

>

## Cobweb (diagramme en toile d'araignée)

Toile d'araignée de dynamique logistique avec  $r=1$

```
> #####  
Conditions initiales f,x0  
restart: N:=10:t0:=0:x0:=0.5:  
Digits:=4:t1:=3: h:=(t1-t0)/N:  
#####  
f:=Y*(1-Y):G:=f-Y:  
#####  
#####  
#####  
x := array(1..N+1):  
nn := array(1..N+1):  
t := array(1..N+1):  
Y(1):=x0: T(1):=t0:  
nn[1]:=0:  
t[1]:=T(1):  
x[1]:=Y(1):  
fn:=eval(G, [T=T[1],Y=T(1)]):  
for n from 1 to N do  
T(n+1):= T(n)+h; fn:=eval(G,  
[Y=Y(n),T=T(n)]);  
Y(n+1):=eval(f, [Y=Y(n),T=T(n)]);  
t[n+1]:=T(n+1);  
x[n+1]:=Y(n+1);  
nn[n+1]:=n;  
end do:
```

```

#####
#####
with(plots) :
pointsx:= [seq([nn[n], x[n]], n=1..N+1)]:
R1:=plot(pointsx, style=point,
labels=["n", "x"],
legend="calcul numérique de x[n]"):
R2:=plot(pointsx, color=blue,
labels=["n", "x"],
style=line, thickness=2,
title="Comportement
de solutions", scaling=unconstrained):
#####
with(plottools) :
F:=plot([f, Y], Y=-.2..1.2,
title="diagramme en toile d'araignée

f(x)=x(1-x), x0 = 1/2",
style=[line], color=[black, blue, red,
magenta, green],
labels=[Y, "f"]):
#####
#####
A1 := arrow([Y(1), 0], vector([0, Y(2)]),
.005, .02, .1,
color=red):
A2:=arrow([Y(1), Y(2)],
vector([Y(2)-Y(1), 0]), .005, .02, .1,
color=blue):

```

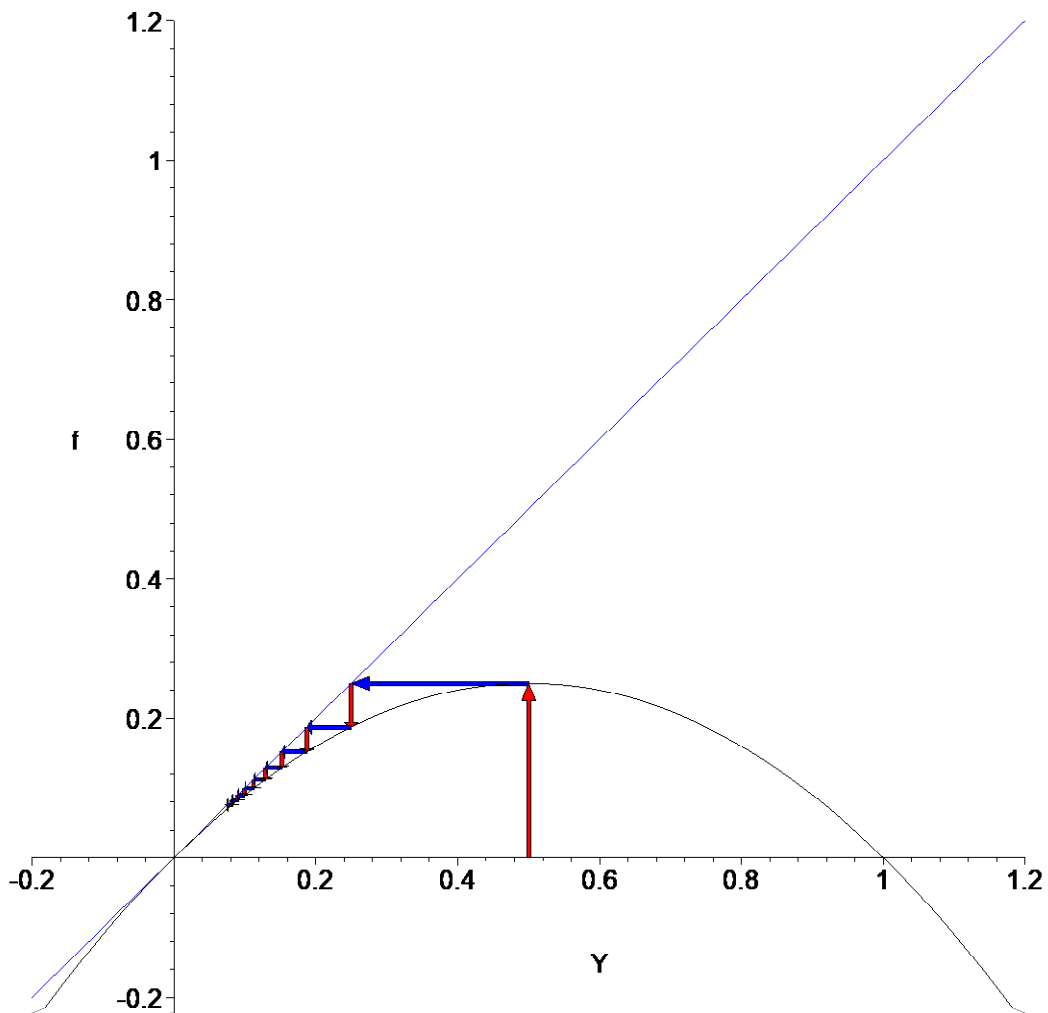
```

A3 := arrow([Y(2), Y(2)],
vector([0, Y(3)-Y(2)]), .005, .02, .1,
color=red):
#A4:=arrow([Y(2), Y(3)],
vector([Y(3)-Y(2), 0]), .005, .02,
.1, color=blue):
#####
A5:=seq(PLOT(arrow([Y(i), Y(i)],
vector([0, Y(i+1)-Y(i)]),
.005, .02, .1, color=red)), i=2..N-1):
A6:=seq(PLOT(arrow([Y(i), Y(i+1)],
vector([Y(i+1)-Y(i), 0]),
.005, .02, .1, color=blue)), i=2..N-1):
plots[display](F,
A1, A2, A3, A5, A6);
print("Les valeurs de [n, x_n]=",
pointsx, "points fixes=", solve(f=Y));
display(R1, R2);
> #####

```

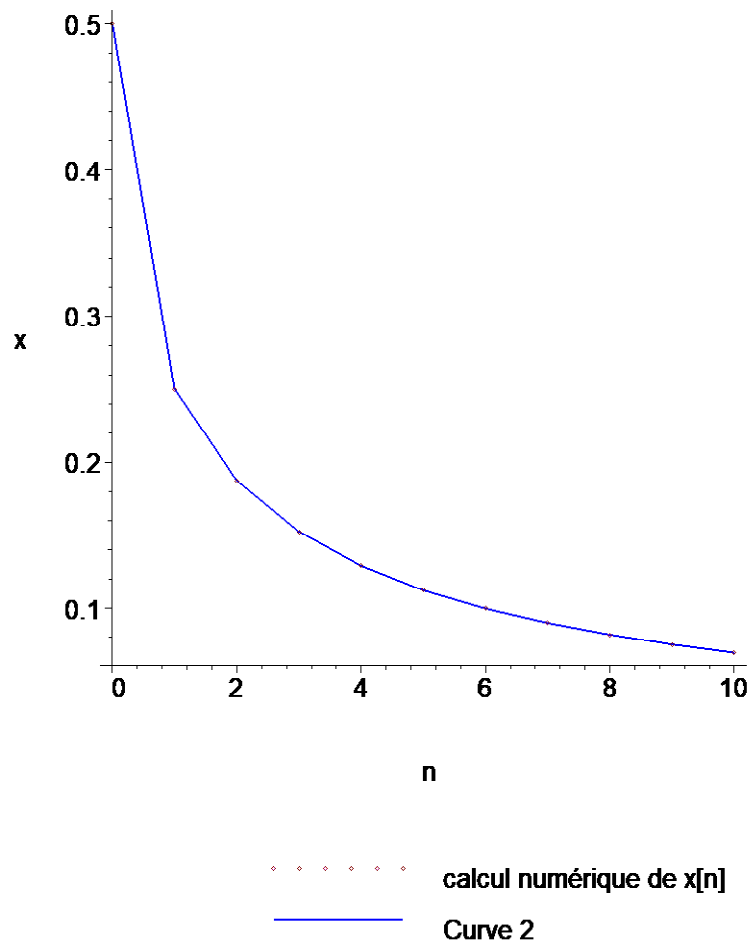
diagramme en toile d'araignée

$$f(x)=x(1-x), x_0 = 1/2$$



"Les valeurs de  $[n, x_n]=$ ",  $[[0, 0.5], [1, 0.25], [2, 0.1875],$   
 $[3, 0.1523], [4, 0.1291], [5, 0.1124], [6, 0.09977],$   
 $[7, 0.08981], [8, 0.08175], [9, 0.07506], [10, 0.06942]]$ ,  
"points fixes=",  $0, 0$

