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THE PERNICIOUS INFLUENCE OF MATHEMATICS UPON PHILOSOPHY*

ABSTRACT. We shall argue that the attempt carried out by certain philosophers in this century to parrot the language, the method, and the results of mathematics has harmed philosophy. Such an attempt results from a misunderstanding of both mathematics and philosophy, and has harmed both subjects.

1. THE DOUBLE LIFE OF MATHEMATICS

Are mathematical ideas invented or discovered? This question has been repeatedly posed by philosophers through the ages, and will probably be with us forever. We shall not be concerned with the answer: what matters is that by asking the question, we acknowledge the fact that mathematics has been leading a double life.

In the first of its lives, mathematics deals with facts like any other science. It is a fact that the altitutes of a triangle meet at a point; it is a fact that there are only seventeen kinds of symmetry in the plane; it is a fact that there are only five non-linear differential equations with fixed singularities; it is a fact that every finite group of odd order is solvable. The work of a mathematician consists in dealing with these facts in various ways. When mathematicians talk to each other, they tell the facts of mathematics. In their research work, mathematicians study the facts of mathematics with a taxonomic zeal similar to that of the botanist who studies the properties of some rare plant.

The facts of mathematics are as useful as the facts of any other science. No matter how abstruse they may appear at first, sooner or later they find their way back to practical applications. The facts of group theory, for example, may appear abstract and remote, but the practical applications of group theory have been numerous, and they have occurred in ways that no one could have anticipated. The facts of today's mathematics are the springboard for the science of tomorrow.

In its second life, mathematics deals with proofs. A mathematical theory begins with definitions, and derives its results from clearly agreed upon rules of inference. Every fact of mathematics must be ensconced in an axiomatic theory and formally proved if it is to be accepted as

Synthese **88**: 165–178, 1991.

true. Axiomatic exposition is indispensable in mathematics because the facts of mathematics, unlike the facts of physics, are not amenable to experimental verification.

The axiomatic method of mathematics is one of the great achievements of our culture. However, it is only a method. Whereas the facts of mathematics, once discovered, will never change, the method by which these facts are verified has changed many times in the past, and it would be foolhardy not to expect that it will not change again at some future date.

2. THE DOUBLE LIFE OF PHILOSOPHY

The success of mathematics in leading a double life has long been the envy of philosophy, another field which also is blessed – or maybe we should say cursed – to live in two worlds, but which has not been quite as comfortable with its double life.

In the first of its lives, philosophy sets to itself the task of telling us how to look at the world. Philosophy is effective at correcting and redirecting our thinking. It helps us do away with glaring prejudices and unwarranted assumptions. Philosophy lays bare contradictions that we would rather avoid facing up to. Philosophical descriptions make us aware of phenomena that lie at the other end of the spectrum of rationality, phenomena which science will not and cannot deal with.

The assertions of philosophy are less reliable than the assertions of mathematics, but they run deeper into the roots of our existence.

The philosophical assertions of today will be part of the common sense of tomorrow.

In its second life, philosophy, like mathematics, relies on a method of argumentation that seems to follow the rules of some logic or other. But the method of philosophical reasoning, unlike the method of mathematical reasoning, has never been clearly agreed upon by philosophers, and much philosophical discussion since the beginnings in Greece has been spent on discussions of method. Philosophy's relationship with Goddess Reason is closer to a forced cohabitation than to the romantic liaison that has always existed between Goddess Reason and mathematics.

The assertions of philosophy are tentative and partial. It is not even

clear what it is that philosophy deals with. It used to be said that philosophy was 'purely speculative', and this used to be an expression of praise. But lately the word 'speculative' has become a *bad word*.

Philosophical arguments are emotion-laden to a greater degree than mathematical arguments. Philosophy is often written in a style which is more reminiscent of a shameful admission than of a dispassionate description. Behind every question of philosophy there lurks a gnarl of unacknowledged emotional cravings, which act as powerful motivation for conclusions in which reason plays at best a supporting role. To bring such hidden emotional cravings out into the open, as philosophers have felt it their duty to do, is to call for trouble. Philosophical disclosures are frequently met with the anger that we reserve for the betrayal of our family secrets.

