

Statistiques quantiques

$$\textcircled{1} \quad p(a_j) = \sum_i p_i \cdot P_i(a_j | \psi_i)$$
$$= \sum_i p_i \frac{\langle \psi_i | P_{\varphi_j} \psi_i \rangle}{\langle \psi_i | \psi_i \rangle}$$

$$= \sum_i \text{Tr} \left(p_i \frac{|\psi_i\rangle\langle\psi_i|}{\langle\psi_i|\psi_i\rangle} P_{\varphi_j} \right)$$
$$= \text{Tr} \left(\hat{\rho} P_{\varphi_j} \right)$$

donc

$$\langle A \rangle := \sum_j p(a_j) a_j \quad = \text{moyenne statistique}$$

$$= \sum_j \text{Tr}(\hat{\rho} P_{\varphi_j}) a_j = \text{Tr} \left(\hat{\rho} \sum_j P_{\varphi_j} a_j \right)$$
$$= \text{Tr}(\hat{\rho} \hat{A})$$

$$\text{car } \hat{A} = \sum_j a_j P_{\varphi_j}$$

② entropie :

$$S(\vec{p}, \hat{A}) := - \sum_j p(a_j) \log p(a_j)$$

si $\hat{A} = \vec{f}$, alors,

$$\begin{aligned} S(\vec{f}) &:= S(\vec{p}, \vec{f}) = - \sum p_i \log p_i \\ &:= - \text{Tr}(\vec{f} \log \vec{f}) \end{aligned}$$

Si $\vec{f} = |\psi\rangle\langle\psi|$, alors $(p_i)_i = (1, 0, \dots, 0)$

$$S(\vec{f}) = 0.$$