Comportement  
de solutions



## Cobweb (diagramme en toile d'araignée)

Toile d'araignée de dynamique logistique avec  $r = 3/2$

```
> #####  
Conditions initiales f,x0  
restart: N:=10:t0:=0:x0:=0.2:  
Digits:=4:t1:=3: h:=(t1-t0)/N:  
#####  
f:=(3/2)*Y*(1-Y):G:=f-Y:
```

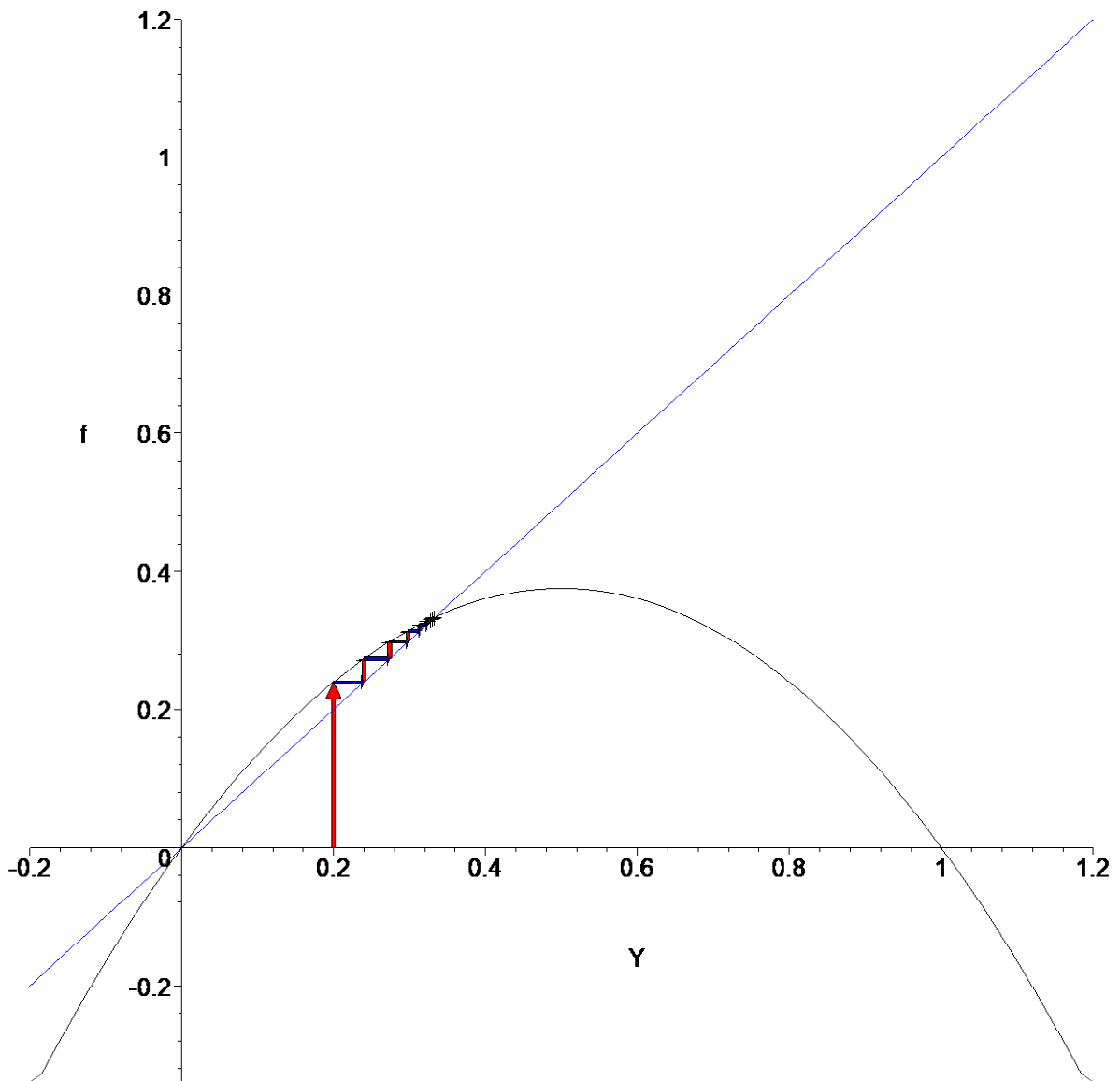
```

vector([Y(i+1)-Y(i), 0]),
.005, .02, .1, color=blue), i=2..N-1):
plots[display](F,
A1,A2,A3, A5,A6);
print("Les valeurs de [n, x_n]=",
pointsx, "points fixes=", solve(f=Y));
display(R1,R2);

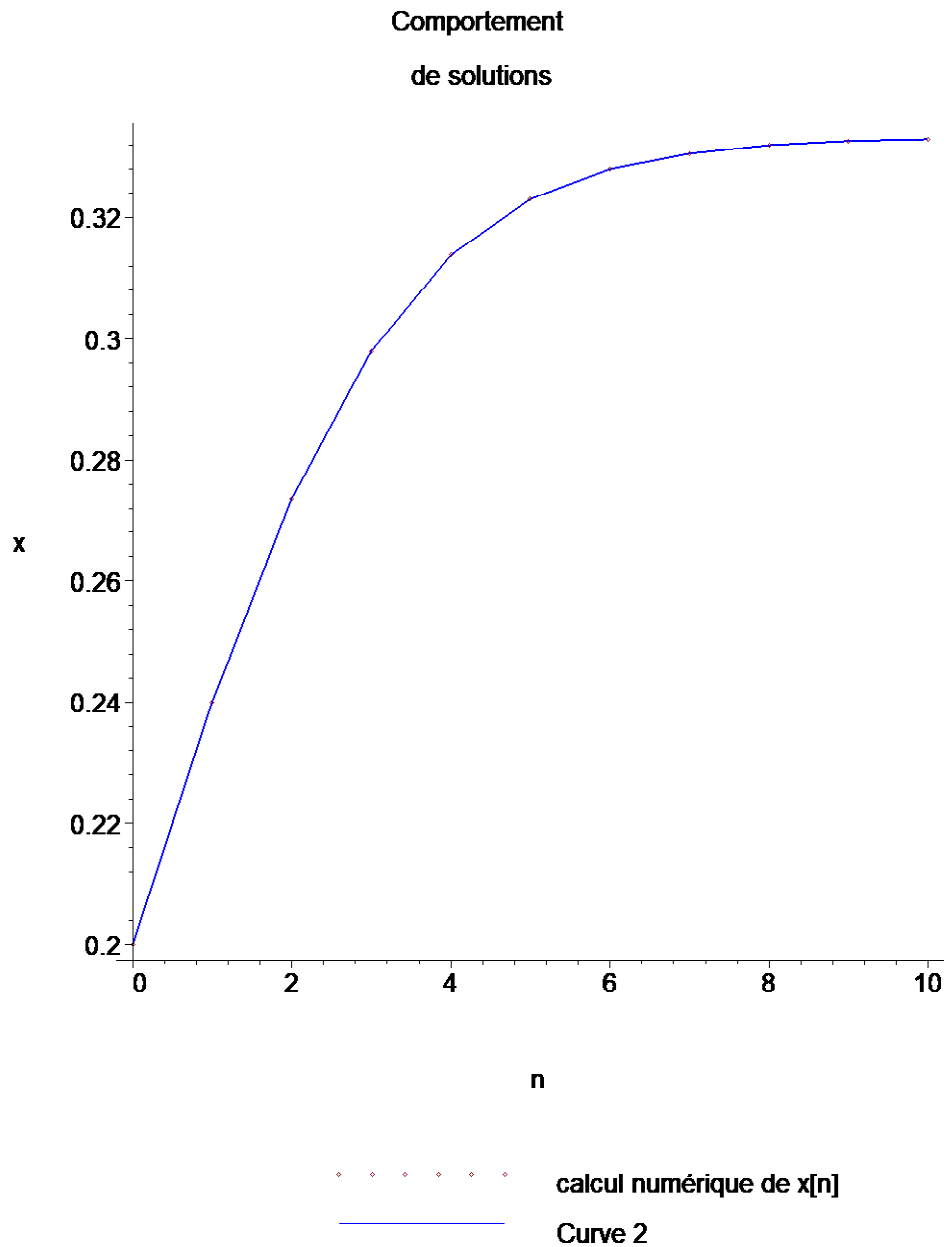
```

diagramme en toile d'araignée

$$f(x) = 1.5x(1-x), x_0 = 0.2$$



"Les valeurs de  $[n, x_n]=$ ",  $[[0, 0.2], [1, 0.2400], [2, 0.2736], [3, 0.2980], [4, 0.3138], [5, 0.3230], [6, 0.3280], [7, 0.3306], [8, 0.3320], [9, 0.3327], [10, 0.3330]]$ , "points fixes=",  $0, \frac{1}{3}$

