

Elasticité d'un brin de laine

$$(1) \quad L = N_\alpha a + (N - N_\alpha) b = N_\alpha(a-b) + Nb$$

$$(2) \quad E = N_\alpha E_\alpha + (N - N_\alpha) E_\beta - FL$$

$$= N_\alpha (E_\alpha - E_\beta) + N E_\beta$$

$$- F N_\alpha (a-b) - F N b$$

$$= N_\alpha (E_\alpha - E_\beta - F(a-b)) + N (E_\beta - Fb)$$

$$\left(\frac{\partial E}{\partial N_\alpha} \right)_N = E_\alpha - E_\beta - F(a-b)$$

$$(3) \quad \mathcal{P} = \binom{N}{N_\alpha} = \frac{N!}{N_\alpha! (N - N_\alpha)!}$$

$$(4) \quad S = k \ln(\mathcal{P}) = k \left(\ln N! - \ln N_\alpha! - \ln (N - N_\alpha)! \right)$$

$$= k \left(N \ln N - N_\alpha \ln N_\alpha - (N - N_\alpha) \ln (N - N_\alpha) \right)$$

$$\left(\frac{\partial S}{\partial N_\alpha} \right)_N = k \left(- \ln N_\alpha - \frac{N_\alpha}{N_\alpha} + \ln (N - N_\alpha) + \frac{(N - N_\alpha)}{N - N_\alpha} \right)$$

$$= k \left(- \ln N_\alpha + \ln (N - N_\alpha) \right)$$

$$\stackrel{N_\alpha \gg 1}{=} k \ln \left(\frac{N}{N_\alpha} - 1 \right)$$

$$\textcircled{5} \quad \frac{1}{T} = \frac{\partial S}{\partial E} \quad ; \quad \text{définition de } T$$

$$= \left(\frac{\partial S}{\partial N_\alpha} \right) \left(\frac{\partial N_\alpha}{\partial E} \right)$$

$$= k \ln \left(\frac{N}{N_\alpha} - 1 \right) \frac{1}{(E_\alpha - E_\beta - F(a-b))}$$

$$\textcircled{6} \quad \text{D'après } \textcircled{1} \quad N_\alpha = \frac{L - Nb}{a-b} \quad ,$$

et d'après $\textcircled{5}$,

$$\frac{1}{T} = \frac{k \ln \left(\frac{N(a-b)}{L-Nb} - 1 \right)}{E_\alpha - E_\beta - F(a-b)}$$

on pose

$$\left. \begin{aligned} \Delta &= \frac{E_\alpha - E_\beta}{kT} \\ \gamma &= \frac{F(a-b)}{kT} \end{aligned} \right\} \quad ; \quad \text{paramètre sans dimension} \\ \text{qui compare } kT \text{ à} \\ \text{la différence d'énergie} \\ \text{des monomères}$$

$$\text{call} \frac{1}{T} = \frac{h \ln \left(\frac{N(a-b)}{L-Nb} \right)}{hT(\Delta - \gamma)}$$

$$= N_{\alpha} \left(E_{\alpha} - E_{\beta} - F(a-b) \right) + N \left(E_{\beta} - Fb \right)$$