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Homeomorphic non-diffeomorphic surfaces with small invariants

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1. Introduction

A compact complex algebraic surface, for short *surface*, has a natural structure of an oriented differentiable four-manifold. For simply connected surfaces Freedman's fundamental result [5] implies that the topological type is completely determined by $H^2(S,\mathbb{Z})$ together with its intersection form $q_S \in \mathrm{Sym}^2 H_2(S,\mathbb{Z})$. Hirzebruch's signature formula $1/3(c_1^2(S) - 2c_2(S))$ shows that in addition to c_2 also c_1^2 is a topological invariant.

Recall that a unimodular indefinite lattice is classified by its parity, rank and signature. If S has an almost complex structure, Wu's formula implies that q_S is even if $c_1(S)$ is 2-divisible in $H^2(S,\mathbb{Z})$ and odd otherwise. It follows that for a simply connected complex surface S with indefinite intersection form, the Chern numbers together with the divisibility of c_1 mod 2 fix the topological type of S.

Of crucial importance is to find some differentiable invariant which is finer than a topological invariant. A natural candidate for invariance is the canonical class. However, if S is a non-minimal surface, the exceptional curve E defines a reflection σ_E in cohomology ($\sigma_E(x) = x + 2q_S(x,E)E$) which is known to be realizable as a diffeomorphism. But c_1 is sent to $c_1 - 2E$ and therefore is not preserved. For this reason we only consider minimal surfaces in the sequel.

Using the Donaldson polynomials [2], Friedman, Morgan and Moishezon [6] showed that for certain types of minimal surfaces the canonical class c_1 is indeed a differentiable invariant up to sign. These surfaces however form a very limited class.

Our contribution consists of two parts.

First we show that by a combination of Lefschetz theory and an elementary number theoretic observation we can compare surfaces whose c_1 is known to be invariant by [6] with surfaces in products of projective spaces and weighted projective three space, thereby enlarging considerably the class of surfaces to look for examples.

Secondly, we observe that invariants of simply connected algebraic surfaces can be produced in prodigious numbers by considering complete intersections in multiprojective spaces. In particular there will be many types of surfaces crowded into slots with relatively small invariants. Although this observation may seem inane, it has provided the major motivation for the work of this paper, and actually subsumed most of it as well in the time-consuming process of actual programming, a tiny portion of its output being added to the end of the paper.

Using the list and the first observation we find indeed several dozens of small invariants which occur at least for two homeomorphic surfaces which cannot be diffeomorphic. The second observation also shows that this phenomenon is quite common in the world of surfaces and occurs for many -even small- invariants, contrary to what one might expect from [3], [10] or [13]. For the sake of record-hunters, of course the lowest world record of invariants is $(c_1, c_1) = 0$ and $p_g = 0$ and is realised by the Dolgachev surface and the 9-fold blown up plane. For pairs one of which is of general type the record is $(c_1, c_1) = 1$ and $p_g = 0$ and is realised by the Barlow surface and the 8-fold blown up plane. See [8]. The rest of the paper exclusively deals with pairs of surfaces of general type.

In [3] Ebeling uses this method for complete intersections, in [10] it is applied to repeated triple covers of $\mathbb{P}^1 \times \mathbb{P}^1$ and in [13] to repeated double covers of $\mathbb{P}^2$. The examples found have large characteristic numbers $c_1^2(S)$ and $\chi(S)$. Ebeling has examples with $c_1^2 = 2^2 \cdot 3^9 \cdot 5 \cdot 7^2 = 19,289,340$ (these have been discovered independently before him by Libgober and Wood [9]). The lowest first Chern class a surface in [10] has is $2^5 \cdot 3^6 \cdot 11^2 = 2,822,688$. Salvetti's examples have Chern numbers starting in a smaller range of magnitude $c_1^2 = 5^2 \cdot 3^4 = 2,025$. Our surfaces have very small Chern numbers. They are all complete intersections in multiprojective spaces or smooth hypersurfaces in weighted projective spaces.

Our list of genuine examples is at the moment rather small, but can be extended to all of the pairs in the tables at the end of this note if indeed the canonical class would be a differentiable invariant up to sign. In fact, our list of examples would be too big to be published, and we may even venture to guess that almost all Chern-invariants occur as examples of the phenomena of homeomorphic but non-diffeomorphic surfaces. In any case, the list contains the beginning of at least one infinite sequence of examples. This was mentioned to us by Kotschick. See Remark 2.