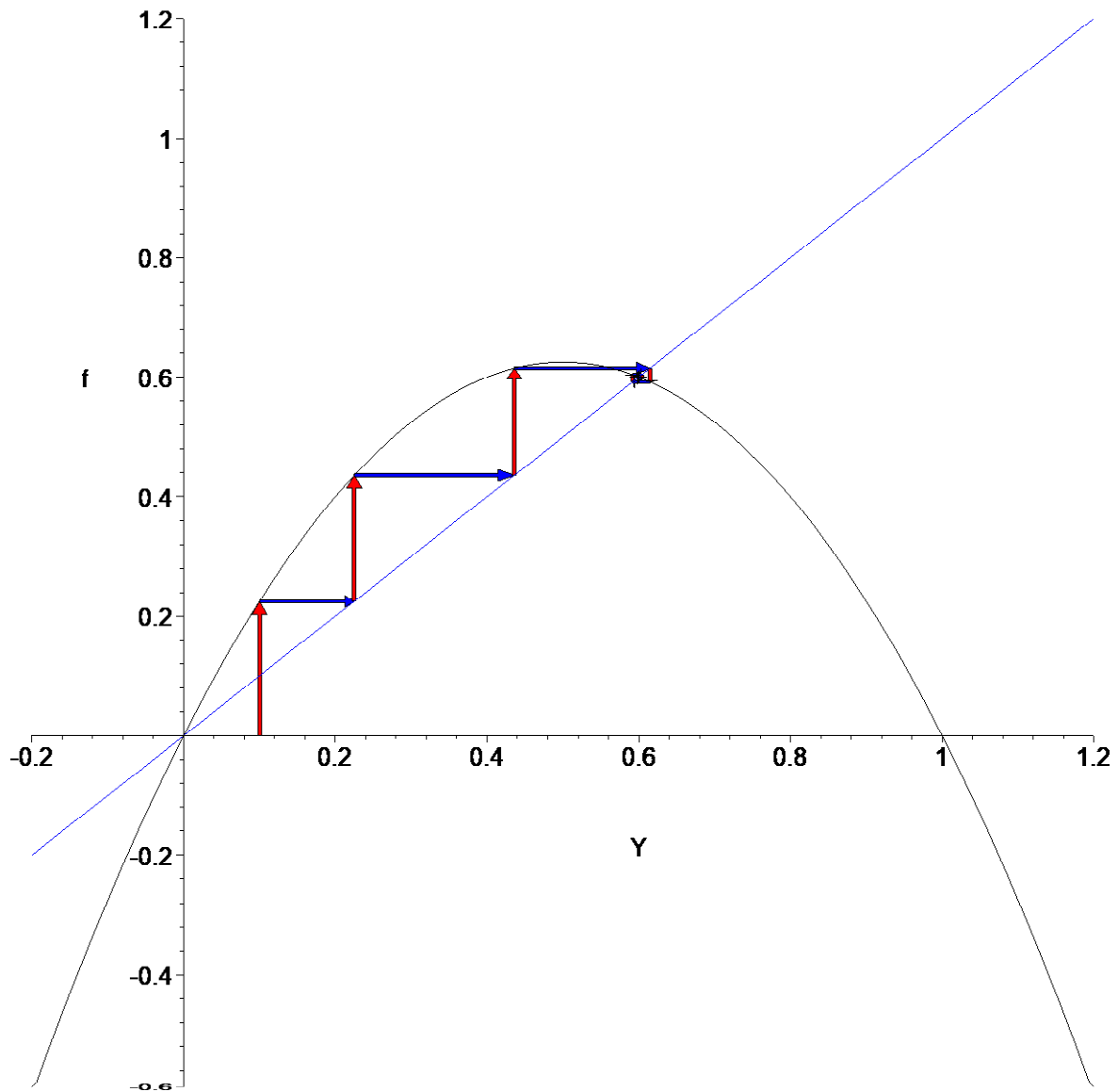


> #####

**Cobweb (diagramme en toile d'araignée)**

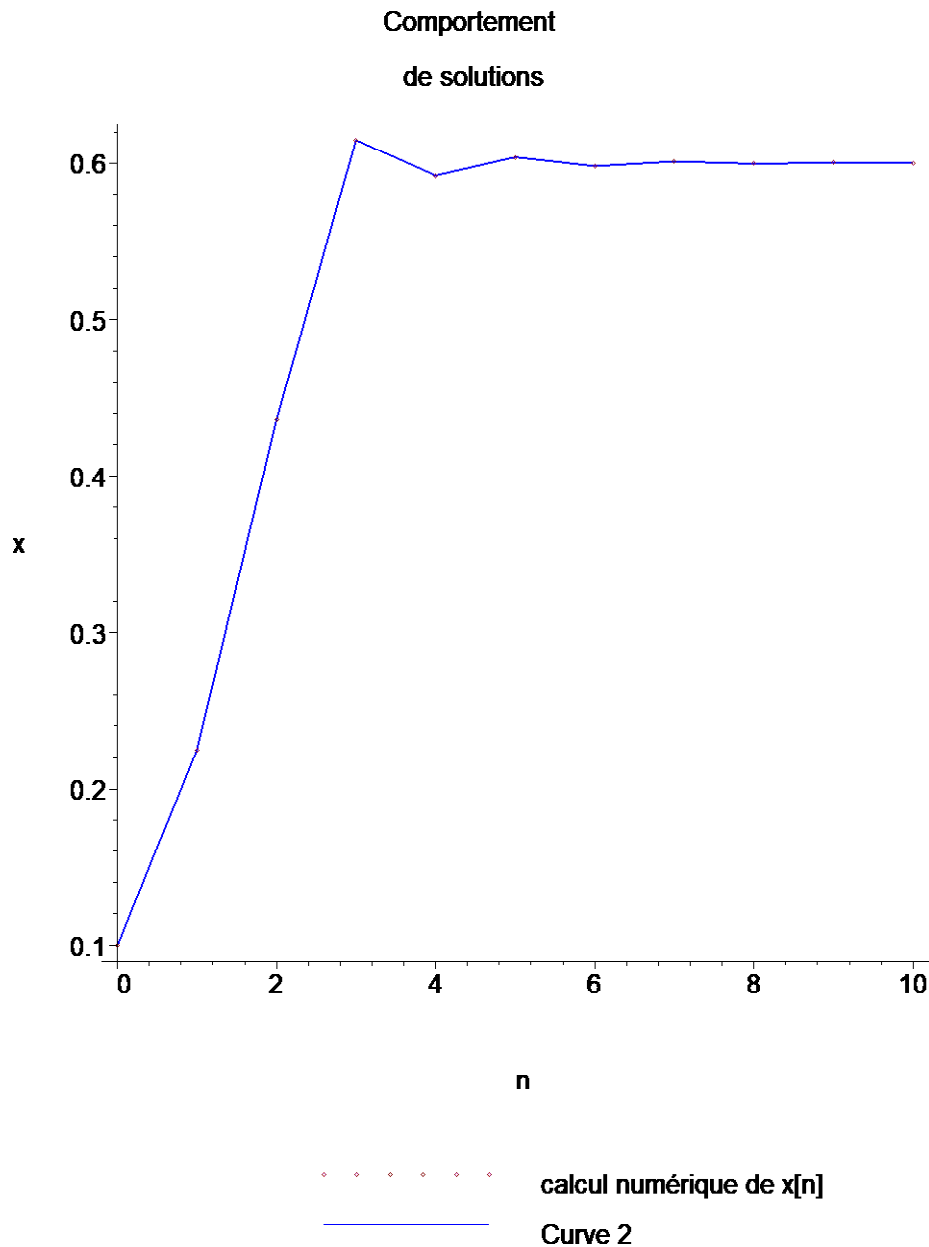
diagramme en toile d'araignée

$$f(x) = \frac{5}{2}x(1-x), x_0 = 0.1$$



"Les valeurs de  $[n, x_n]$ =",  $[[0, 0.1], [1, 0.2250], [2, 0.4360], [3, 0.6148], [4, 0.5920], [5, 0.6038], [6, 0.5980], [7, 0.6010], [8, 0.5995], [9, 0.6002], [10, 0.6000]]$ , "points fixes=",  $0, \frac{3}{5}$





