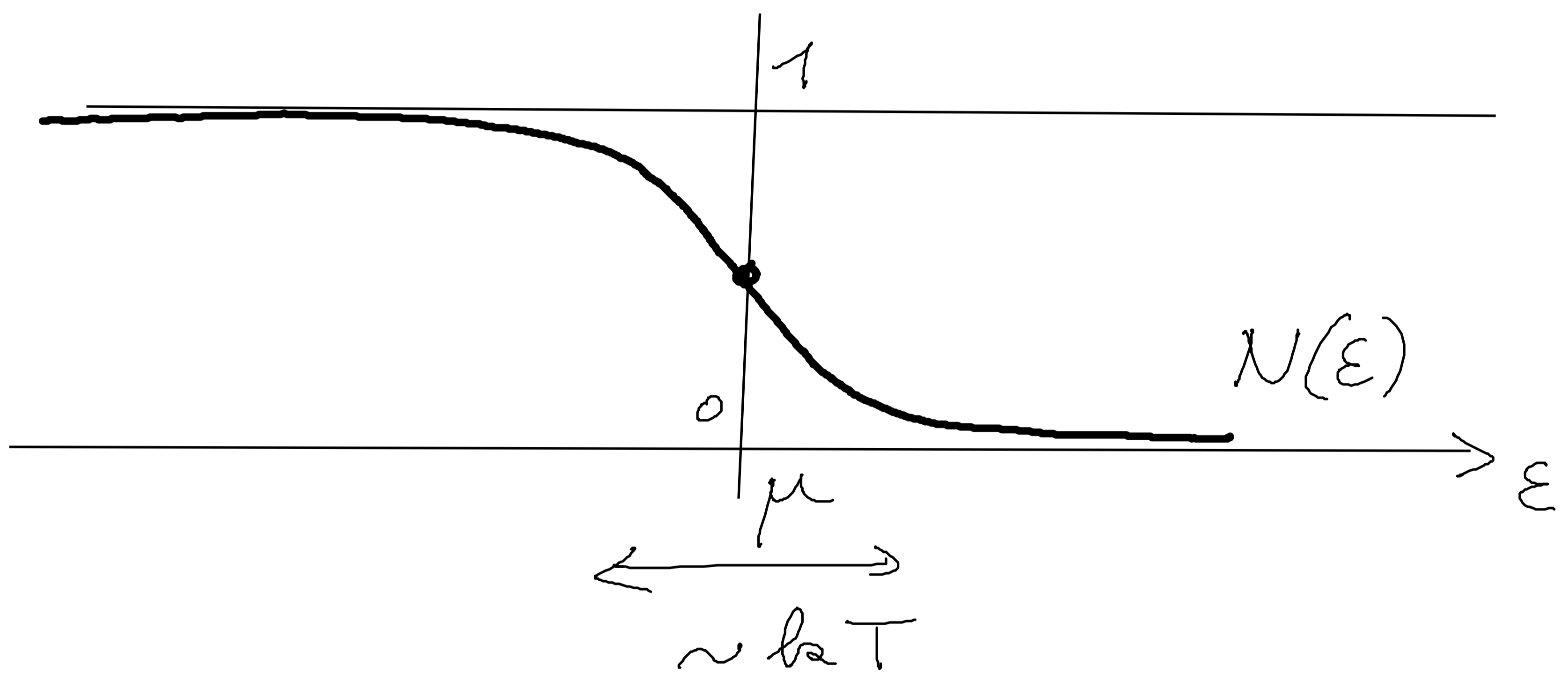


Expansion de Sommerfeld

μ : potentiel chimique

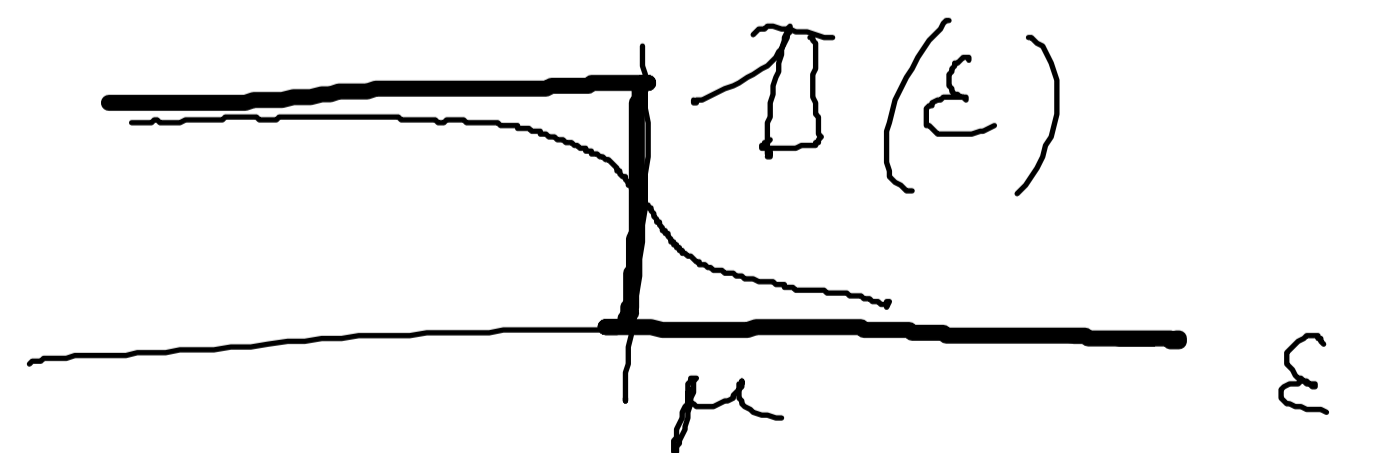
$$\beta = \frac{1}{k_B T}$$

$$N(\varepsilon) = \frac{1}{e^{\beta(\varepsilon - \mu)} + 1} \quad : \text{Distribution de Fermi}$$

