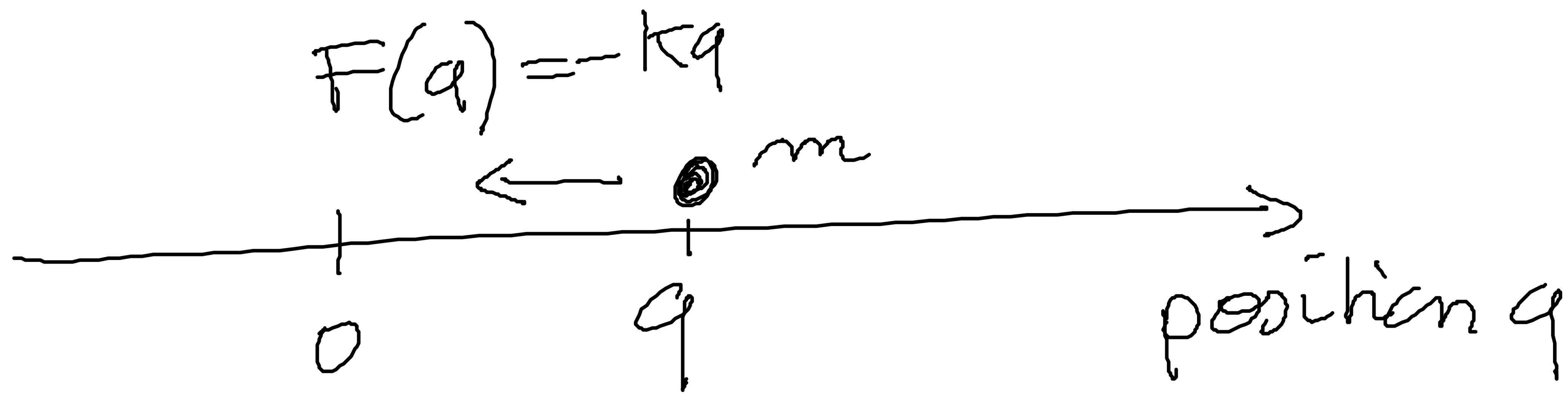
23/12/19

Oscillateur Harmonique et formule de Weyl



① $$E = H(q,p) = \frac{p^2}{2m} + \frac{1}{2} k q^2$$

$$\begin{cases} \dot{q} = \dfrac{\partial H}{\partial p} = \dfrac{p}{m} \\ \dot{p} = -\dfrac{\partial H}{\partial q} = -kq = F(q) \end{cases}$$: équ. de Hamilton