This confused state of affairs makes philosophical reasoning more difficult, but far more rewarding. Although philosophical arguments are blended with emotion, although philosophy seldom reaches a firm conclusion, although the method of philosophy has never been clearly agreed upon, nonetheless, the assertions of philosophy, tentative and partial as they are, come far closer to the truth of our existence than the proofs of mathematics.

3. THE LOSS OF AUTONOMY

Philosophers of all times, beginning with Thales and Socrates, have suffered from the recurring suspicions about the soundness of their work and have responded to them as best they could.

The latest reaction against the criticism of philosophy began around the turn of the twentieth century and is still very much with us.

Today's philosophers (not all of them, fortunately) have become great believers in mathematization. They have rewritten Galileo's famous sentence to read, "The great book of philosophy is written in the language of mathematics".

"Mathematics calls attention to itself", wrote Jack Schwartz in a famous paper on another kind of misunderstanding.[1] Philosophers in this century have suffered more than ever from the dictatorship of definitiveness. The illusion of the final answer, what two thousand years of Western philosophy failed to accomplish, was thought in this century to have come at last within reach by the slavish imitation of mathematics.

Mathematizing philosophers have claimed that philosophy should be made factual and precise. They have given guidelines to philosophical argument which are based upon mathematical logic. They have contended that the eternal riddles of philosophy can be definitively solved by pure reasoning, unencumbered by the weight of history. Confident in their faith in the power of pure thought, they have cut all ties to the past, on the claim that the messages of past philosophers are now 'obsolete'.

Mathematizing philosophers will agree that traditional philosophical reasoning is radically different from mathematical reasoning. But this difference, rather than being viewed as strong evidence for the heterogeneity of philosophy and mathematics, is taken instead as a reason for doing away with non-mathematical philosophy altogether.

In one area of philosophy the program of mathematization has succeeded. Logic is nowadays no longer a part of philosophy. Under the name of mathematical logic, it is now a successful and respected branch of mathematics, one that has found substantial practical applications in computer science, more so than any other branch of mathematics.

But logic has become mathematical at a price. Mathematical logic has given up all claims to give a foundation to mathematics. Very few logicians of our day now believe that mathematical logic has anything to do with the way we think.

Mathematicians are therefore mystified by the spectacle of philosophers pretending to re-inject philosophical sense into the language of mathematical logic. A hygienic cleansing of every trace of philosophical reference had been the price of admission of logic into the mathematical fold. Mathematical logic is now just another branch of mathematics, like topology and probability. The philosophical aspects of mathematical logic are qualitatively no different from the philosophical aspects of topology or the theory of functions, aside from a curious terminology which, by an accident of chance going back to Leibniz's reading of Suárez, goes back to the Middle Ages.

The fake philosophical terminology of mathematical logic has misled philosophers into believing that mathematical logic deals with the truth in the philosophical sense. But this is a mistake. Mathematical logic does not deal with the truth, but only with the game of truth. The snobbish symbol-dropping one finds nowadays in philosophical papers raises eyebrows among mathematicians. It is as if you were at the

grocery store and you watched someone trying to pay his bill with Monopoly money.

4. MATHEMATICS AND PHILOSOPHY: SUCCESS AND FAILURE

By all accounts, mathematics is the most successful intellectual undertaking of mankind. Every problem of mathematics gets solved, sooner or later. Once it is solved, a mathematical problem is forever finished: no later event will disprove a correct solution. As mathematics progresses, problems that were once difficult become easy enough to be assigned to school children. Thus, Euclidean geometry is now taught in the second year of high school. Similarly, the mathematics that mathematicians of my generation learned in graduate school has now descended to the undergraduate level, and the time is not far when it may be taught in the high schools.

