$\boxed{\text{Ex23}}$

a) $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-\sqrt{1-x}}{x} = \lim\limits_{x\to 0} \dfrac{(\sqrt{1+x}-\sqrt{1-x})(\sqrt{1+x}+\sqrt{1-x})}{x(\sqrt{1+x}+\sqrt{1-x})} =$

$= \lim\limits_{x\to 0} \dfrac{1+x-(1-x)}{x(\sqrt{1+x}+\sqrt{1-x})} = \lim\limits_{x\to 0} \dfrac{2x}{x(\sqrt{1+x}+\sqrt{1-x})} = \dfrac{2}{\sqrt{1+0}+\sqrt{1-0}} = 1$

b) $\lim\limits_{x\to 0+} \dfrac{\ln(3x+1)}{2x} \underset{\text{l'Hôpital}}{=} \lim\limits_{x\to 0+} \dfrac{(\ln(3x+1))'}{(2x)'} = \lim\limits_{x\to 0+} \dfrac{3}{3x+1}\cdot\dfrac{1}{2} = \dfrac{3}{2}$

d) $\lim\limits_{x\to 0} x\cdot\sin\dfrac{1}{x} = 0$, car $\underset{\downarrow \atop 0}{-|x|} \le x\cdot\sin\dfrac{1}{x} \le \underset{\downarrow \atop 0}{|x|}$ $\left(-1\le\sin\dfrac{1}{x}\le 1\right) \Rightarrow$

$x\sin\dfrac{1}{x} \xrightarrow[x\to 0]{} 0$

e) $\lim\limits_{x\to +\infty} \dfrac{x\,e^{\cos x}}{x^2+1} = 0$, car $\underset{\downarrow \atop 0}{0} < \dfrac{x\cdot e^{\cos x}}{x^2+1} \le \underset{\downarrow \atop 0}{\dfrac{x\cdot e^{1}}{x^2+1}}$ $(\cos x \le 1) \Rightarrow$

$\dfrac{x\,e^{\cos x}}{x^2+1} \xrightarrow[x\to +\infty]{} 0$

f) $\lim\limits_{x\to +\infty} (x\ln x - x\ln(x+2)) = \lim\limits_{x\to +\infty} x\cdot\ln\dfrac{x}{x+2} = \lim\limits_{x\to +\infty} \dfrac{\ln\frac{x}{x+2}}{\frac{1}{x}} =$

$\underset{\text{l'Hôpital}}{=} \lim\limits_{x\to +\infty} \dfrac{\left(\ln\frac{x}{x+2}\right)'}{\left(\frac{1}{x}\right)'} = \lim\limits_{x\to +\infty} \dfrac{\frac{x+2}{x}\cdot\frac{2}{(x+2)^2}}{-\frac{1}{x^2}} = \lim\limits_{x\to +\infty} \dfrac{-2x^2}{x(x+2)} =$

$= \lim\limits_{x\to +\infty} \dfrac{-2x^2}{x^2+2x} = \lim\limits_{x\to +\infty} \dfrac{-2}{1+\frac{2}{x}} = -2$

g) $\lim\limits_{x\to 0} \dfrac{|x|}{x}$? Si $x>0$ $|x|=x \Rightarrow \lim\limits_{x\to 0+} \dfrac{|x|}{x} = \lim\limits_{x\to 0+} \dfrac{x}{x} = 1$

Si $x<0$ $|x|=-x \Rightarrow \lim\limits_{x\to 0-} \dfrac{|x|}{x} = \lim\limits_{x\to 0-} \dfrac{-x}{x} = -1.$

h) $\lim\limits_{x\to 2} \dfrac{x^2-5x+6}{x-2} \underset{\text{l'Hôpital}}{=} \lim\limits_{x\to 2} \dfrac{(x^2-5x+6)'}{(x-2)'} = \lim\limits_{x\to 2} \dfrac{2x-5}{1} = -1$

i) $\lim_{x\to 3} \frac{x-3}{\sqrt{x}-\sqrt{3}} = \lim_{x\to 3} \frac{(\sqrt{x}-\sqrt{3})(\sqrt{x}+\sqrt{3})}{\sqrt{x}-\sqrt{3}} = \lim_{x\to 3}(\sqrt{x}+\sqrt{3}) = 2\sqrt{3}$

j) $\lim_{x\to +\infty} \frac{4x^5+4x^3+x^2}{-3x^2+50x} = \lim_{x\to+\infty} \frac{x^5\left(4+\frac{4}{x^2}+\frac{1}{x^3}\right)}{x^2\left(-3+\frac{50}{x}\right)} = -\infty$

$\downarrow +\infty \qquad \downarrow -\frac{4}{3}$

k) $\lim_{x\to 0} \frac{4x^5+4x^3+x^2}{-3x^2+50x} = \lim_{x\to 0} \frac{x^2(4x^3+4x+1)}{x(-3x+50)} = \lim_{x\to 0} \frac{x(4x^3+4x+1)}{-3x+50} = 0$

l) $\lim_{x\to 0} \frac{x^2+|x|}{x}$? Si $x>0$ $|x|=x$ $\Rightarrow \lim_{x\to 0+} \frac{x^2+|x|}{x} = \lim_{x\to 0+} \frac{x^2+x}{x} =$

$= \lim_{x\to 0+} \frac{x(x+1)}{x} = \lim_{x\to 0+}(x+1) = 1.$

Si $x<0$ $|x|=-x \Rightarrow \lim_{x\to 0-} \frac{x^2+|x|}{x} = \lim_{x\to 0-} \frac{x^2-x}{x} = \lim_{x\to 0-} x-1 = -1.$

m) $\lim_{x\to+\infty}(\sqrt{x+3}-\sqrt{x+2}) = \lim_{x\to+\infty} \frac{(\sqrt{x+3}-\sqrt{x+2})(\sqrt{x+3}+\sqrt{x+2})}{\sqrt{x+3}+\sqrt{x+2}} =$

$= \lim_{x\to+\infty} \frac{x+3-(x+2)}{\sqrt{x+3}+\sqrt{x+2}} = \lim_{x\to+\infty} \frac{1}{\sqrt{x+3}+\sqrt{x+2}} = \frac{1}{+\infty} = 0$

o) $\lim_{x\to 0+} \frac{x+2}{x^2 \ln x} = \lim_{x\to 0+}(x+2) \cdot \lim_{x\to 0+} \frac{1}{x^2 \ln x} = 2 \cdot \frac{1}{0-} = -\infty$

car $\lim_{x\to 0+} x^2 \ln x = 0_-$