> #####

## Cobweb (diagramme en toile d'araignée)

Toile d'araignée de dynamique logistique avec  $r = 5/2$

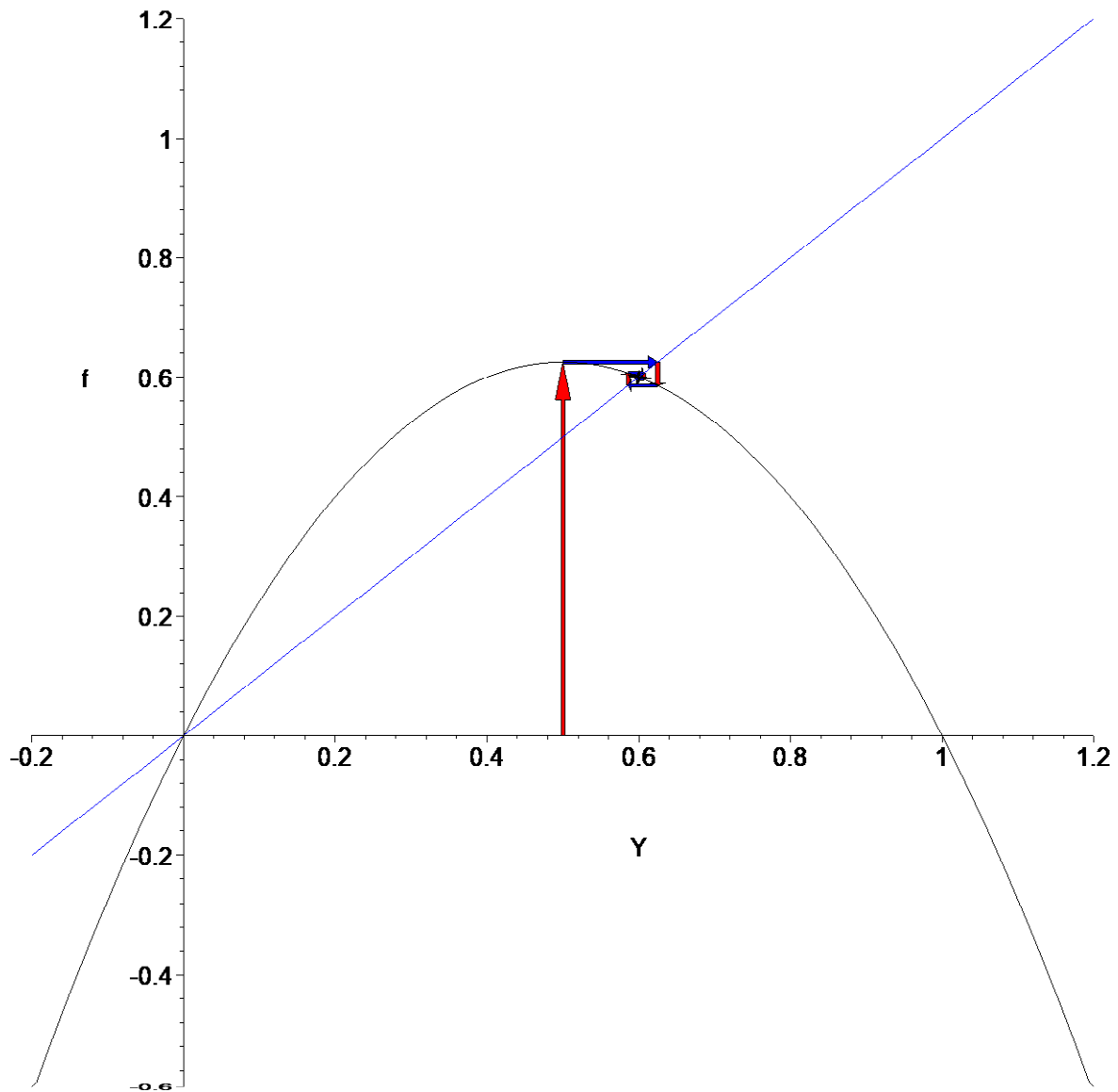
> #####

Conditions initiales f, x0

restart: N:=10:t0:=0:x0:=0.5:

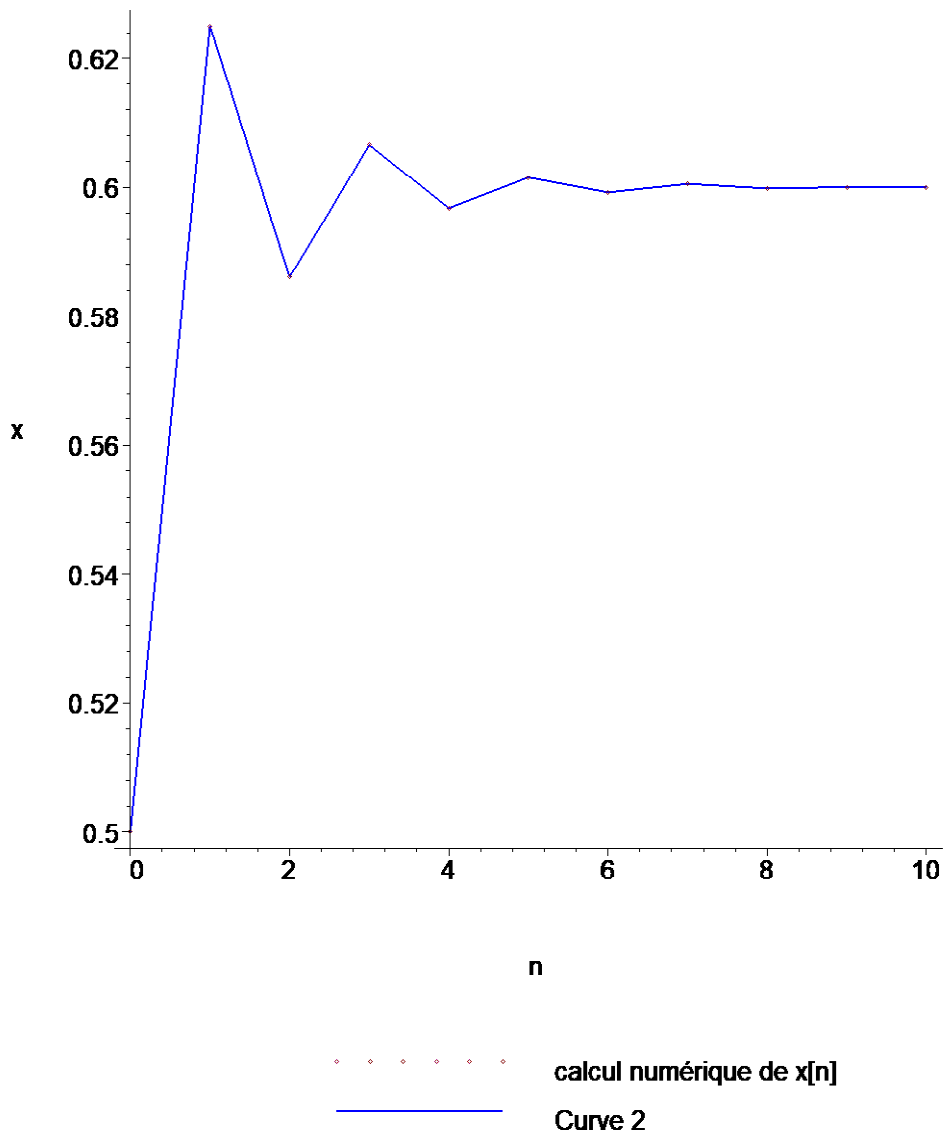
diagramme en toile d'araignée

$$f(x) = \frac{5}{2}x(1-x), x_0 = 0.5$$



"Les valeurs de  $[n, x_n]$ =",  $[[0, 0.5], [1, 0.6250], [2, 0.5860], [3, 0.6065], [4, 0.5968], [5, 0.6015], [6, 0.5992], [7, 0.6005], [8, 0.5998], [9, 0.6000], [10, 0.6000]]$ , "points fixes=",  $0, \frac{3}{5}$