Not only is every mathematical problem solved, but eventually, every mathematical problem is proved trivial. The quest for ultimate triviality is characteristic of the mathematical enterprise.

When we look at the problems of philosophy, another picture emerges. Philosophy can be described as the study of a few problems whose statements have changed little since the Greeks: the mind-body problem, or the problem or reality, to recall only two. A dispassionate look at the history of philosophy discloses two contradictory features: first, these problems have in no way been solved, nor are they likely to be solved as long as philosophy survives; and second, every philosopher who has ever worked on any of these problems has proposed his own "definitive solution", which has all invariably been rejected as false by his successors.

Such crushing historical evidence forces us to the conclusion that these two paradoxical features must be an inescapable concomitant of the philosophical enterprise. The failure to conclude has been an outstanding characteristic of philosophy throughout its history.

Philosophers of the past have repeatedly stressed the essential role of failure in philosophy. José Ortega y Gasset, for example, used to describe philosophy as "a constant shipwreck". However, the fear of failure did not stop him or any other philosopher from doing philosophy.

Philosophers' failure to reach any kind of agreement does not make

their writings any less relevant to the problems of our day. We reread with interest the mutually contradictory theories of mind that Plato, Aristotle, Kant and Comte have bequeathed to us, and we find their opinions timely and enlightening, even in problems of artificial intelligence.

But unfortunately, the latter-day mathematizers of philosophy are unable to face up to the inevitability of failure. Borrowing from the world of business, they have embraced the ideal of success. Philosophy had better be successful, or else it should be given up, like any business.

5. THE MYTH OF PRECISION

Since mathematical concepts are precise, and since mathematics has been successful, our darling philosophers mistakenly infer that philosophy would be better off if it dealt with precise concepts and inequivocal statements. Philosophy will have a better chance at being successful, if it becomes precise.

The prejudice that a concept must be precisely defined in order to be meaningful, or that an argument must be precisely stated in order to make sense, is one of the most insidious of the twentieth century. The best-known expression of this prejudice appears at the end of Ludwig Wittgenstein's *Tractatus*, and the author's later writings, in particular *Philosophical Investigations*, is a loud and repeated retraction of his earlier *gaffe*.

Looked at from the vantage point of ordinary experience, the ideal of precision appears preposterous. Our everyday reasoning is not precise, yet it is effective. Nature itself, from the cosmos to the gene, is approximate and inaccurate.

The concepts of philosophy are among the least precise. The mind, perception, memory, cognition, are words that do not have any fixed or clear meaning. Yet, they do have meaning. We misunderstand these concepts when we force them to be precise. To use an image due to Wittgenstein, philosophical concepts are like the winding streets of an old city, which we must accept as they are, and which we must familiarize ourselves with by strolling through them, while admiring their historical heritage. Like a Carpathian dictator, the advocates of precision would raze the city to the ground and replace it with a straight and wide Avenue of Precision.

The ideal of precision in philosophy has its roots in a misunder-

standing of the notion of rigor. It has not occurred to our mathematizing philosophers that philosophy might be endowed with its own kind of rigor, a rigor that philosophers should dispassionately describe and codify, as mathematicians did with their own kind of rigor a long time ago. Bewitched as they are by the success of mathematics, they remain enslaved by the prejudice that the only possible rigor is that of mathematics, and that philosophy has no choice but to imitate it.

6. THE MISUNDERSTANDING OF THE AXIOMATIC METHOD

The facts of mathematics are verified and presented by the axiomatic method. One must guard, however, against confusing the *presentation* of mathematics with the *content* of mathematics. An axiomatic presentation of a mathematical fact differs from the fact that is being presented, as medicine differs from food. It is true that this particular medicine is necessary to keep the mathematician at a safe distance from the self-delusions of the mind. Nonetheless, understanding mathematics means being able to forget the medicine, and to enjoy the food. Confusing mathematics with the axiomatic method for its presentation is as preposterous as confusing the music of Johann Sebastian Bach with the techniques for counterpoint in the Baroque age.

