Problem 1. *Prove that* $\widehat{i}(a, b)$ *depends only on the homology classes of the curves* a *and* b *in the surface* S.

Problem 2. *Show that* $\mathrm{Homeo}_0(S, \partial S)$*, the path component of the identity in* $\mathrm{Homeo}^+(S, \partial S)$*, is a normal subgroup.*

Problem 3. *Carefully show that the following are equivalent for* $\phi, \psi \in Homeo^+(S, \partial S)$*, quoting theorems from the lecture:*

- ϕ *and* ψ *are in the same path component of* $\mathrm{Homeo}+(S, \partial S)$ *in the compact-open topology.*
- ϕ *and* ψ *are in the same coset of* $\mathrm{Homeo}_0(S, \partial S)$*, the path component of the identity.*
- ϕ *and* ψ *are homotopic.*

Problem 4. *Show that if* S *is a surface with nonempty boundary, then every homeomorphism of* S *that restricts to the identity on* ∂S *must also preserve the orientation of* S.

Problem 5. *Let* S^1 *be the unit circle in the plane. Show that every element of* $\mathrm{Homeo}(S^1)$ *is homotopic to either the identity or reflection about the* x*–axis. What does this tell us about the mapping class group of* S^1*?*

Problem 6. *An arc on a surface* S *is a map of an interval* I *to* S*. We demand that endpoints of* I *map to* ∂S*, and if an endpoint is missing, then the map is proper (so the missing "endpoint" maps to a puncture of* S*). Arcs are considered up to proper homotopy relative to endpoints. State and prove a bigon criterion for arcs, and for an arc and a curve.*

Problem 7. *State and prove a criterion for checking whether a single curve has the minimal number of self- intersections for its homotopy class.*

Problem 8. *Prove that any two nonseparating simple closed curves in a surface differ by a homeomorphism of the surface (a simple closed curve is nonseparating if it does not divide the surface into two pieces).*

Problem 9. *Prove the assertion that any two pairs of simple closed curves that intersect once differ by a homeomorphism of the surface.*

Problem 10. *Let* $\mathcal{MCG}^{\pm}(S)$ *be the version of the mapping class group that includes classes of orientation-reversing homeomorphisms, up to equivalence by isotopy. Show that* $\mathcal{MCG}^{\pm}(S)$ *is a semidirect product of* $\mathbb{Z}/2\mathbb{Z}$ *acting on* $\mathcal{MCG}(S)$*. Describe the action of* $\mathbb{Z}/2\mathbb{Z}$ *on the set of Dehn twists coming from some such decomposition.*