On the topological side, we note that the fact that the invariants are small is potentially of interest for topologists since they could try to explicitly find some standard topological operation which passes from one member of a pair in the table to the other one, like the logarithmic transformations in case of the elliptic surfaces. Also, it would be interesting to know whether both members of a pair have a big diffeomorphism group. It seems to us that this question is not likely to be resolved by algebro-geometric means only.

If one is willing to leave the realm of complex surfaces, the recent results of Fintushel and Stern [7] imply that many of the examples of surfaces with big diffeomorphism groups have infinitely many different differentiable structures. But the exotic structures are probably all non-algebraic. It remains a challenging problem to determine how our examples fit in with theirs. Their constructions are of a differential topological nature and there is a priori no way of comparing the examples.

Last, but not least, we want to thank the various mathematicians whose advice has benefited us greatly. Foremost of those is of course D. Kotschik, who not only supplied the examples of Remark 2 and kindly allowed us to present them in this paper; but also read through the manuscript (although slips and mistakes still remain our sole responsibility) and contributed many a comment that saved us from potential embarrassment. Conversations with R. Stern have also been of great value. Finally we also like to thank C. T. C. Wall for supplying some references.

2. Big diffeomorphism groups

The classical way of obtaining diffeomorphisms of an algebraic surface is to look at those induced by monodromy. Thus if $S = \mathcal{S}_t$ sits as a fibre in a family

$$\mathcal{S} \to P$$

those cycles of S that extend to $\mathcal{S}$ will be invariant under monodromy (and under additional assumptions we may conclude that only those will be fixed). In particular we see that the canonical class is preserved. At this point it may be worthwhile to recall some facts from Lefschetz theory dealing with this situation.

Let $Y \subset \mathbb{P}^m$ be a smooth algebraic variety of dimension $2+r$ and consider all complete intersection surfaces of Y with r hypersurfaces of fixed degrees. These will be parametrised by some quasi-projective manifold M. Loops in M based at the point corresponding to S define the monodromy representation on $H^2(S)$. The fixed lattice by definition is $H^2_{\text{fixed}} = \operatorname{Im}(H^2(Y) \to H^2(S))$ and it is acted upon trivially by monodromy. Lefschetz theory shows that there is an orthogonal direct sum decomposition

$$H^2(S,\mathbb{Q}) = H^2_{\text{fixed}}(S,\mathbb{Q}) \oplus H^2_{\text{var}}(S,\mathbb{Q})$$

into *irreducible* modules for the monodromy representation. Moreover, the variable part does not contain any classes fixed by monodromy.

Assume now that $H^2(Y,\mathbb{Z})$ has rank one, that $c_1(S)^2 > 0$ and that p_g is odd. In this case the main result from [FMM, section 3] states that $c_1(S)$ is a differentiable invariant up to sign. together with earlier results from Ebeling and Beauville (see e.g. [4] for references) imply that $c_1(S)$ is a differentiable invariant up to sign.

A much more detailed analysis actually reveals that one can dispense with the condition p_g even under some additional hypothesis as we show now. If p_g is odd so that the degree $d(k)$ of the Donaldson polynomials φ_k is even, O'Grady in [12] defines polynomials of degree $d(k)$ and shows that the coefficient of $q_S^{d(k)/2-1}c_1(S)$ is positive as soon as the surface contains a base point free pencil of curves of genus > 2. The main result of Morgan from [11] implies that the Donaldson-polynomials can be computed using Donaldson's prescription, but now carried out on Gieseker's compactification. But this is exactly the way O'Grady defined his polynomials. This implies as in [6] that c_1 is a differentiable invariant (up to sign). In case of odd p_g and odd c_1^2 one can use the $SO(3)$-polynomials instead as shown in [4]. See also [15].

We can summarize the main point

(2.1) **Proposition** *If a minimal surface S with positive p_g and with $c_1^2 > 0$ is a complete intersection surface in a projective manifold Y with $b_2(Y) = 1$, then $c_1(S)$ is invariant (up to sign) by any orientation preserving diffeomorphism in each of the following cases*

i. *p_g is even,*

ii. *p_g is odd and S has a base point free pencil of curves of genus > 2,*

iii. *p_g is odd and c_1^2 is odd.*

Note also that even in the case of big monodromy we cannot conclude that every diffeomorphism fixes (up to sign) every conceivable canonical class for algebraic realizations of the underlying differentiable manifold. But proposition 2.1 will allow us to use the following Lemma.