This is not, however, the opinion held by our mathematizing philosophers. They are convinced that the axiomatic method is a basic instrument for discovery. They mistakenly believe that mathematicians use the axiomatic method in solving problems and proving theorems. To the misunderstanding of the role of the method they have added the absurd pretense that this presumed method should be adopted in philosophy. Systematically confusing food with medicine, they have pretended to replace the food of philosophical thought with the medicine of axiomatics.

This mistake betrays the philosophers' pessimistic view of their own field. Unable or afraid as they are of singling out, describing and analyzing the structure of philosophical reasoning, they seek help from the proven technique of another field, a field that is the object of their envy and veneration. Secretly disbelieving in the power of autonomous philosophical reasoning to arrive at the truth, they have surrendered to a slavish and superficial imitation of the truth of mathematics.

The negative opinion that many philosophers hold of their own field has caused damage to philosophy. The mathematician's contempt at

the philosopher's exaggerated estimation of a method of mathematical exposition feeds back onto philosophers' inferiority complex, and further decreases the philosophers' confidence.

7. "DEFINE YOUR TERMS!"

This old injunction has become a platitude in everyday discussions. What could be healthier than a clear statement, right at the beginning, of what it is that we are talking about? Doesn't mathematics start with definitions and then develop the properties of the objects that have been defined, by an admirable and inexorable logic?

Salutary as this injunction may be in mathematics, it has had disastrous consequences when carried over to philosophy. Whereas mathematics *starts* with a definition, philosophy *ends* with a definition. A clear statement of what it is we are talking about is not only missing in philosophy, such a statement would be the end of all philosophy. If we could define our terms, then we would dispense with philosophical argument.

Actually, the 'define your terms' imperative is deeply flawed in more than one way. While reading a formal mathematical argument, we are given to believe that the 'undefined terms', or the 'basic definitions' have been whimsically chosen out of a variety of possibilities. Mathematicians take mischievous pleasure in faking the arbitrariness of definition. In actual fact, no mathematical definition is arbitrary. The theorems of mathematics motivate the definitions as much as the definitions motivate the theorems. A good definition is 'justified' by the theorems one can prove with it, just like the proof of a theorem is 'justified' by appealing to a previously given definition.

There is, thus, a hidden circularity in formal mathematical exposition. The theorems are proved starting with definitions, but the definitions themselves are motivated by the theorems that we have previously decided ought to be right.

Instead of focussing on this strange circularity, philosophers have pretended it does not exist, as if the axiomatic method, proceeding linearly from definition to theorem, were endowed with a definitiveness which is instead, as every mathematician knows, a subtle fakery to be debunked.

Perform the following thought experiment. Suppose that you are given two formal presentations of the same mathematical theory. The

definitions of the first presentation are the theorems of the second, and vice versa. This situation frequently occurs in mathematics. Which of the two presentations makes the theory 'true'? Neither, evidently: what we have is two presentations of the *same* theory.

This thought experiment shows that mathematical truth is not brought into being by a formal presentation; rather, formal presentation is only a technique for displaying mathematical truth. The truth of a mathematical theory is distinct from the correctness of any axiomatic method that may be chosen for the presentation of the theory.

Mathematizing philosophers have missed this distinction.

8. THE APPEAL TO PSYCHOLOGY

What will happen to the philosopher who insists on precise statements and clear definitions? Realizing after futile trials that philosophy resists such a treatment, the philosopher will proclaim that most problems previously thought to belong to philosophy are heretofore to be excluded from consideration. He will claim that they are 'meaningless', or at best, that they can be settled by an analysis of their statements that will eventually show them to be vacuous.

This is not an exaggeration. The classical problems of philosophy have become forbidden topics in many philosophy departments. The mere mention of one such problem by a graduate student or by a junior colleague will result in raised eyebrows, followed by severe penalties. In this dictatorial regime, we have witnessed the shrinking of philosophical activity to an impoverished *problématique*, mainly dealing with language.

