Tropical Geometry, Berkovich Spaces, Arithmetic D-Modules and p-adic Local Systems

Imperial College of London

8-9-10 December 2020

Note about the format of the conference. The format of the conference changed (due to the conditions related to the pandemic). Unfortunately, for the same reasons, some of the original speakers have been obliged to canceled their participation.

In order to increase the connections between the participants, and the visibility of the event, the new format includes chats and virtual rooms where everybody can ask questions about the talks. The talks are uploaded or performed in streaming.

SCHEDULE

Time is expressed in London time zone.

Tuesday, 8 December 2020

12 :45 - Opening

13 :00 - Mattias Jonsson
14 :00 - Alex Fink
15 :00 - Hélène Esnault

Wednesday, 9 December 2020

13 :00 - Christine Huyghe
14 :00 - Masha Vlasenko
15 :00 - Jeffrey Giansiracusa

Thursday, 10 December 2020

13 :00 - Tomoyuki Abe
14 :00 - Wiesława Nizioł
15 :00 - Roman Bezrukavnikov

Abstracts of the confirmed talks

Kiran S. Kedlaya
Drinfeld's lemma for F-isocrystals.

The term "Drinfeld's lemma" refers to several statements that relate the geometry of a scheme over $\mathbb{F}_p$ to the result of base-extending to an algebraically closed field, then formally dividing by the "partial Frobenius" action on the field (fixing the original scheme). One such statement asserts that for any prime $\ell \neq p$, lisse ℓ-adic sheaves on the original scheme are the same as on this formal quotient. This result plays a pivotal role in the approach to the Langlands correspondence for a reductive group over the function field of a curve over a finite field, pioneered by Drinfeld for the group GL(2) and subsequently extended by L. Lafforgue and then V. Lafforgue.

In this lecture, we describe an analogue of this statement involving "lisse p-adic sheaves", meaning overconvergent F-isocrystals. The hope is that this can be used to upgrade Abe's proof of the Langlands correspondence for $\mathrm{GL}(n)$ (based on L. Lafforgue's method) to more general reductive groups.

Since overconvergent F-isocrystals are not directly described by representations of the profinite étale fundamental group, the p-adic statement is not an immediate corollary of the original Drinfeld's lemma. We deduce it by building up some structural properties of "Φ-isocrystals" (isocrystals with multiple Frobenius structures), particularly the Newton polygon variation and slope filtration, so that we can eventually reduce to the prior result.

Michel Gros
Twisted local Simpson correspondence and crystals on the q-crystalline and prismatic sites.

We will explain how the twisted Simpson correspondence elaborated these last years in collaboration with B. Le Stum and A.Quiroós can be interpreted in the language of crystals on the q-crystalline and prismatic sites recently considered by B. Bhatt and P. Scholze, giving in the same time very explicit examples of these objects.

Michael Temkin
Tropical reduction and lifting theorems.

One can often assign to a geometric object over a non-archimedean field geometric objects over the group of values – a skeleton or a tropicalization, and over the residue field – a reduction. Sometimes one can combine them in a single object, that we call tropical reduction. In simple cases it is something well known, like a log variety, but often the definition is still rather ad hoc.

In my talk I will discuss two cases which reveal a surprising similarity : a wildly ramified cover of curves with minimal wild ramification (joint work with U. Brezner), and a curve with a differential form when the residue characteristic is zero (joint work with I. Tyomkin). In both cases there is a lifting theorem indicating that our definition is the correct one and we do not lose any residual/tropical information : any compatible tropical and residual data can be lifted to an object over the non-archimedean field.

Veronika Ertl
Poincaré duality in rigid analytic Hyodo-Kato theory.

Hyodo-Kato theory plays an important role in different parts of arithmetic geometry, for example in p-adic Hodge theory or the research of p-adic L-fnctions.Especially for the latter it is of advantage to describe explicitly certain cohomology classes, both in the usual Hyodo-Kato theory and the compactly supported Hyodo-Kato theory. I will talk about a rigid analytic construction of Hyodo-Kato theory developed together with Kazuki Yamada (Keio University) and its compatibility with Poincaré duality, which is suited for such explicit computations.

Andreas Bode
Bornological D-modules on rigid analytic spaces.

Ardakov-Wadsley introduced p-adic D-cap-modules on rigid analytic spaces in order to study p-adic representations geometrically, in analogy to the theory of Beilinson-Bernstein localization over the complex numbers. In this talk, we report on an ongoing project to extend their framework to the (derived) category of complete bornological D-cap-modules, which allows us to define analogues of the usual six operations. We then consider a subcategory playing the role of $D_{coh}^b(D)$ and prove a number of stability results.