(2.2) **Lemma** *Suppose that S and S' are two minimal simply connected surfaces of general type with $p_g(S)$ and $p_g(S')$ positive. Suppose $c_1(S)$ is invariant up to sign under the group of diffeomorphisms of S. Let $\Lambda' \subset H^2(S',\mathbb{Z})$ be the largest sublattice acted upon trivially by a group of diffeomorphisms of S'. Write $c_1(S) = \gamma \cdot c$ with c primitive. If there is no primitive element $c' \in \Lambda'$ for which $(c', c') = (c, c)$, then S and S' are not oriented diffeomorphic.*

Proof Suppose that there is an oriented diffeomorphism $f : S' \to S$. Then $c' = f^*(c_1(S))$ is left invariant by all oriented diffeomorphisms of S' and hence in particular it sits in the sublattice Λ'. But this contradicts our assumptions.

Let us apply this to the case of the preceding proposition. We find:

(2.3) **Corollary** *Let S be a minimal simply connected surface with $c_1^2(S) > 0$ and $p_g(S)$ positive. Assume that $c_1(S)$ is invariant (up to sign) under the group of diffeomorphisms. Let S' be a simply connected surface which is the complete intersection of very ample divisors of a smooth projective variety. Write $c_1(S) = \gamma \cdot c$ with c primitive. Suppose that $H^2_{\text{fixed}}(S')$ does not contain a primitive element c' with $(c', c') = (c, c)$. Then S and S' are not oriented diffeomorphic.*

Remark 1. In particular, if $b_2(Y) = 1$, the statement reduces to the assertion that the divisibility of the first Chern classes being distinct implies that the surfaces cannot be diffeomorphic. This is the situation of [FMM, section 3].

3. Homeomorphic non-diffeomorphic surfaces

We are going to apply Corollary 2.3. The candidate surfaces for S will be complete intersections of n hypersurfaces in projective space $\mathbb{P}^{n+2}$ or surfaces of degree dpq in weighted projective space of type $\mathbb{P}(1, 1, p, q)$ with p and q relatively prime. The particular kind of weighted projective surfaces are smooth and the usual Lefschetz theory applies by [14]. Work of Cox in [1¿] indicates that this can also be done for more general weighted hypersurfaces, but as those involve the resolution of singularities to compute the Chern numbers we have not bothered to treat those.

The candidates for S' are of the same type or else they can be complete intersections of n hypersurfaces in multiple projective spaces $\mathbb{P}^{a_1} \times \mathbb{P}^{a_2} \times \ldots \times \mathbb{P}^{a_k}$ with $n+2 = \sum a_j$.

It is straightforward to compute c_1^2 and the divisibility of the canonical class. The intersection form on the fixed part is also easy to compute, using as generators the hyperplane classes of the factors of the multiprojective spaces. Finally, the geometric genus $p_g(S) = H^0(\mathcal{O}(K_S))$ can be computed from the standard Koszul-resolution of the structure sheaf of a complete intersection S by hypersurfaces of multidegrees $(d_{j1}, d_{j2}, \ldots, d_{jk})$, $j = 1, \ldots, n$. In the following tables we collected some potentially interesting examples, found by computer.

In the tables $\chi = p_g + 1$, type $(3)(p,q)$ means weighted projective space $\mathbb{P}(1,1,p,q)$ and $(a_1, \ldots, a_k)$ means a product $\mathbb{P}^{a_1} \times \mathbb{P}^{a_2} \times \ldots \times \mathbb{P}^{a_k}$. The multidegree of any hypersurface in such a product is given by a k-tuple $(d_1, \ldots, d_k)$ and they are collected in a n by k matrix. The last entry gives $q_S \mid H^2_{\text{fixed}}(S)$ and $q_S \mid H^2_{\text{fixed}}(S')$.

Observe that proposition 2.1 can be applied to the surfaces of table 2. See e.g. [O-G] for the complete intersection case. The weighted projective case is more subtle, but here only the examples 2, 3 and 5 have even c_1^2 and here a modification of the argument works.