In order to justify their neglect of most of the old and substantial questions of philosophy, our mathematizing philosophers have resorted to the ruse of claiming that many questions, formerly thought to be philosophical, are instead 'purely psychological' and that they should be dealt with in the psychology department.

If the psychology department of any university were to consider only one-tenth of the problems that philosophers are palming off on them, then psychology would without question be the most fascinating of all subjects. Maybe it is. But the fact is that psychologists have no intention of dealing with problems abandoned by philosophers who have been derelict in their duties.

One cannot do away with problems by decree. The classical problems of philosophy are now coming back with a vengeance in the forefront of science. For example, the Kantian problem of the conditions of possibility of vision, after years of neglect, is now again rearing its old head in brain science.

Experimental psychology, neurophysiology and computer science may turn out to be the best friends of traditional philosophy. The awesome complexities of the phenomena that are being studied in these sciences have convinced scientists (well in advance of the philosophical establishment) that progress in science will crucially depend on philosophical research of the most classical vein.

9. THE REDUCTIONIST CONCEPT OF THE MIND

What does a mathematician do when trying to work on a mathematical problem? An adequate description of this event might take a thick volume. We shall be content with recalling an old saying, probably going back to the mathematician George Pólya: "Few mathematical problems are ever solved directly".

Every mathematician will agree that an important step in solving a mathematical problem, perhaps the most important step, consists in analyzing other attempts, either attempts that have been previously carried out or else attempts that one imagines might have been carried out, with a view to discovering how such 'previous' attempts were misled. In short, no mathematician will ever dream of attacking a substantial mathematical problem without first becoming acquainted with the history of the problem, whether the real history or an ideal history that a gifted mathematician might reconstruct. The solution of a mathematical problem goes hand-in-hand with the discovery of the inadequacy of previous attempts, with the enthusiasm that sees through and does away with layers of irrelevancies inherited from the past, which cloud the real nature of the problem. In philosophical terms, a mathematician who solves a problem cannot avoid facing up to the historicity of the problem. Mathematics is nothing if not a historical subject par excellence.

Every philosopher since Heraclitus has stressed with striking uniformity the lesson that all thought is constitutively historical. Until,

that is, our mathematizing philosophers came along, claiming that the mind is nothing but a complex thinking machine, not to be polluted by the inconclusive ramblings of bygone ages. Historical thought has been dealt a *coup de grâce* by those who today occupy some of the chairs of our philosophy departments. Graduate school requirements in the history of philosophy have been dropped, together with language requirements, and in their place we find required courses in mathematical logic.

It is important to uncover the myth that underlies such drastic revision of the concept of mind, that is, the myth that the mind is a mechanical device. This myth has been repeatedly and successfully attacked by the best philosophers of our time (Husserl, John Dewey, Wittgenstein, Austin, Ryle, to name only a few).

According to this myth, the process of reasoning is viewed as the functioning of a vending machine which, by setting into motion a complex mechanism reminiscent of those we saw in Charlie Chaplin's film 'Modern Times', grinds out solutions to problems, like so many Hershey bars. Believers in the theory of the mind as a vending machine, will rate human beings according to 'degrees' of intelligence, the more intelligent ones being those endowed with bigger and better gears in their brains, as can of course be verified by administering I.Q. tests.

Philosophers believing in the mechanistic myth believe that the solution of a problem is obtained in just one way: by thinking hard about it. They will go as far as asserting that acquaintance with previous contributions to a problem may bias the well-geared mind. A blank mind, they believe, is better geared up to initiate the solution process than an informed mind.

This outrageous proposition originates from a misconception of how mathematicians work. Our mathematizing philosophers behave like failed mathematicians. They gape at working mathematicians in wide-eyed admiration, like movie fans gaping at posters of Joan Crawford and Bette Davis. Mathematicians are superminds who turn out solutions of one problem after another by dint of pure brain power, simply by staring at a blank piece of paper in intense concentration.