Martin Ulirsch
Non-Archimedean uniformization and tropicalization : Teichmueller space and the moduli space of curves.

The theories of uniformization for maximally degenerate curves over non-Archimedean curves (by Mumford) and for abelian varieties (by Raynaud) are one of the big achievements of modern arithmetic algebraic geometry. In recent years it has become clear that this story also has a tropical aspect. In fact, one may think of the construction as a two-step process : first construct a tropical uniformization, then use the combinatorial data of this tropical uniformization to build the non-Archimedean uniformization. In this talk, I will illustrate this principle in the case of the moduli space of curves. In particular I will explain how to build a non-Archimedean uniformization of the moduli space of stable algebraic curves that is closely connected to the non-Archimedean Schottky space for Mumford curves constructed by Gerritzen and Herrlich. Our approach will, in particular, exhibit tropical Teichmueller space, a simplicial compactification of Culler-Vogtmann Outer space, as a strong deformation retract of non-Archimedean Teichmueller space. The crucial technical ingredient will be the theory of Artin fans which will allow us to lift tropical data to algebraic moduli functors.

Annette Werner
On p-adic vector bundles and local systems.

Correspondences between vector bundles on p-adic varieties and local systems arise in the context of p-adic Simpson theory for vanishing Higgs field. I will explain recent contributions to

this theory obtained jointly with Lucas Mann. We use Scholze's framework of diamonds to prove a category equivalence on any proper adic space of finite type over $\mathbb{C}_p$ relating $\mathbb{C}_p$-local systems with integral models to certain modules under the structure sheaf for the v-topology.

Tobias Schmidt
D-modules and irreducibility results for locally analytic representations.

Let G be a split reductive p-adic group and let P be a parabolic subgroup of G. Let X be the rigid analytic flag variety of G and Y a P-stable closed subset. After a brief review of the theory of equivariant D-modules on rigid analytic spaces, I will discuss the geometric induction functor, due to K. Ardakov, which relates P-equivariant D_X-modules M with support Y to G-equivariant D_X-modules $ind(M)$ with support GY. I will then explain how to establish the irreducibility of the induced module $ind(M)$ for M the (push forward of the) structure sheaf on certain smooth Schubert varieties Y. As an application, we reprove geometrically some irreducibility results for locally analytic G-representations in the image of the Orlik-Strauch functor. This is work in progress with Konstantin Ardakov.

Mattias Jonsson
K-stability from a non-Archimedean perspective.

K-stability was introduced in complex differential geometry as a conjectural criterion (the YTD conjecture) for the existence of special Kähler metrics, such as Kähler-Einstein metrics, or constant scalar curvature Kähler metrics. There has been a great deal of recent activity around K-stability : the YTD conjecture is solved in many cases, a nice picture for K-stable Fano varieties has emerged, and there is an increased understanding of the notion of K-stability itself. In my talk, I will explain how K-stability can be viewed through the lens of non-Archimedean geometry. This is based on joint work with Berman, Blum, Boucksom, and Hisamoto.

Hélène Esnault
Connections and symmetric differentials.

If X is a smooth complex projective variety without differential form, every local system has finite monodromy (Brunebarbe-Klingler-Totaro). In work in progress with *Michael Groechenig*, we develop a positive characteristic analog. We'll discuss the rank 2 case and how ideally it relates to isocrystals.

Alex Fink
Tropical differential equations and their solutions.

Tropical differential equations were introduced in 2015 by Dima Grigoriev, whose interest was in the computational complexity of finding the supports of power series solutions of systems of linear ODEs. The next year, Fuensanta Aroca, Cristhian Garay and Zeinab Toghani proved a

Fundamental Theorem showing that every tropical solution of a differential ideal lifts to a solution over a field. I'll present this theory and some of the developments Zeinab, I and others have achieved since, such as extensions to PDEs and connections to Groebner theory.

Richard Crew
Integral p-adic cohomology theories.

Suppose R is a complete local ring with residue field k. The prospects for a cohomology theory on the category of separated k-schemes of finite type with values in finitely generated R-modules are not good. This is joint work with Tomoyuki Abe.

Christian Liedtke
p-adic Tate conjectures and abeloid varieties.

We study Tate-type conjectures over p-adic fields and in particular, a conjecture of Raskind for smooth and projective varieties over p-adic fields that have totally degenerate reduction. In the case of abeloid varieties, we translate Raskind's conjecture into a question about filtered (φ, N)-modules and use this to construct counter-examples. This is joint work with Oliver Gregory.

