

Problem 1. *Classify all separating curves on a surface S_g up to homeomorphism.*

Problem 2. *Show that if $i(a, b) \neq 0$ and with T_a, T_b nontrivial. Verify that $[T_a, T_b] \neq 1$. Do this by drawing pictures.*

Problem 3. *Given a surface S with n punctures. Let Σ_n be the permutation group. Recall a mapping class of S is allowed to permute the punctures. Prove that the natural map $\mathcal{MCG}(S) \rightarrow \Sigma_n$ is a well-defined surjective homomorphism. The kernel is called the pure mapping class group of S .*

Problem 4. *Verify the claim that if a and c are nonseparating curves with $i(a, c)$ equal to 0 or 2 then there is a curve b with $i(a, b) = i(b, c) = 1$.*

Problem 5. *What conditions must a set of Dehn twists satisfy in order for the intersection of their centralizers to be trivial?*

Problem 6. *Show that the center of every finite-index subgroup of $\mathcal{MCG}(S_g)$ is trivial for $g \geq 3$.*

Problem 7. *Suppose that a group G acts on a connected graph X by automorphisms, transitively on vertices and edges. Fix two vertices $u, v \in X$ and let $h \in G$ be such that $h(v) = u$. Show G is generated by G_u (the stabilizer of u) and h .*

Problem 8. *Prove that the mapping class group of the torus is isomorphic to $\mathrm{SL}_2(\mathbb{Z})$. Also do this for the mapping class group of the once-punctured torus.*

Problem 9. *For a surface S of genus $g \geq 2$, prove there are at most $3g - 3$ pairwise disjoint (free homotopy class of) essential simple closed curves on S .*

Problem 10. *For the Klein bottle K^2 , find all the free homotopy classes of essential simple closed curves. Hint: there are only finitely many! What is the curve graph of K^2 ?*

Problem 11. *In the curve graph of a surface, what does it mean for two vertices to have distance 2? Find a path of length 3 in the curve graph of S_2 .*