Fibonacci Sequence and the Golden Ratio

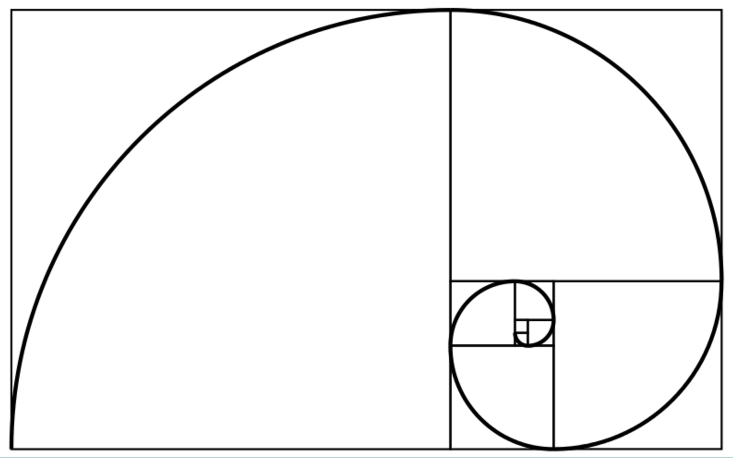
Killian Rolland & Hubert Villuendas

Introduction

- Leonardo Fibonacci
- *Liber Abaci* (1202)
  - Growth of a rabbit population

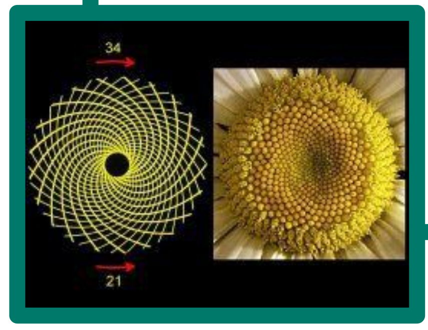
Fibonacci Numbers & Spiral

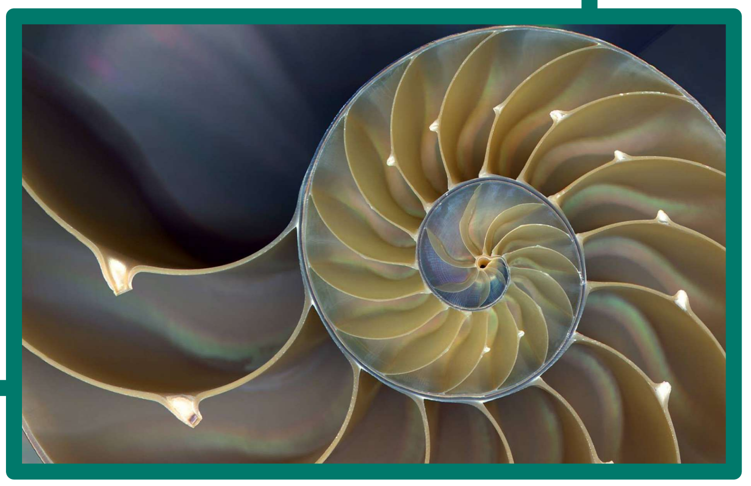
- $F_{n+2} = F_{n+1} + F_n \qquad F_0 = F_1 = 1$
- Spiral :



Appearance in Nature

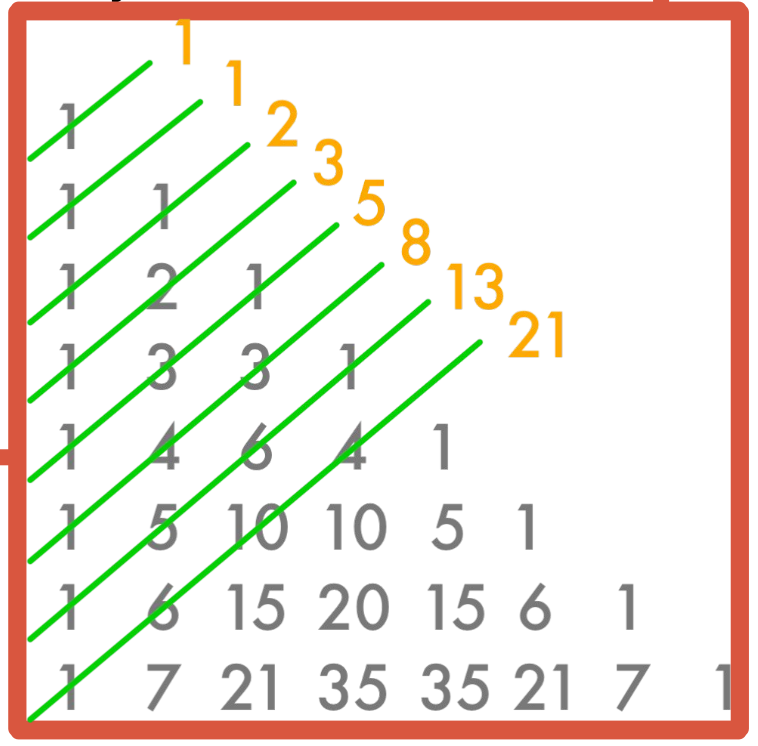
- Flowers Shapes
- Shells Shapes





Fibonacci and Pascal's Triangle

- Pascal's triangle : triangular array of the binomial coefficients.
- The sum of the coefficients of the n^{th} coefficients makes appear the n^{th} Fibonacci numbers.



The Golden Ratio

- Unique positive solution of $x^2 - x - 1 = 0$
- limite des quotients

Newton's method

- Newton's method consists in approaching the roots of a function by approximating the function with its tangents
- Used to approach the Golden Ratio as a root of $f: x \mapsto x^2 - x - 1$

Explicit Formula

- Using the Golden Ratio and the characteristic equation of the Fibonacci Sequence, we prove that :

$$\frac{1}{\sqrt{5}}\left(\phi^n - \left(-\frac{1}{\phi}\right)^n\right)$$

Sources

- https://www.researchgate.net/publication/334015286_Fibonacci_Numbers_and_Golden_Ratio_in_Mathematics_and_Science
- https://www.hindawi.com/journals/jmath/2013/204674/