Comportement  
de solutions



> #####

> #####

## Cobweb (diagramme en toile d'araignée)

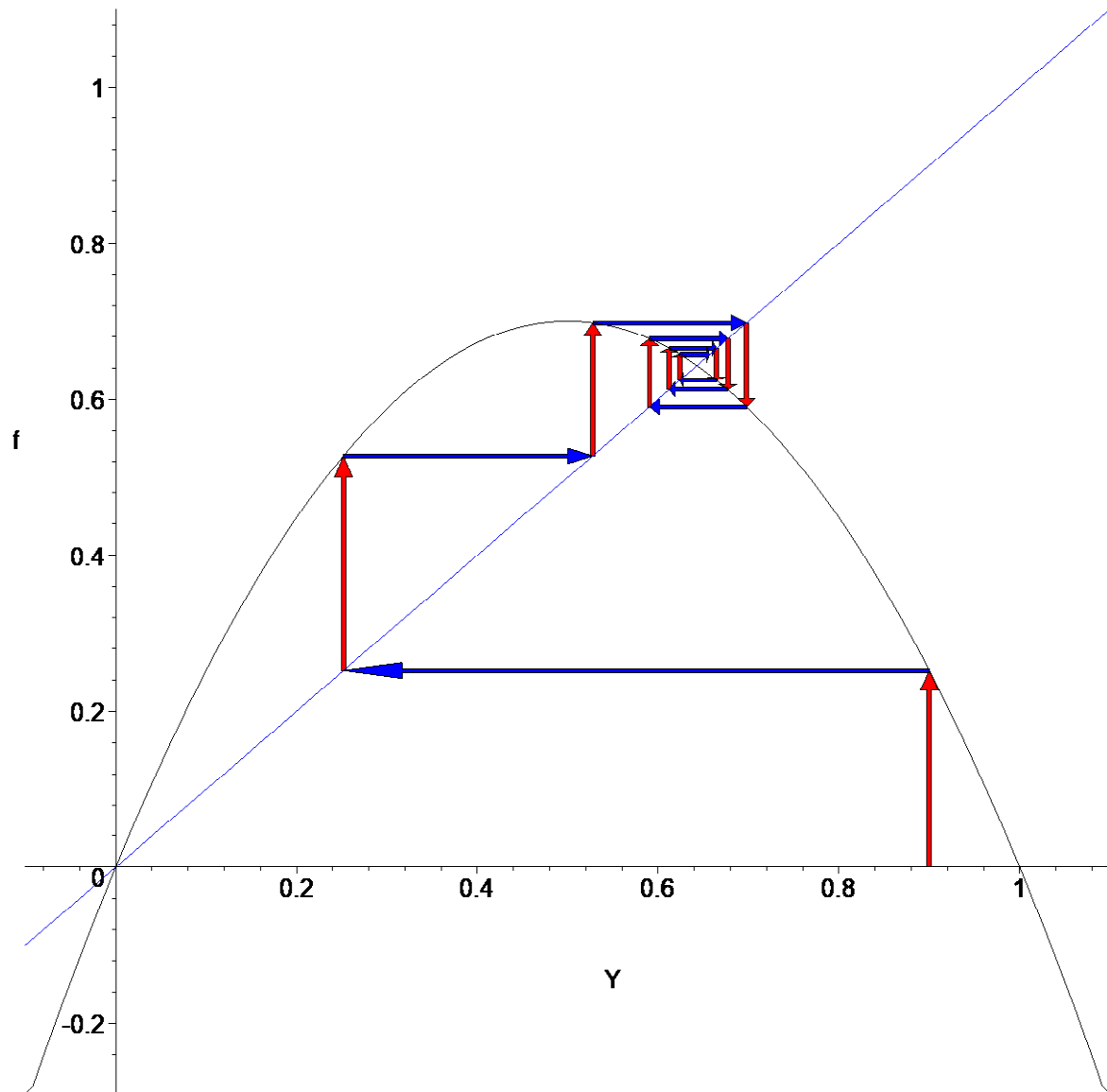
Toile d'araignée de dynamique logistique avec  $r = 2.8$

> #####

Conditions initiales  $f, x_0$

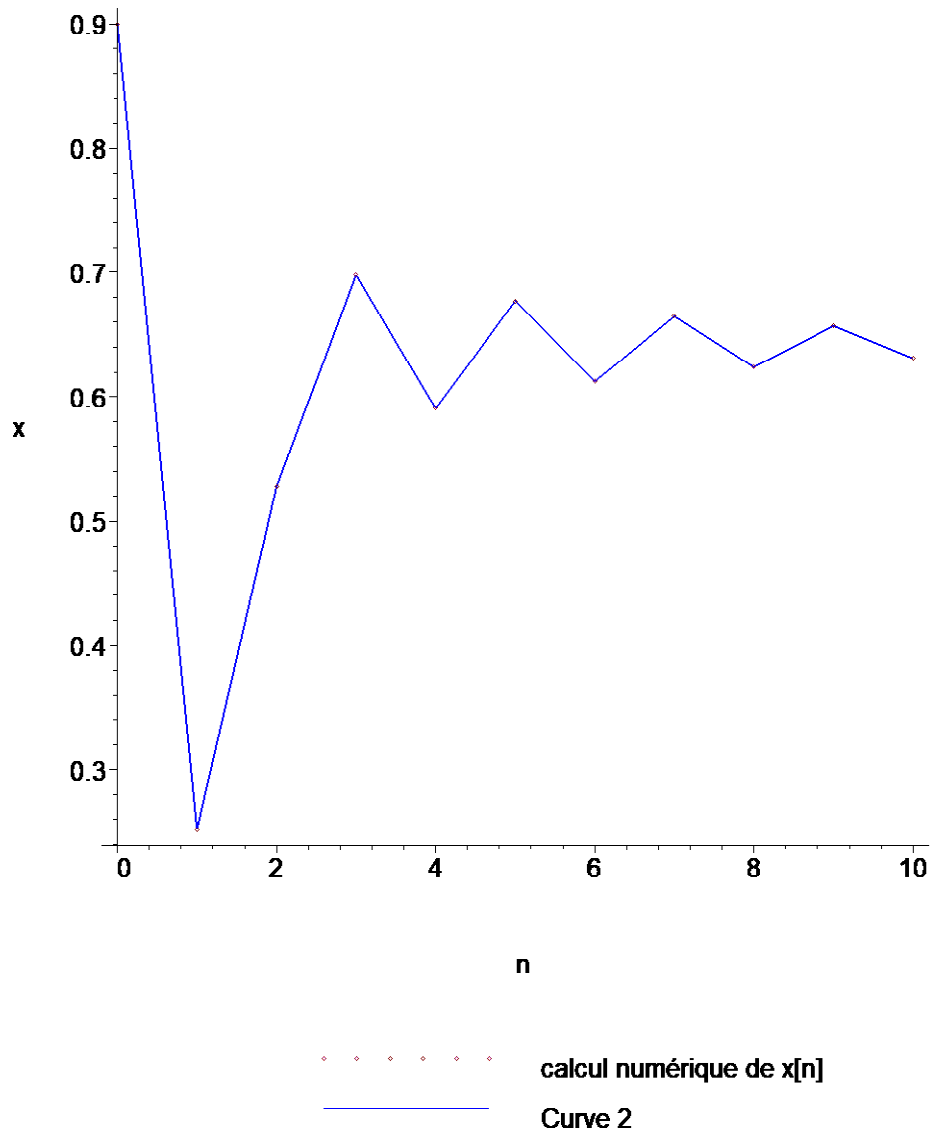
diagramme en toile d'araignée

$$f(x)=2.8x(1-x), x_0 = 0.8$$



"Les valeurs de  $[n, x_n]=$ ",  $[[0, 0.9], [1, 0.252], [2, 0.5278],$   
 $[3, 0.6978], [4, 0.5905], [5, 0.6770], [6, 0.6124], [7, 0.6647],$   
 $[8, 0.6241], [9, 0.6569], [10, 0.6311]]$ , "points fixes=", 0.,  
0.6429

