Géométries – Geometrias

Université Joseph Fourier – IMPA

http://www-fourier.ujf-grenoble.fr/∼pberard/brfr09.html

Contents

1 Mini-courses at IMPA (April, 13 – 15)

Schedule: Check the last schedule on the "Evento" section of the site of IMPA.

Spectral problems on Riemannian manifolds

Pierre Bérard (Université Joseph Fourier, Grenoble)

1. Introduction to the spectrum of a closed Riemannian manifold using the variational characterization of eigenvalues.
2. Examples. Eigenvalue estimates (Cheng-Gromov, Cheeger, Faber-Krahn).
3. Heat kernel on a closed Riemannian manifold. Uniqueness, existence. Asymptotics. The asymptotics will be used in Part 4.
4. Embedding closed Riemannian manifolds in a Hilbert space using the eigenfunctions of the Laplacian or the heat kernel (joint work with G. Besson and S. Gallot, to be used in Besson's lectures). Applications.

Rigidity theorem and applications, from Mostow's theorem to the growth of groups

Gérard Besson, (CNRS Institut Fourier, Grenoble)

1. The starting point is an embedding into a Hilbert space using the eigenfunctions of the Laplacian (joint work with P. Bérard and S. Gallot). This will be addressed in more details in Pierre Bérard's course. It relies on the use of the heat kernel. When the time parameter goes to zero the embedding, in the compact case, converges to a point. However, in the non compact case, it converges to an embedding in a Hilbert space using the Poisson kernel. This is the starting point of the work by the author together with G. Courtois and S. Gallot. We shall finish this first lecture by presenting the situation and the problem.
2. In this lecture we shall give a simple and quick proof of Mostow's rigidity theorem for rank one locally symmetric spaces. This is an approach which differs from the original theory by G. D. Mostow which will be presented by H. Pajot.

3. Then we shall describe extension of the previous technique, which allows to construct maps between manifolds (one of which is locally symmetric) with nice properties with respect to the volume. This gives rise to a sharp inequality on the entropy of the geodesic flow, answering several conjectures by M. Gromov and A. Katok. In dimension three it should be compared to a result following from the work of G. Perelman on the geometrisation of 3-manifolds involving the scalar curvature.

4. This technique can be further modified to yield a construction which allows to describe the structure of the fundamental group of compact real hyperbolic manifolds. We show that if such a group is amalgamated, then the group on which the amalgamation is taken has to be big, in a sense related to the critical exponent of the Poincaré series. The complex case is still conjectural and shall be discussed.

5. Finally, another modification of the technique, allows to get results on the growth and uniform growth of certain groups. The context shall be described as well as open questions.

Quasiconformal geometry and Mostow rigidity

Hervé Pajot (Université Joseph Fourier, Grenoble)

A famous theorem of Mostow states that if Γ_1, Γ_2 are two lattices in $\mathrm{Isom}(\mathbb{H}^n)$ (the group of isometries of the hyperbolic n-space), then any isomorphism $\phi : \Gamma_1 \to \Gamma_2$ is a conjugation by an element of $\mathrm{Isom}(\mathbb{H}^n)$. In these lectures, we will present an adaptation of the original proof of Mostow due to Gromov and Pansu. This proof is based on tools from quasiconformal geometry that we will describe in a very general setting. This will allows us to extend Mostow rigidity result to non compact symmetric spaces of rank one (for instance, quaternionic hyperbolic spaces) or to Fuchsian buildings.

1. We will discuss some tools from geometric analysis: Modulus of a curve family, conformal capacity, the Loewner condition and the Poincaré inequality in metric measure spaces.

2. We will define quasiconformal/quasisymmetric homeomorphims and describe their properties in metric spaces with bounded geometry. We will also prove the Rademacher-Stepanov theorem (about differentability of quasiconformal mappings) in $\mathbb{R}^n$.

3. We will prove the Efremovich-Tihomirova theorem: Any isometry of $\mathbb{H}^n$ extends to a quasiconformal homeomorphism on the boundary $\partial\mathbb{H}^n = S^{n-1}$ of the hyperbolic space. Extensions of this result to general Gromov hyperbolic spaces will be also discussed.

4. We will use results of the previous lectures to prove the theorem of Mostow and also related results due to Sullivan and Tukia.

5. We will discuss Mostow type results in the setting of symmetric spaces (Pansu) and Fuchsian buildings (Bourdon-Pajot).

2 Conferences at IMPA (Thursday April, 16)

Schedule: Check the last schedule on the "Evento" section of the site of IMPA.

Entire curves and algebraic differential equations

Jean-Pierre Demailly (Université Joseph Fourier, Grenoble)

Summary: The goal of the talk will be to present some basic facts concerning non constant entire holomorphic maps from the complex line into a complex algebraic variety X, namely a subvariety of complex projective space defined by a collection of homogeneous polynomials. In dimension 1, these varieties are compact complex curves (i.e. compact Riemann surfaces). They are classified according to their genus. When $g = 0$, the curve is biholomorphic to the Riemann sphere and has positive curvature, when $g = 1$ we have an elliptic curve (zero curvature), and when $g > 1$, the curve is covered by the disk and can be equipped with a metric of constant negative curvature. In higher dimension, a fundamental question is to study geometric properties which are capable of distinguishing the various "parts" of different curvature signs. One such property is the existence of an entire curves into the given variety, i.e. holomomorphic maps defined over the whole complex line. Such a curve cannot exist when the curvature is too negative - this is a consequence of a well-known lemme due to Ahlfors. In case there are no such curves the variety is hyperbolic in the sense of Kobayashi and possesses a natural extension of the Poincaré metric. We will prove a recent criterion of a different nature: every entire curve must be a solution of certain global algebraic differential equations, which somehow generalize foliations to higher

order tangencies. The proof relies in an essential way on Nevanlinna theory in one variable. It has deep consequences in the study of projective algebraic varieties and also potentially in number theory.