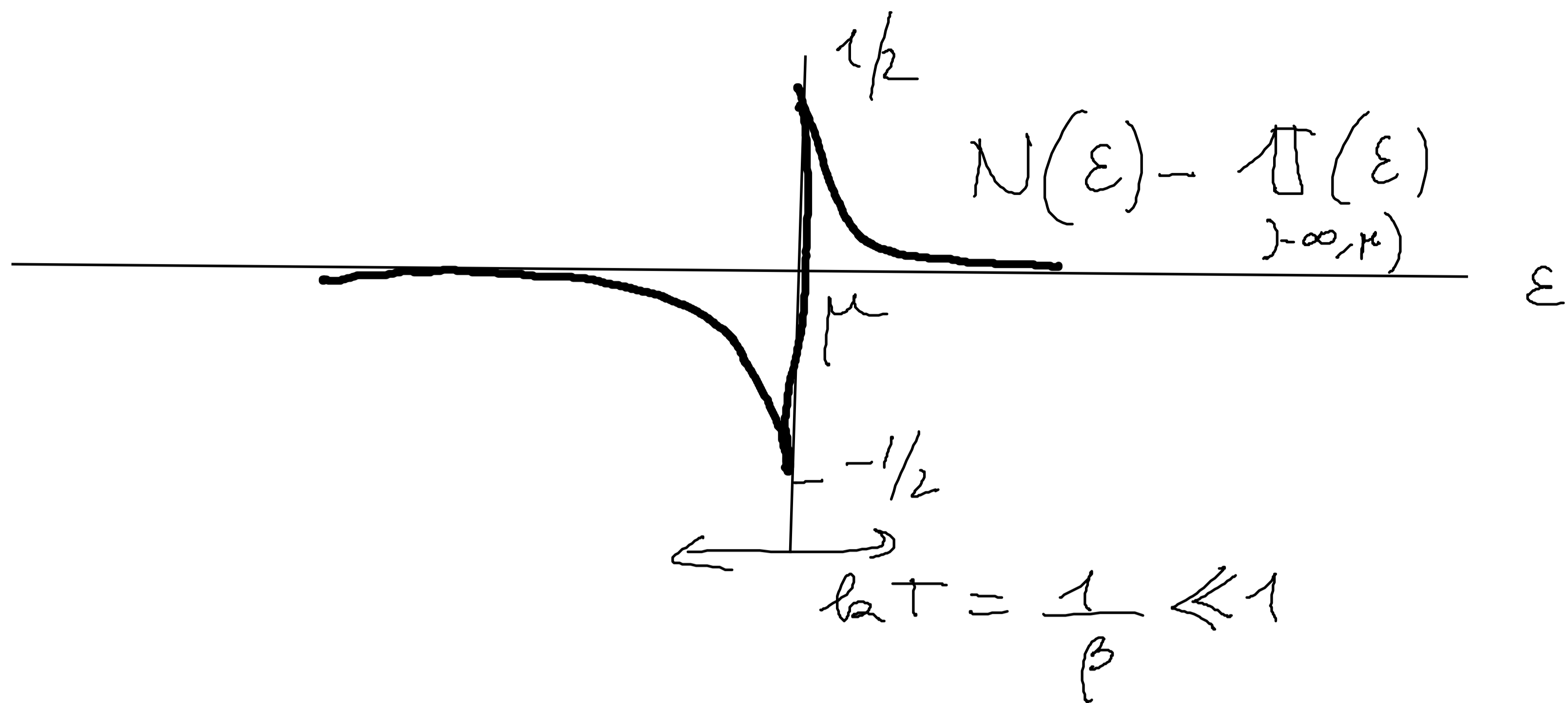


Pour $\beta \rightarrow \infty \Leftrightarrow T \rightarrow 0$,

$$N(\varepsilon) = \Pi_{-\infty, \mu}(\varepsilon).$$



(appelée fonction indicatrice)