Inspecting this table, Corollary 2.5 and a little elementary number theory shows:

(3.1) **Theorem** *The pairs of surfaces numbered 3, 5, 7, 10, 12, 17 and 23 in the first table and the first seven pairs in the second table are homeomorphic but not diffeomorphic.*

Remark 2. Kotschick observes that the pair numbered 3 occurs in an infinite family. One the one hand one has a surface of degree $3k+7$ in $\mathbb{P}^3$ and on the other hand we have a surface in $\mathbb{P}^1 \times \mathbb{P}^2$ of bidegree $(k+3, 3k+6)$. One can check that these give genuine examples of this kind, if $k \equiv 0, 2 \bmod 3$ and $k \not\equiv 3 \bmod 4$ (to ensure even p_g).

Remark 3. As observed before, all pairs in the tables would give genuine examples if it could be shown that c_1 is a differentiable invariant (up to sign), at least for the sort of surfaces we are considering here.

At the time of writing P. Kronheimer and T. Mrowka announce a proof of a form of a conjecture due to R. Thom:

Let S be a surface with p_g odd and such that $|K_S|$ contains a smooth curve of genus ≥ 2 and let C be any algebraic curve with $(C,C) > 0$. Then any compact differentiable surface differentiably embedded in S and homologous to C must have genus at least as big as the genus of C.

It seems reasonable to expect that the same methods give a slightly stronger statement where we allow C to be any effective divisor (where 'genus' now means arithmetic genus). If so, it is not hard to deduce from this that indeed c_1 (up to sign) is a differentiable invariants for all surfaces in the second list. Of course, if the condition "p_g odd" can be removed a similar remark applies to the first list.

no.	c_1^2	χ	div	type	degrees	form
1	25	11	5	(3)(5,3)	(15)	(1)
			1	(2,1,1)	$\begin{pmatrix}3&2&3\\1&1&0\end{pmatrix}$	$\begin{pmatrix}3&3&5\\3&0&3\\3&5&0\end{pmatrix}$
2	50	17	5	(3)(3,2)	(12)	(2)
			1	(3,1)	$\begin{pmatrix}3&6\\2&1\end{pmatrix}$	$\begin{pmatrix}14&6\\6&0\end{pmatrix}$
3	63	21	3	(3)	(7)	(7)
			1	(2,1)	(6,3)	$\begin{pmatrix}3&6\\6&0\end{pmatrix}$
4	75	25	5	(3)(4,1)	(12)	(3)
			1	(3,1)	$\begin{pmatrix}3&6\\2&1\end{pmatrix}$	$\begin{pmatrix}15&6\\6&0\end{pmatrix}$
5	81	25	9	(3)(5,4)	(20)	(1)
			1	(3,1)	$\begin{pmatrix}2&7\\3&0\end{pmatrix}$	$\begin{pmatrix}21&6\\6&0\end{pmatrix}$

Table of interesting Surfaces (p_g even).