The myth of the vending machine that grinds solutions out of nothing may perhaps appropriately describe the way to solve the linguistic puzzles of today's impoverished philosophy, but this myth is wide of the mark in describing the work of mathematicians, or any other serious work.

The fundamental error is one of reductionism. The *process* of the working of the mind, which may be of interest to physicians but is of no interest to mathematicians, is confused with the *progress* of thought that is required in the solution of any problem.

This catastrophic misunderstanding of the nature of knowledge is the heritage of one hundred-odd years of pseudo-mathematization of philosophy.

10. THE ILLUSION OF DEFINITIVENESS

The results of mathematics are definitive. No one will every improve on a sorting algorithm which has been proved best possible. No one will ever discover a new finite simple group, now that the list has been drawn, after a century of research. Mathematics is forever.

We could classify the sciences by how close their results come to being definitive. At the top of the list we would find the sciences of lesser philosophical interest, such as mechanics, organic chemistry, botany. At the bottom of the list we would find the more philosophically inclined sciences, such as cosmology and evolutionary biology.

The old problems of philosophy, such as mind and matter, reality, perception, are least likely to have 'solutions'. In fact, we would be hard put to spell out what might be acceptable as a 'solution'. The term 'solution' is borrowed from mathematics, and tacitly presupposes an analogy between problems of philosophy and problems of mathematics that is seriously misleading. Perhaps the use of the word 'problem' in philosophy raised expectations that philosophy could not fulfill.

Philosophers of our day go one step farther in their mis-analogies between philosophy and mathematics. Driven by a misplaced belief in definitiveness measured in terms of problems solved, and realizing the futility of any attempt to produce definitive solutions to any of the classical problems, they have had to change the problems. And where do they think to have found problems worthy of them? Why, in the world of facts!

Science deals with facts. Whatever it is that traditional philosophy deals with, it is not facts in the scientific sense. Therefore, traditional philosophy is worthless.

This syllogism, wrong on several counts, is predicated on the assumption that no statement is of any value, unless it is a statement of fact. Instead of realizing the absurdity of this assumption, philosophers have

swallowed it, hook, line and sinker, and have busied themselves in making their living on facts.

But previous philosophers had never been equipped to deal directly with facts, nor had they ever considered facts to be any of their business. Nobody turns to philosophy to learn facts. Facts are the domain of science, not of philosophy. And so, a new slogan had to be coined: philosophy *should* be dealing with facts.

This 'should' comes at the end of a long line of other 'should's'. Philosophy should be precise; it should follow the rules of mathematical logic; it should define its terms carefully; it should ignore the lessons of the past; it should be successful at solving its problems; it should produce definitive solutions.

"Pigs should fly", as the old saying goes.

But what is the standing of such 'should's', flatly negated as they are by two thousand years of philosophy? Are we to believe the not so subtle insinuation that the royal road to right reasoning will at last be found if we follow these imperatives?

There is a more plausible explanation of this barrage of should's. The reality we live in is constituted by a myriad contradictions, which traditional philosophy has taken pains to describe with courageous realism. But contradiction cannot be confronted by minds who have put their salvation in precision and definitiveness. The real world is filled with absences, with absurdities, with abnormalities, with aberrances, with abominations, with abuses, with *Abgrund*. But our latter-day philosophers are not concerned with facing up to these unpleasant features of the world, nor, to be sure, to any real features whatsoever. They would rather tell us what the world should be like. They find it safer to escape from distasteful description of what is into pointless prescription of what isn't. Like ostriches with their heads in the ground, they will meet the fate of those who refuse to acknowledge the lessons of the past and to meet the challenge of our difficult present: increasing irrelevance followed by eventual extinction.

NOTES

* Portions of the present text have previously appeared in *The Review of Metaphysics* **44** (1990), 259–271, are reprinted with permission.

[1] Kac, M., G-C. Rota, and J. T. Schwartz: 1986, *Discrete Thoughts*: *Essays on Mathematics Science and Philosophy*, Birkhäuser, Boston, pp. 19–25.

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