$$\Longleftrightarrow \quad m\ddot{q} = \dot{p} = F(q)$$: loi de Newton

② L'énergie $E(t) = H(q(t), p(t))$ est conservée

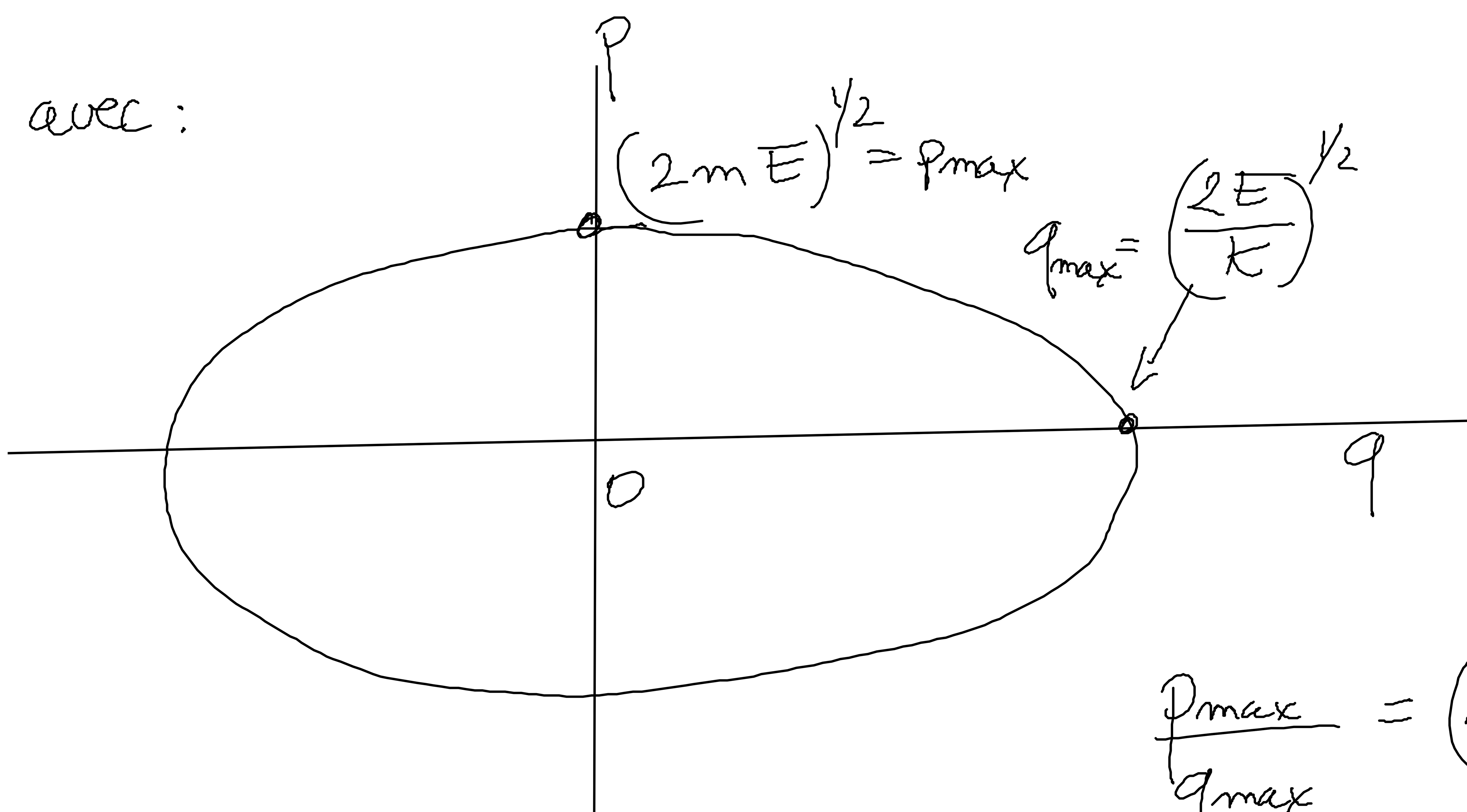
car $\dfrac{\partial E}{\partial t} = (\partial_q H)\,\dot q + (\partial_p H)\,\dot p = (\partial_q H)(\partial_p H) - (\partial_p H)(\partial_q H)$

$= 0$

donc $E = \dfrac{p^2}{2m} + \dfrac{1}{2} K q^2 = \text{cte}/t$: equation d'une ellipse

$$\Longleftrightarrow \left(\frac{p}{p_{max}}\right)^2 + \left(\frac{q}{q_{max}}\right)^2 = 1$$

avec :



$$\frac{p_{max}}{q_{max}} = (mK)^{1/2}$$

<u>Surface de l'ellipse :</u>

$$\mathrm{Vol}(\{q, p \text{ tq } H(q,p) \leq E\}) = \pi\, q_{max}\, p_{max}$$

$$= \pi\, 2E \sqrt{\frac{m}{K}}$$

$$= \frac{2\pi E}{\omega} \quad \text{avec } \omega := \sqrt{\frac{K}{m}}$$

on montre ci dessous que ω est la fréquence

Posons $X := \dfrac{q}{q_{max}}$, $Y := \dfrac{p}{p_{max}}$

$$z = X + iY \in \mathbb{C}$$

alors $\dot{z} = \dfrac{\dot{q}}{q_{max}} + i\dfrac{\dot{p}}{p_{max}} = \dfrac{p}{m q_{max}} - i\dfrac{kq}{p_{max}}$

$$= \frac{p_{max}}{m\, q_{max}} Y - i\frac{k q_{max}}{p_{max}} X$$

$=$