ADDENDUM TO
"AN EXOTIC DEFORMATION OF THE HYPERBOLIC SPACE"

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Our proof of Proposition 5.4 in [1] glossed over one special case (but the statement of the proposition is correct). We shall give below the additional argument that is needed. First, we recall the statement.

Proposition (Proposition 5.4 in [1]). *Let (G, K) be a Gelfand pair and $k = \mathbf{R}$ or $\mathbf{C}$. Let π be a continuous k-linear representation of G on a k-Hilbert space $\mathscr{H}$ preserving a continuous, strongly nondegenerate (sesqui)linear form of finite index. If π is irreducible, then the space $\mathscr{H}^K$ of K-invariant vectors has k-dimension at most 1 if $k = \mathbf{C}$ and at most 2 if $k = \mathbf{R}$.*

We keep all notation and terminology from [1]. It is proved in [1] that the restriction of B to $\mathscr{H}^K$ is nondegenerate. The rest of the proof given there applies when the index q of this restriction is positive.

We now address the case $q = 0$; thus, $\mathscr{H}^K$ is a k-Hilbert space and $\mathscr{A}$ is a commutative self-adjoint algebra acting irreducibly on V. In particular, if $k = \mathbf{C}$, the usual Schur lemma implies $\dim_{\mathbf{C}} \mathscr{H}^K \leq 1$.

It remains to consider $k = \mathbf{R}$, in which case self-adjoint algebras are often called *symmetric*. There is also a real version of Schur's lemma:

Theorem. *Let V be a real Hilbert space and $\mathscr{A}$ any symmetric algebra of bounded operators of V, acting irreducibly on V. Then the commutant of $\mathscr{A}$ is isomorphic as an algebra to $\mathbf{R}$, $\mathbf{C}$ or $\mathbf{H}$ (the quaternions).*

A proof of this theorem can be found for instance in [2]; it can also be deduced from the classical complex Schur lemma. We now return to the proof of the proposition, where $\mathscr{A}$ is abelian and $V = \mathscr{H}^K$. The theorem implies that $\mathscr{A}$ is finite dimensional. Since the action of $\mathscr{A}$ on $\mathscr{H}^K$ is irreducible, this implies that $\mathscr{H}^K$ is finite dimensional. This is enough to conclude as in [1].

References

1. Nicolas Monod and Pierre Py, *An exotic deformation of the hyperbolic space*, Amer. J. Math. **136** (2014), no. 5, 1249–1299.
2. Jonathan Rosenberg, *Structure and applications of real C^*-algebras*, Contemp. Math., vol. 671, pp. 235–258, Amer. Math. Soc., providence, RI, 2016.

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