



PLAY NIM AND WIN

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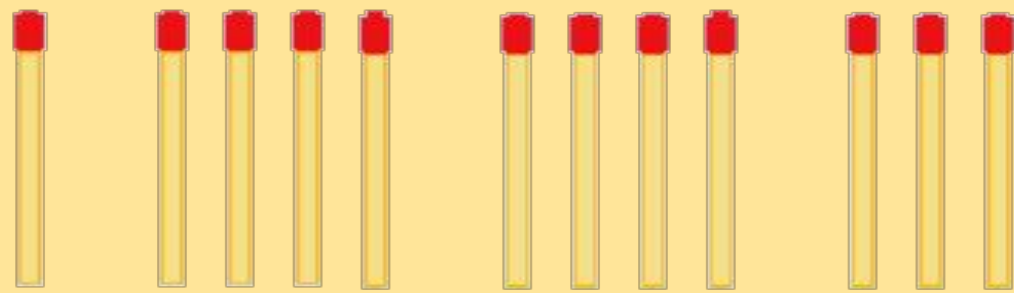
The NIM game

- A two player game
- A winning strategy game

NIM \longleftrightarrow WIN

- With matches, marble, counter...

The classical Nim game



Rules : Choose 1, 2 or 3 matches !

Goal : Don't take the last one !

Strategy : By using congruence

$$n - x \equiv 1 [4]$$

n = number of matches on the table
 x = number of matches to remove

\rightarrow What is the initial configuration to win every time ?

A variant of Nim game

(Classical Nim game with three piles)

Rules : Choose any matches as you want from a single pile.

Goal : Don't take the last one !

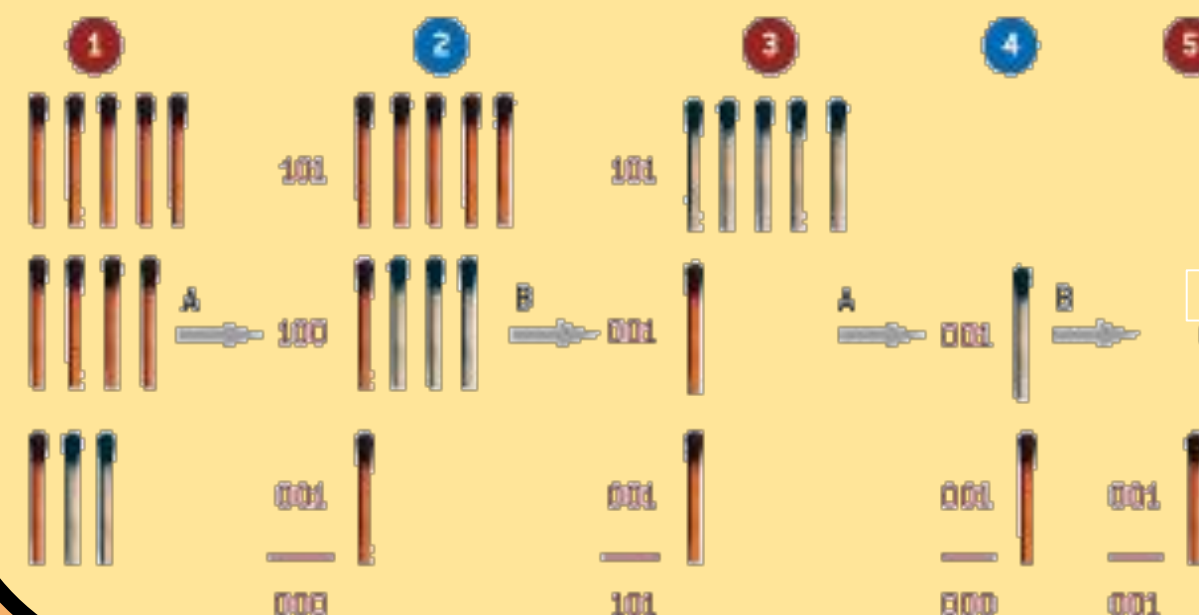
Strategy :

- Be the first to make a **safe combination** :
 \rightarrow In binary, the sum of number of matches per pile is equal to 0.

Theorem 1 : Opponent can't leave a safe combination after.

- Your turn : make a safe combination again

Theorem 2 : We can always diminish one of the piles, and leave a safe combination.



Methods

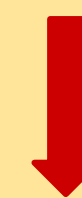
Two initial configurations

(A)

Safe combination

(B)

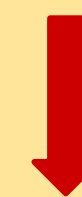
Unsafe combination



Opponent start



You start



You WIN !



You WIN !

$$\mathbb{P}(A) = \frac{2^{n-1} - 1}{2^{n-1}(2^{n-1} + 1)}$$

2^n = maximal number of matches per pile

Source

Nim, A Game with a Complete Mathematical Theory, Charles Leonard Bouton, 1901