• Posons le changement de variables

$$\varepsilon \rightarrow x = \beta(\varepsilon - \mu) \Leftrightarrow \varepsilon = \frac{x}{\beta} + \mu$$

et $D(x) = N(\varepsilon) - \Pi(\varepsilon)$

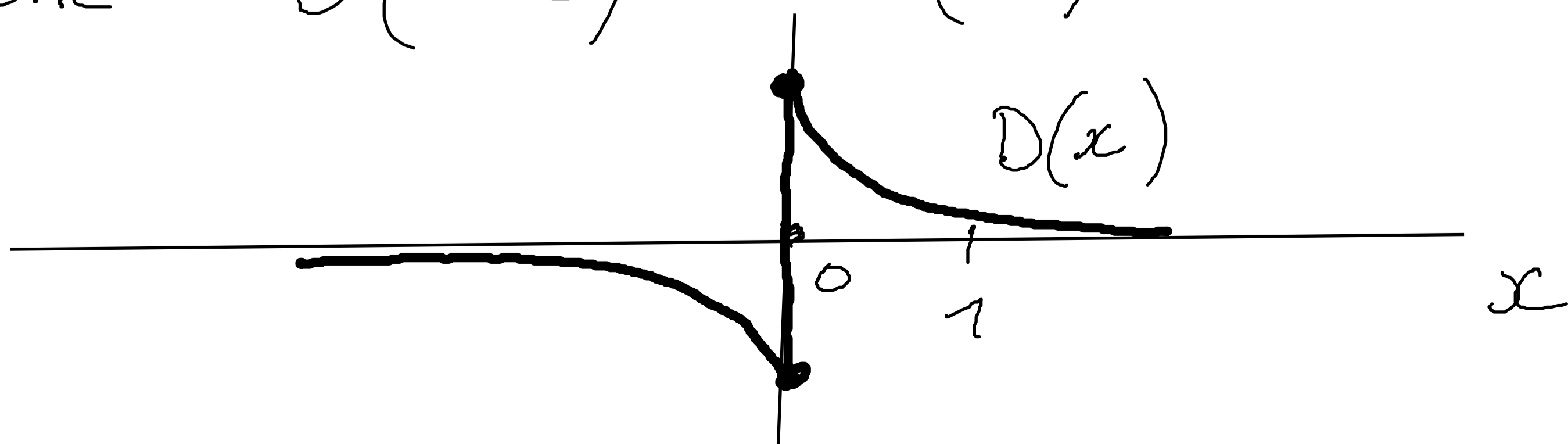
si $x < 0$,

$$D(x) = \frac{1}{e^x + 1} - 1 = \frac{-e^x}{e^x + 1} = \frac{-1}{1 + e^{-x}}$$

si $x > 0$,

$$D(x) = \frac{1}{1 + e^x}$$

Donc $D(-x) = -D(x)$: fonction impaire



• Soit $u(\varepsilon)$ une fonction "test"

$$\int u(\varepsilon) N(\varepsilon) d\varepsilon = \int_{-\infty, \mu}^{\mu} u(\varepsilon) \Pi(\varepsilon) d\varepsilon + \int u(\varepsilon) D(x) d\varepsilon$$
$$= \int_{-\infty}^{\mu} u(\varepsilon) d\varepsilon + \int_{\mathbb{R}} u\left(\frac{x}{\beta} + \mu\right) D(x) \frac{dx}{\beta}$$

Considérons le 2^{ème} terme.

Pour $\beta \gg 1, \Leftrightarrow \frac{1}{\beta} \ll 1,$

$$u\left(\mu + \frac{x}{\beta}\right) = u(\mu) + u'(\mu) \frac{x}{\beta} + O\left(\frac{1}{\beta^2}\right)$$

alors

$$\int_{\mathbb{R}} u\left(\frac{x}{\beta} + \mu\right) D(x) \frac{dx}{\beta} = \frac{u(\mu)}{\beta} \underbrace{\int D(x) dx}_{=0 \text{ car D impaire}}$$
$$+ \frac{u'(\mu)}{\beta^2} \int x D(x) dx + O\left(\frac{1}{\beta}\right)$$

or $\int_0^{\infty} \frac{x}{1+e} dx$