



Elementary Proof that e is Irrational

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Source: *The American Mathematical Monthly*, Vol. 60, No. 7 (Aug. - Sep., 1953), p. 474

Published by: [Mathematical Association of America](#)

Stable URL: <http://www.jstor.org/stable/2308411>

Accessed: 07/10/2010 03:30

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sets of conjugate diameters of Q .

Again if we join the vertices of any triangle from (ii) to the centre of Q , we get a set of three mutually perpendicular lines.

COROLLARY 1. C passes through the three principal axes of Q . For e passes through A, B, C which are the traces of principal axes.

COROLLARY 2. If P is the central plane of which p is the trace then the normal to P at the center and the diameter of which P is the diametral plane lie on 'C.'

From the definition it can be proved that the pole of p w.r.t. any conic of the family $q + \lambda\Omega = 0$ lies on e . As the pole of p w.r.t. Ω is the trace of any normal to P , the normal at the center lies on C . Again the pole of p w.r.t. g , which must lie on e , is the trace of the diameter D of which P is the diametral plane. Hence D must lie on C .

MATHEMATICAL NOTES

EDITED BY F. A. FICKEN, University of Tennessee

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ELEMENTARY PROOF THAT e IS IRRATIONAL

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The following variation on the usual proof of the irrationality of e is perhaps slightly simpler. Suppose that e is rational, say $e = a/b$. Then

$$b/a = e^{-1} = \sum_{n=0}^{\infty} (-1)^n/n!,$$

and multiplication by $(-1)^{a+1}a!$ and transposition of terms gives

$$\begin{aligned} & (-1)^{a+1} \left\{ b(a-1)! - \sum_{n=0}^a (-1)^n a!/n! \right\} \\ &= \frac{1}{(a+1)} - \frac{1}{(a+1)(a+2)} + \frac{1}{(a+1)(a+2)(a+3)} - \dots \end{aligned}$$

The right side has a value between 0 and 1 since the alternating series clearly converges to a value between its first term and the sum of its first two terms. But the left side is an integer, so we have a contradiction.