Comportement  
de solutions



>

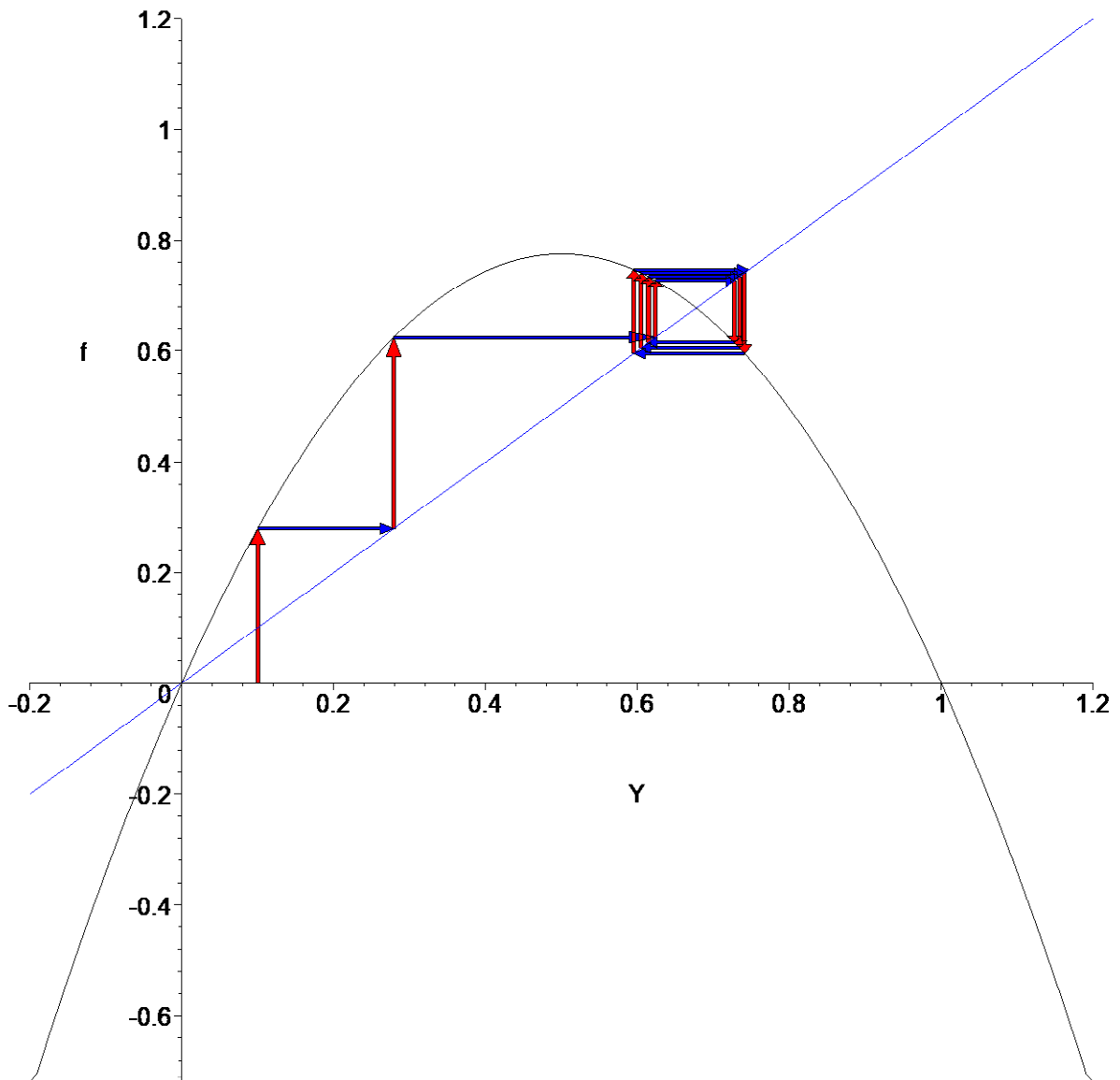
## Cobweb (diagramme en toile d'araignée)

Toile d'araignée de dynamique logistique avec  $r = 3,1$   
la suite oscille, pour  $n$  assez grand, entre deux valeurs

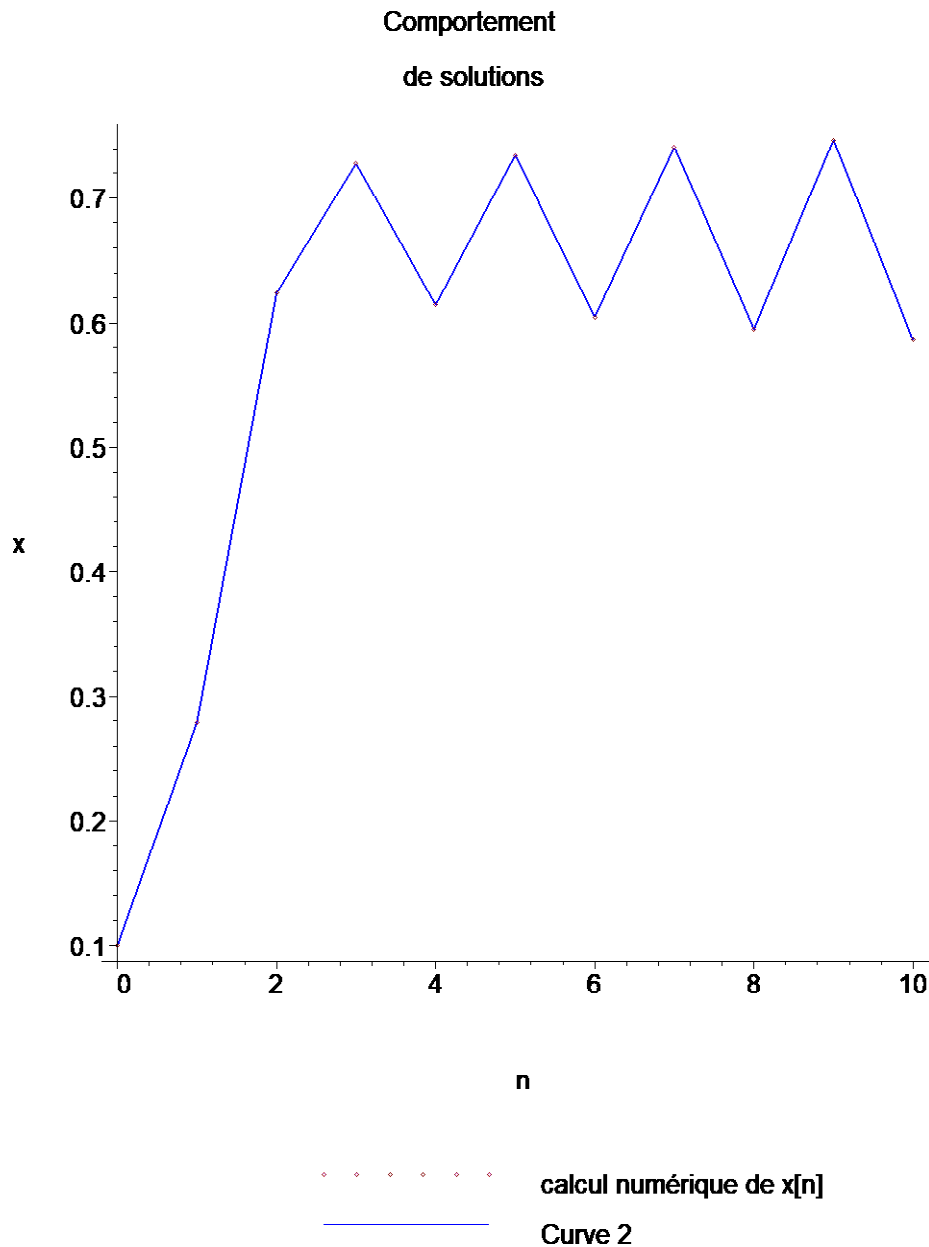
> #####  
Conditions initiales  $f, x_0$

diagramme en toile d'araignée

$$f(x) = 3.1 \cdot x(1-x), x_0 = 0.1$$



"Les valeurs de  $[n, x_n]$ =",  $[[0, 0.1], [1, 0.279], [2, 0.6237],$   
 $[3, 0.7276], [4, 0.6144], [5, 0.7344], [6, 0.6048], [7, 0.7409],$   
 $[8, 0.5952], [9, 0.7468], [10, 0.5862]]$ , "points fixes=", 0.,  
0.6774