Uniformization in several complex variables

Philippe Eyssidieux (Université Joseph Fourier, Grenoble)

Summary:

In this talk we will review the basic facts, theorems and conjectures concerning the universal covering space of a compact Kähler manifold. These problems can be studied effectively using Corlette-Simpson's non abelian Hodge Theory as a tool to analyze the linear representations of the fundamental group of the manifold.

Arithmetics under influence

Emmanuel Peyre (Université Joseph Fourier, Grenoble)

Summary: Exhibing strange relations between integers is as old as the notion of numbers. The identity $4^2 + 3^2 = 5^2$ was already used during antiquity to construct right angles with a rope. The Fermat equation

$$X^p + Y^p = Z^p$$

has fascinated many would-be mathematicians for 300 years.
With computers, it is possible to get quite surprising relations like

$$95800^4 + 217519^4 + 414560^4 = 422481^4$$

or

$$2682440^4 + 15365639^4 + 18796760^4 = 20615673^4$$

with which Noam Elkies disproved a long standing conjecture of Euler.

Therefore one would like to have criteria to determine from an explicit system of polynomial equations with integer coefficients:

1. Whether the system has solutions with integer coordinates;
2. How "small" these solutions can be;

3. How many such solutions one may find with coordinates smaller than a real number B.

The first question is known as Hilbert's tenth problem. It was proven to be not solvable by Matiyasevich: no algorithm can determine in finite time whether a system of polynomial equations is solvable over the integers.
On the bright side, it turned out that the set of solutions was under the influence of the geometry of the complex variety defined by the same equations. One of the most famous result in that direction is a theorem of Faltings: If the geometric genus of a complex curve, which may be seen as a Riemann surface, is bigger or equal to two, there exists only a finite numbers of points with rational coordinates on the curve.

The aim of this talk is to show on examples how differential geometry seems to control the integral solutions of diophantine equations.

3 Conferences at PUC-Rio (Friday April, 17)

Schedule: Check the last schedule on the site `http://www.mat.puc-rio.br/`

The differentiable sphere theorem (after Brendle-Schoen and Boehm-Wilking)

Gérard Besson (CNRS, Institut Fourier, Grenoble)

Summary: We shall describe the proof given by S. Brendle and R. Schoen of the so-called differentiable sphere theorem, which states that a quarter pinched positively curved Riemannian manifold is diffeomorphic to a sphere. The proof uses the Ricci flow and ideas developped by Ch. Boehm and B. Wilking. The talk is essentially the one given at the Bourbaki seminar (Paris, March 2009).

Quasi-conformal geometry and hyperbolic groups. The Cannon conjecture

Hervé Pajot (Université Joseph Fourier, Grenoble)

Summary: The Cannon conjecture could be stated as follows: Let G be a Gromov hyperbolic group whose boundary at infinity is homeomorphic to the standard 2-sphere S^2. Then, G has a geometric action on the hyperbolic space H^3. We will explain an approach of this conjecture through quasiconformal geometry.

Bibliography.
Bruce Kleiner, The asymptotic geometry of negatively curved spaces: uniformization, geometrization, and rigidity, Proceedings of the International Congress of Mathematicians (2006).
Peter Haissinsky, Géométrie quasiconforme, analyse au bord des espaces métriques hyperboliques et rigidités (d'après Pansu, Bourdon, Pajot, Bonk, Kleiner), Séminaire Bourbaki (2008).

Schedule: Check the last schedule on the "Evento" section of the site of IMPA.

4 Bolsas para uma temporada de pesquisa em Grenoble

Three followships for a 3-month research visit at Institut Fourier (Université Joseph Fourier – CNRS, Grenoble) and five fellowships to participate in the Summer school "Optimal transportation: Theory and applications" (Institut Fourier, Grenoble) will be offered.

- Três de bolsas (viagem e estadia de no máximo três meses) serão oferecidas para uma temporada de pesquisa em Grenoble entre Junho e Novembre de 2009. Candidaturas abertas para os alunos que participarão dos mini-cursos e das conferências.

- Cinco bolsas (viagem e estadia por três semanas) para participar da " École d'été de mathématiques 2009 ", 15 de junho a 3 de julho 2009. Programa
  `http://www-fourier.ujf-grenoble.fr/-2009-.html`

  Candidaturas abertas para alunos de doutorado e pesquisadores (com prioridade para jovens pesquisadores), possivelmente alunos avançados de mestrado.

Candidaturas : Contactar o Prof. Pierre Bérard até 30 de Abril de 2009. Mandar uma carta de motivação, uma carta de recomendação, um CV e um histórico escolar (em inglês ou francês).
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"França.Br 2009" l'année de la France au Brésil (21 avril - 15 novembre) est organisée : En France, par le Commissariat général français, le Ministère des Affaires étrangères et européennes, le Ministère de la Culture et de la communications et Culturesfrance. Au Brésil, par le Commissariat général brésilien, le Ministère de la culture et le Ministère des Relations extérieures.

"França.Br 2009" Ano da França no Brasil (21 de abril a 15 de novembro) é organizada : Na França, pelo Commissariado geral francês, pleo Ministério das Relações exteriores e européias, pleo Ministério da Cultura e da comunicação e por Culturesfrance. No Brasil, pelo Comissariado geral brasileiro, pelo Ministério da Cultura e pelo Ministério das Relações exteriores.