no.	c_1^2	χ	div	type	degrees	form
6	98	37	7	(3)(10,1)	(20)	(2)
			1	(2,1,1)	$\begin{pmatrix} 2 & 8 & 2 \\ 2 & 5 & 0 \end{pmatrix}$	$\begin{pmatrix} 10 & 4 & 26 \\ 4 & 0 & 4 \\ 26 & 4 & 0 \end{pmatrix}$
7	108	31	3	(4)	$\begin{pmatrix} 6 \\ 2 \end{pmatrix}$	(12)
			1	(2,1)	(6,4)	$\begin{pmatrix} 4 & 6 \\ 6 & 0 \end{pmatrix}$
8	125	35	5	(3)(2,1)	(10)	(5)
			1	(3,1,1)	$\begin{pmatrix} 3 & 2 & 4 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 13 & 7 & 8 \\ 7 & 0 & 3 \\ 8 & 3 & 0 \end{pmatrix}$
9	147	43	7	(3)	(15)	(3)
			1	(2,1,1)	$\begin{pmatrix} 0 & 5 & 2 \\ 4 & 3 & 1 \end{pmatrix}$	$\begin{pmatrix} 11 & 8 & 20 \\ 8 & 0 & 0 \\ 20 & 0 & 0 \end{pmatrix}$
10	180	45	3	(5)	$\begin{pmatrix} 5 \\ 2 \\ 2 \end{pmatrix}$	(20)
			1	(2,2)	$\begin{pmatrix} 3 & 3 \\ 4 & 1 \end{pmatrix}$	$\begin{pmatrix} 3 & 15 \\ 15 & 12 \end{pmatrix}$
11	225	57	5	(3)	(9)	(9)
			1	(2,1,1)	$\begin{pmatrix} 3 & 2 & 5 \\ 1 & 3 & 1 \end{pmatrix}$	$\begin{pmatrix} 17 & 8 & 11 \\ 8 & 0 & 3 \\ 11 & 3 & 0 \end{pmatrix}$
12	225	61	15	(3)(10,3)	(30)	(1)
			1	(3,2)	$\begin{pmatrix} 2 & 5 \\ 0 & 3 \\ 3 & 0 \end{pmatrix}$	$\begin{pmatrix} 45 & 18 \\ 18 & 0 \end{pmatrix}$
13	242	61	11	(3)(5,2)	(20)	(2)
			1	(2,1,1)	$\begin{pmatrix} 1 & 6 & 2 \\ 3 & 5 & 1 \end{pmatrix}$	$\begin{pmatrix} 16 & 7 & 23 \\ 7 & 0 & 3 \\ 23 & 3 & 0 \end{pmatrix}$
14	289	71	17	(3)(6,5)	(30)	(1)
			1	(3,1,1)	$\begin{pmatrix} 2 & 7 & 0 \\ 2 & 3 & 3 \\ 1 & 2 & 0 \end{pmatrix}$	$\begin{pmatrix} 33 & 6 & 28 \\ 6 & 0 & 4 \\ 28 & 4 & 0 \end{pmatrix}$
15	294	71	7	(3)(2,1)	(12)	(6)
			1	(2,1)	$\begin{pmatrix} 1 & 7 & 1 \\ 2 & 3 & 2 \\ 2 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 34 & 8 & 34 \\ 8 & 0 & 4 \\ 34 & 4 & 0 \end{pmatrix}$
16	360	85	6	(3)	(10)	(10)
			2	(2,1)	(9,4)	$\begin{pmatrix} 4 & 9 \\ 9 & 0 \end{pmatrix}$
17	405	95	9	(3)(3,1)	(15)	(5)
			1	(3,1,1)	$\begin{pmatrix} 1 & 1 & 4 \\ 3 & 3 & 3 \\ 1 & 2 & 0 \end{pmatrix}$	$\begin{pmatrix} 45 & 15 & 12 \\ 15 & 0 & 3 \\ 12 & 3 & 0 \end{pmatrix}$
18	441	103	6	(3)(7,5)	(35)	(1)
			2	(2,2,1)	$\begin{pmatrix} 0 & 1 & 2 \\ 5 & 3 & 0 \\ 1 & 1 & 1 \end{pmatrix}$	$\begin{pmatrix} 9 & 21 & 8 \\ 21 & 10 & 5 \\ 8 & 5 & 0 \end{pmatrix}$

Table of interesting Surfaces (p_g even) (II)