> #####

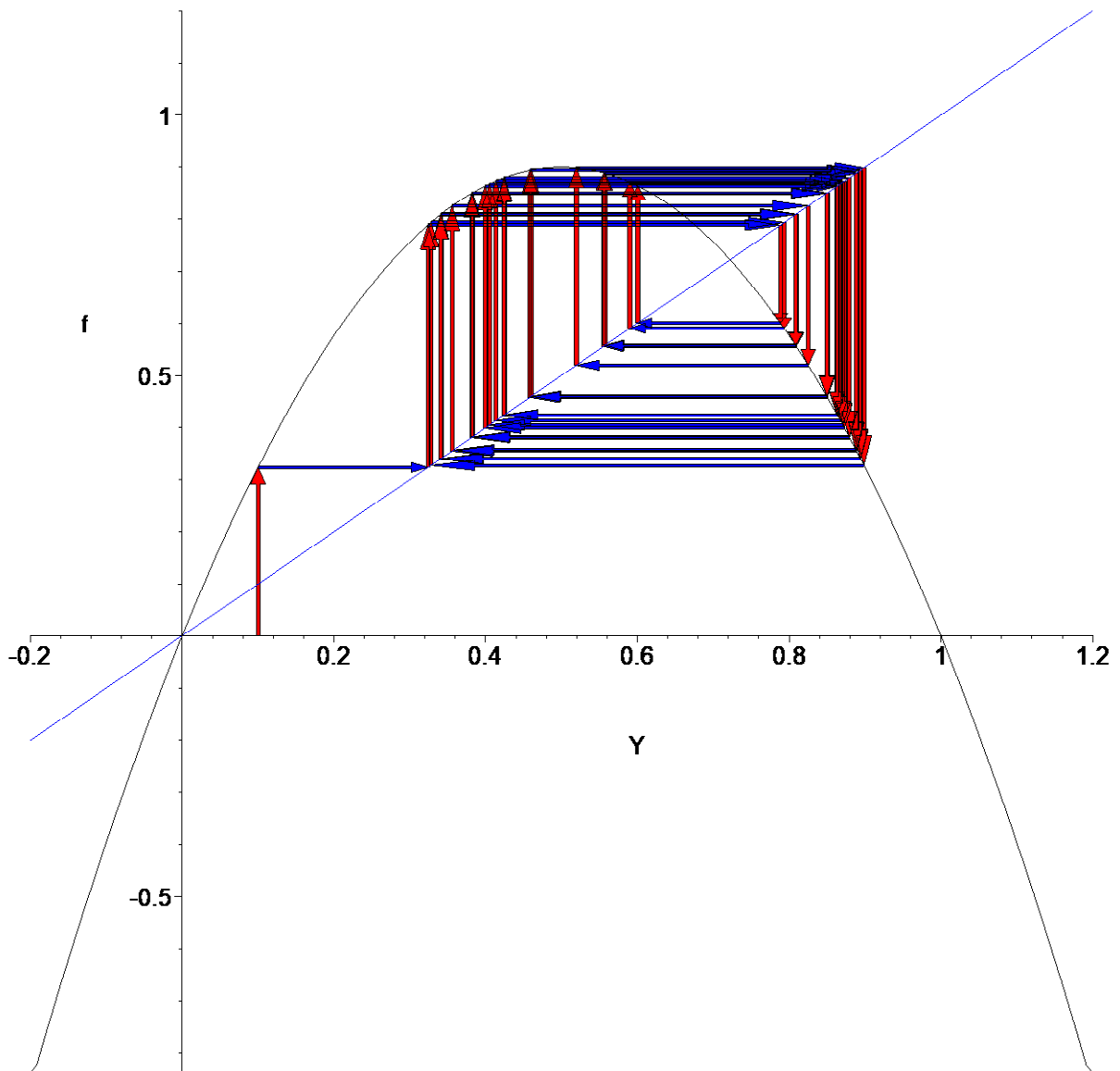
## Cobweb (diagramme en toile d'araignée)

Toile d'araignée de dynamique logistique avec  $r = 3,6$  :  
la suite oscille, pour  $n$  assez grand, entre quatre  
valeurs

> #####

diagramme en toile d'araignée

$$f(x)=3,6x(1-x), x_0 = 0.1$$

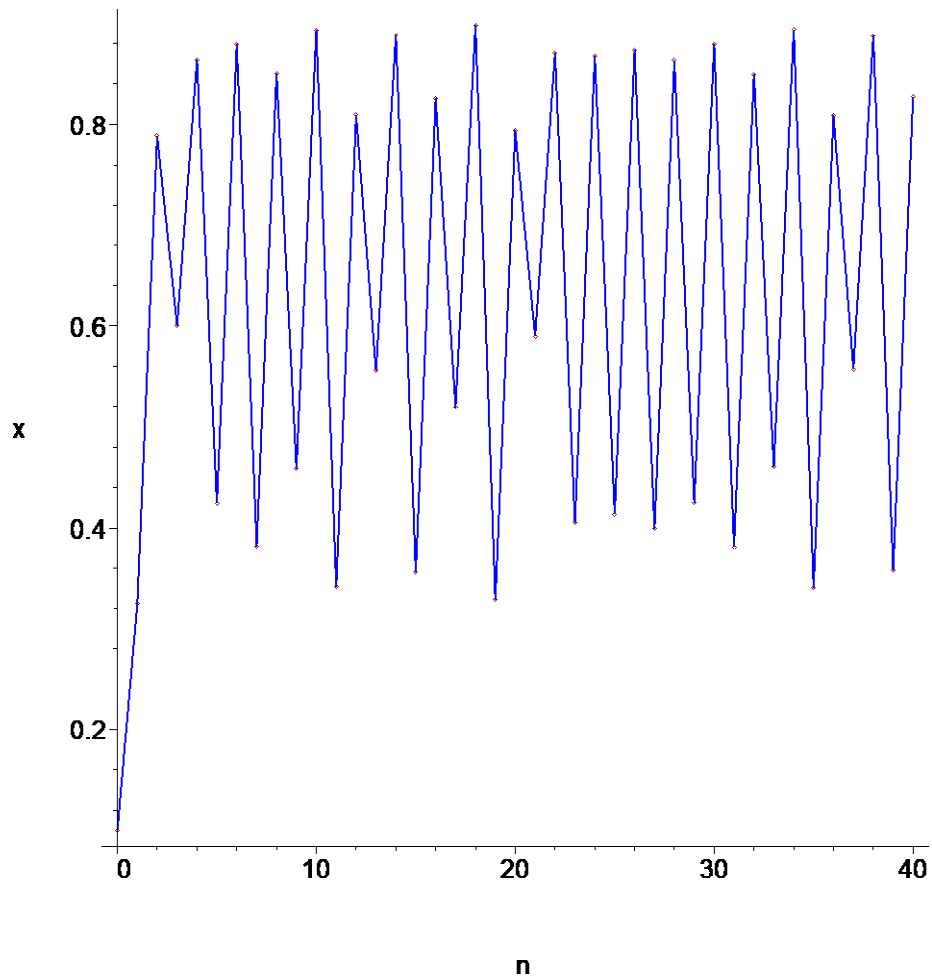


"Les valeurs de  $[n, x_n]=$ ",  $[[0, 0.1], [1, 0.324], [2, 0.7884],$   
 $[3, 0.6005], [4, 0.8636], [5, 0.4241], [6, 0.8791], [7, 0.3827],$   
 $[8, 0.8503], [9, 0.4583], [10, 0.8939], [11, 0.3414],$   
 $[12, 0.8093], [13, 0.5555], [14, 0.8888], [15, 0.3558],$   
 $[16, 0.8251], [17, 0.5195], [18, 0.8986], [19, 0.3280],$   
 $[20, 0.7934], [21, 0.5900], [22, 0.8708], [23, 0.4050],$



[24, 0.8676], [25, 0.4136], [26, 0.8730], [27, 0.3992],  
[28, 0.8633], [29, 0.4248], [30, 0.8795], [31, 0.3816],  
[32, 0.8496], [33, 0.4601], [34, 0.8942], [35, 0.3406],  
[36, 0.8086], [37, 0.5573], [38, 0.8881], [39, 0.3578],  
[40, 0.8273]], "points fixes=", 0., 0.7222

