the point pushing map $\pi_1(S_g) \to \mathcal{MCG}(S_{g,1})$ and the action of $\mathcal{MCG}(S_{g,1})$ on $\pi_1(S_g)$.*

Problem 6. *Derive the Dehn-Nielsen-Baer theorem for $S_{g,1}$ from the Dehn-Nielsen-Baer theorem for S_g. Why can't the statement be true for $S_{g,n}$ for $n \geq 2$? Can you come up with a version of Dehn-Nielsen-Baer theorem that could be true for all $S_{g,n}$?*

Problem 7. *Recall the non-separating curve graph $\widetilde{\mathcal{N}}(S)$ has vertices the set of (free homotopy classes of) non-separating simple closed curves on S and two such curves form an edge if they intersect exactly once. Let's construct $\widetilde{\mathcal{N}}(S)$ For the torus $S = T^2$, Represent the torus T^2 by gluing opposite sides of the square $[0,1] \times [0,1]$ by translations.*

- *Show there is a one-to-one correspondence between a simple closed curve on T^2, which is always non-separating, and a fraction $p/q \in \mathbb{Q} \cup \{\infty\}$, where p and q are in reduced form. ($\infty = 1/0$) as a fraction).*
- *Suppose α and β are curves represented by the fractions p/q and r/s respectively. Show $i(\alpha, \beta) = 1$ if and only if $\left|\det \begin{pmatrix} p & r \\ q & s \end{pmatrix}\right| = 1$.*
- *Let $\mathbb{H}^2$ be the hyperbolic plane represented by upper half-space, with ideal boundary $\partial\mathbb{H}^2 = \mathbb{R} \cup \{\infty\}$. It is well know we can embed $\widetilde{\mathcal{N}}(S)$ in $\mathbb{H}^2 \cup \partial\mathbb{H}^2$ as follows. By above, the vertices of $\widetilde{\mathcal{N}}(S)$ can be identified with $\mathbb{Q} \cup \{\infty\} \subset \partial\mathbb{H}^2$. If two vertices of $\widetilde{\mathcal{N}}(S)$ are connected by an edge, we can realize that edge by a hyperbolic geodesic in $\mathbb{H}^2$. The picture we get is what is known as the Farey graph.*

Problem 8. *Prove the singular-value decomposition theorem for 2×2 matrices. That is, if A is 2×2 matrix with positive determinant, then we can write $A = r \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} s$, where r and s are rotations of $\mathbb{R}^2$, and $a, b > 0$. Hint: Consider the symmetric matrix $B = A^T A$ and apply spectral theorem.*

Problem 9. *As above, $A = r \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} s$. Assume $a > b$. Call $d(A) = \frac{a}{b}$ the* dilatation *of A.*

- *By the decomposition, there are unit vectors u and v such that $Au = av$. Show $u = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ if and only if r is the identity matrix.*

- *Given two matrices A and B, show $d(AB) \leq d(A)d(B)$.*
- *Show $d(AB) = d(A)d(B)$ if and only if $s = r'^{-1}$, where $B = r' \begin{pmatrix} a' & 0 \\ 0 & b' \end{pmatrix} s'$.*

Problem 10. *Verify that the Teichmüller metric satisfies the triangle inequality and the symmetry axiom of a metric.*