no.	c_1^2	χ	div	type	degrees	form
19	441	105	21	(3)(9,4)	(36)	(1)
			1	(2,1,1)	$\begin{pmatrix}3&5&5\\1&4&1\end{pmatrix}$	$\begin{pmatrix}25&8&7\\8&0&3\\17&3&0\end{pmatrix}$
20	539	121	7	(3)	(11)	(11)
			1	(2,1,1)	$\begin{pmatrix}1&0&7\\5&3&0\end{pmatrix}$	$\begin{pmatrix}21&35&3\\35&0&3\\3&5&0\end{pmatrix}$
21	578	133	11	(3)(5,2)	(20)	(2)
			1	(2,1,1)	$\begin{pmatrix}3&7&4\\1&4&2\end{pmatrix}$	$\begin{pmatrix}30&10&19\\10&0&3\\19&3&0\end{pmatrix}$
22	625	125	5	(4)	$\begin{pmatrix}5\\5\end{pmatrix}$	(25)
			1	(2,1,1)	$\begin{pmatrix}3&3&2\\5&1&1\end{pmatrix}$	$\begin{pmatrix}5&13&18\\13&0&15\\18&15&0\end{pmatrix}$
23	648	141	6	(4)	$\begin{pmatrix}9\\2\end{pmatrix}$	(18)
			2	(2,1)	(9,6)	$\begin{pmatrix}6&9\\9&0\end{pmatrix}$
24	675	165	15	(3)(9,1)	(27)	(3)
			1	(2,1,1)	$\begin{pmatrix}1&7&0\\5&4&3\end{pmatrix}$	$\begin{pmatrix}21&3&39\\3&0&5\\39&5&0\end{pmatrix}$
25	700	147	5	(5)	$\begin{pmatrix}7\\2\\2\end{pmatrix}$	(28)
			1	(3,1,1)	$\begin{pmatrix}2&6&2\\1&1&3\\2&2&1\end{pmatrix}$	$\begin{pmatrix}64&18&20\\18&0&4\\20&4&0\end{pmatrix}$
26	729	161	27	(3)(7,6)	(42)	(1)
			1	(3,1,1)	$\begin{pmatrix}5&0&3\\1&3&1\\1&2&0\end{pmatrix}$	$\begin{pmatrix}25&8&25\\8&0&5\\25&5&0\end{pmatrix}$
27	841	185	29	(3)(9,5)	(45)	(1)
			1	((2,2,1))	$\begin{pmatrix}3&1&3\\6&3&2\\1&0&1\end{pmatrix}$	$\begin{pmatrix}3&26&3\\26&42&15\\3&15&0\end{pmatrix}$
28	845	185	13	(3)(4,1)	(20)	(5)
			1	(3,1,1)	$\begin{pmatrix}3&5&4\\1&4&3\\1&1&1\end{pmatrix}$	$\begin{pmatrix}61&16&20\\16&0&3\\20&3&0\end{pmatrix}$
29	900	177	5	(5)	$\begin{pmatrix}6\\3\\2\end{pmatrix}$	(36)
			1	(3,1,1)	$\begin{pmatrix}2&5&2\\5&1&0\\2&0&0\end{pmatrix}$	$\begin{pmatrix}4&20&54\\20&0&20\\54&20&0\end{pmatrix}$

Table of interesting Surfaces (p_g even) (III)

no.	c_1^2	χ	div	type	degrees	form
1	9	6	3	(3)(4,3)	(12)	(1)
			1	(4)	$\begin{pmatrix}3\\3\end{pmatrix}$	(9)
2	32	16	4	(3)(7,1)	(14)	(2)
			2	(1,1,1)	$(6\ \ 4\ \ 2)$	$\begin{pmatrix}0&2&4\\2&0&6\\4&6&0\end{pmatrix}$
3	36	14	3	(3)(2,1))	(8)	(4)
			1	(4,1)	$\begin{pmatrix}1&1\\3&3\\2&0\end{pmatrix}$	$\begin{pmatrix}12&6\\6&0\end{pmatrix}$
4	81	26	9	(3)(7,3))	(21)	(1)
			1	(4,1)	$\begin{pmatrix}3&3\\2&3\\1&1\end{pmatrix}$	$\begin{pmatrix}21&6\\6&0\end{pmatrix}$
5	128	46	8	(3)(11,1)	(22)	(2)
			4	(2,1,1)	$\begin{pmatrix}5&6&2\\2&0&0\end{pmatrix}$	$\begin{pmatrix}0&4&12\\4&0&10\\12&10&0\end{pmatrix}$
6	288	66	4	(4)	$\begin{pmatrix}6\\3\end{pmatrix}$	(18)
			2	(4,1)	$\begin{pmatrix}3&6\\2&0\\2&0\end{pmatrix}$	$\begin{pmatrix}24&12\\12&0\end{pmatrix}$
7	320	70	4	(4)	$\begin{pmatrix}5\\4\end{pmatrix}$	(20)
			2	(4,1)	$\begin{pmatrix}2&4\\3&2\\2&0\end{pmatrix}$	$\begin{pmatrix}32&12\\12&0\end{pmatrix}$
8	384	86	4	(5)	$\begin{pmatrix}6\\2\\2\end{pmatrix}$	(24)
			2	(3,1)	$\begin{pmatrix}6&3\\2&1\end{pmatrix}$	$\begin{pmatrix}12&12\\12&0\end{pmatrix}$
9	400	92	5	(4)	$\begin{pmatrix}8\\2\end{pmatrix}$	(16)
			1	(2,1,1)	$\begin{pmatrix}8&2&2\\1&1&1\end{pmatrix}$	$\begin{pmatrix}4&10&10\\10&0&8\\10&8&0\end{pmatrix}$

Table of interesting Surfaces (p_g odd)

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