Comportement  
de solutions



# Cobweb (diagramme en toile d'araignée)

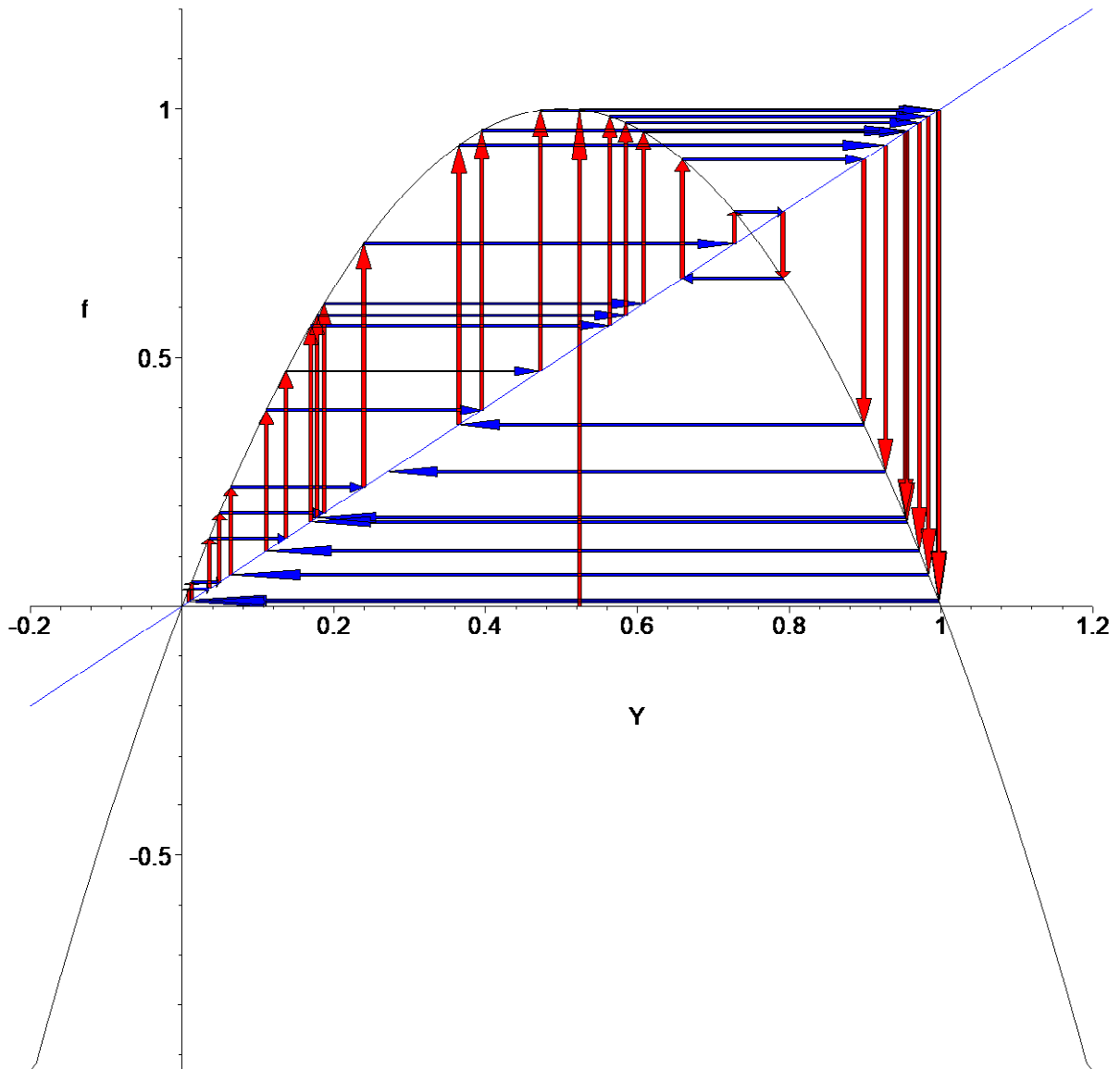
Toile d'araignée de dynamique logistique avec  $r = 4$  :

le comportement est chaotique si le coefficient de reproduction est très grand ( $r > 3,750$ )

```
> #####
Conditions initiales f,x0
restart: N:=30:t0:=0:x0:=Pi/6.:
Digits:=5:t1:=3: h:=(t1-t0)/N:
#####
f:=4*Y*(1-Y):G:=f-Y:
#####
#####
x := array(1..N+1):
nn := array(1..N+1):
t := array(1..N+1):
Y(1):=x0: T(1):=t0:
nn[1]:=0:
t[1]:=T(1):
x[1]:=Y(1):
fn:=eval(G, [T=T[1],Y=T(1)]):
for n from 1 to N do
T(n+1):= T(n)+h; fn:=eval(G,
[Y=Y(n),T=T(n)]);
Y(n+1):=eval(f, [Y=Y(n),T=T(n)]);
t[n+1]:=T(n+1);
x[n+1]:=Y(n+1);
nn[n+1]:=n;
```

diagramme en toile d'araignée

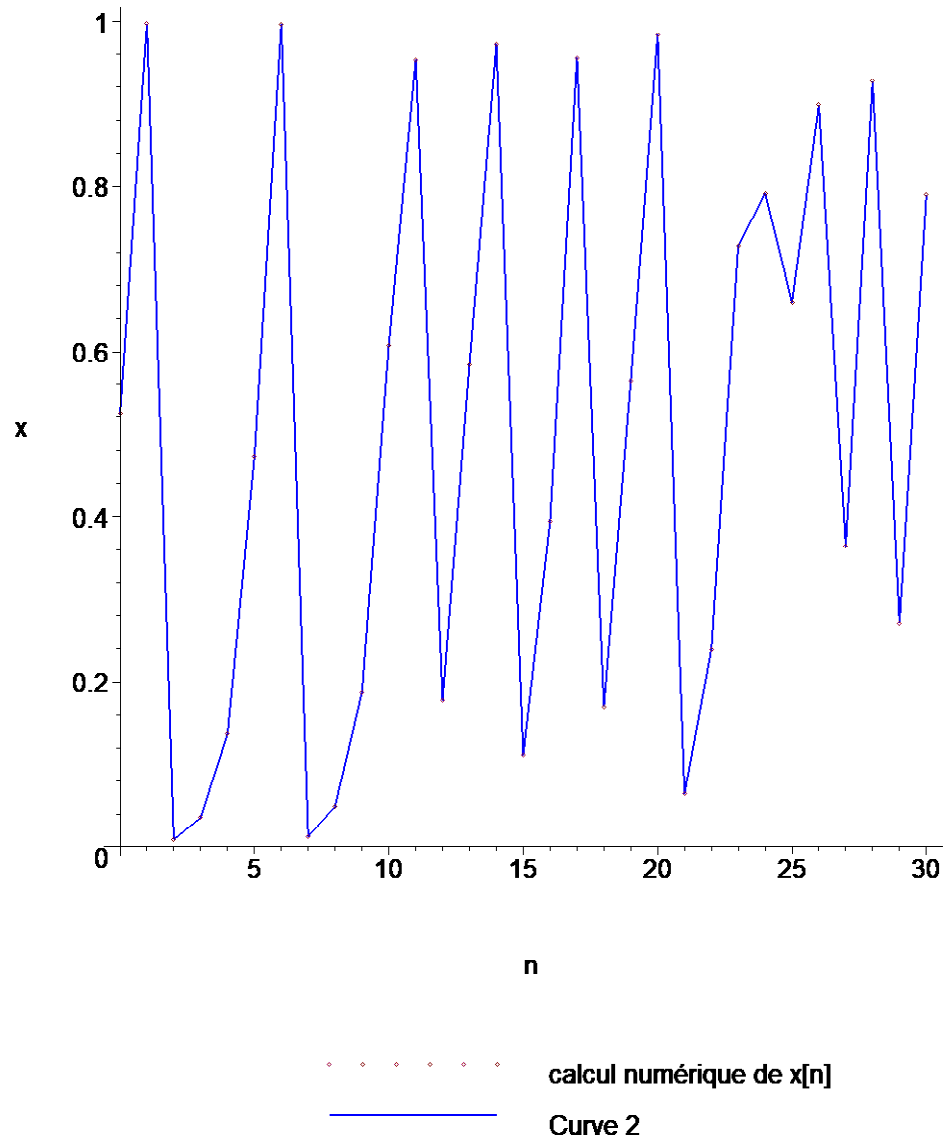
$$f(x)=4x(1-x), x_0 = \pi/6$$



"Les valeurs de  $[n, x_n]=$ ",  $[[0, 0.5235987758], [1, 0.99776], [2, 0.0089400], [3, 0.035440], [4, 0.13674], [5, 0.47216], [6, 0.99688], [7, 0.012441], [8, 0.049144], [9, 0.18692], [10, 0.60792], [11, 0.95340], [12, 0.17771], [13, 0.58452], [14, 0.97144], [15, 0.11098], [16, 0.39465], [17, 0.95560], [18, 0.16972], [19, 0.56368], [20, 0.98376], [21, 0.063904],$

[22, 0.23928], [23, 0.72812], [24, 0.79184], [25, 0.65932],  
[26, 0.89848], [27, 0.36486], [28, 0.92696], [29, 0.27082],  
[30, 0.78992]], "points fixes=",  $0, \frac{3}{4}$

Comportement  
de solutions



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