Christine Huyghe
Arithmetic differential operators with overconvergent coefficients.

The ring of differential operators with overconvergent coefficients plays a central role in Berthelot's theory of arithmetic D-modules over a formal scheme, as well as in Caro's theory of overholonomic modules. It is constructed over the complement of a divisor of some smooth formal scheme. We would like to explain how to do such a construction in the case where we have a non necessary smooth divisorial compactification of some smooth formal scheme.

Wiesława Nizioł
Comparison theorems for p-adic analytic varieties. I will discuss comparison theorems for p-adic

analytic varieties proved recently in a joint work with Pierre Colmez. A key input comes from a fine study of Banach-Colmez spaces appearing in cohomology of analytic varieties.

Masha Vlasenko
Dwork's congruences and p-adic cohomology.

This talk is about some curious p-adic properties of period functions, that were discovered by Bernard Dwork in his work on rationality of zeta functions. I will demonstrate their generalization and explain a cohomological proof. Our proof uses an explicit Dwork-style construction of Cartier operation on differential forms on toric hypersurfaces. This is joint work with Frits Beukers.

Jeffrey Giansiracusa
A general theory of tropical differential equations.

A few years ago Grigoriev introduced the concept of tropicalization of differential equations, and then Aroca et al. proved that there is an analogue of the fundamental theorem of tropical geometry, where any solution at the tropical level can be lifted to a classical solution. Unfortunately, this theory is limited in its applicability, since it only works with trivial valuations. In this talk I will describe the work of Stefano Mereta to build a general framework for tropical differential equations over non-trivially valued rings, thus allowing potential applications to p-adic differential equations. This involves some interesting twists, such as a tropical variant of the Leibniz rule. We support our theory with a differential analogue of Payne's inverse limit theorem.

Tomoyuki Abe
Arithmetic D-modules and rigid cohomologies.

Currently, we have at least two cohomological formalisms for p-adic cohomologies : rigid cohomology and cohomology defined using arithmetic D-modules. It is natural to expect that these coincide, but this is not straightforward. One of the reasons is that the construction of the functor associating arithmetic D-modules to isocrystals, due to Caro, is very complicated. We give alternative construction which enables us to compare the cohomology theory. This is a joint work with Chris Lazda.

Roman Bezrukavnikov
Derived Cartier transform, derived Satake equivalence and cohomology.

Derived localization theorem for modules over $\mathfrak{g} = Lie(G)$ where G is a reductive algebraic group over a filed of positive characteristic relates $\mathfrak{g}$-modules to crystalline D-modules on the flag variety G/B. It can be composed with Cartier transform to relate representations of g to coherent sheaves on the cotangent bundle T^*G/B. A related description of modules over the Frobenius kernel G_r involves coherent sheaves on a certain derived scheme S mapping to T^*G/B. That derived scheme S plays a role in (local) geometric Langlands duality which leads to old and possibly new connections between modular representations and geometric Langlands duality.

POSTERS

Stefano Mereta
A colimit theorem in tropical differential algebra. Extend the setting of the theory of differential

tropical equations as introduced by D.Grigoriev, and F.Aroca-C.Garay-Z.Toghani to encompass the non-trivially valued case, introduce a differential tropicalization functor and prove an inverse limit theorem in a similar fashion to S.Payne and J.H.Giansiracusa & N.Giansiracusa.

Florian Viguier
Fourier-Mukai transform for formal schemes. In 1981, Mukai constructed the Fourier-Mukai trans-

form for abelian varieties over an algebraically closed field, which gives an equivalence of categories between quasi-coherent sheaves over A and the ones over its dual variety. Laumon generalized these results for abelian varieties over a locally noetherian basis. One can then ask the following question : can these results be generalized even more ? What about formal abelian varieties ? And abelian rigid analytic varieties ? The generalization of the Fourier-Mukai transform's construction is based on a simple idea : make the classical construction commute with (derived) inverse limit. Even if the idea seems simple, it implies to clearly understand quasi-coherent sheaves and functors defined over formal varieties. When the formal Fourier-Mukai transform is constructed and its fundamental results has been proved, one can then obtain these results over its generic fiber.

Canceled due to Covid-19 pandemic

– Vladimir Berkovich
– Bruno Chiarellotto
– Angelica Cueto
– Diane Maclagan
– Walter Gubler
– Samuel Payne
– Peter Schneider
– Simon Wadsley