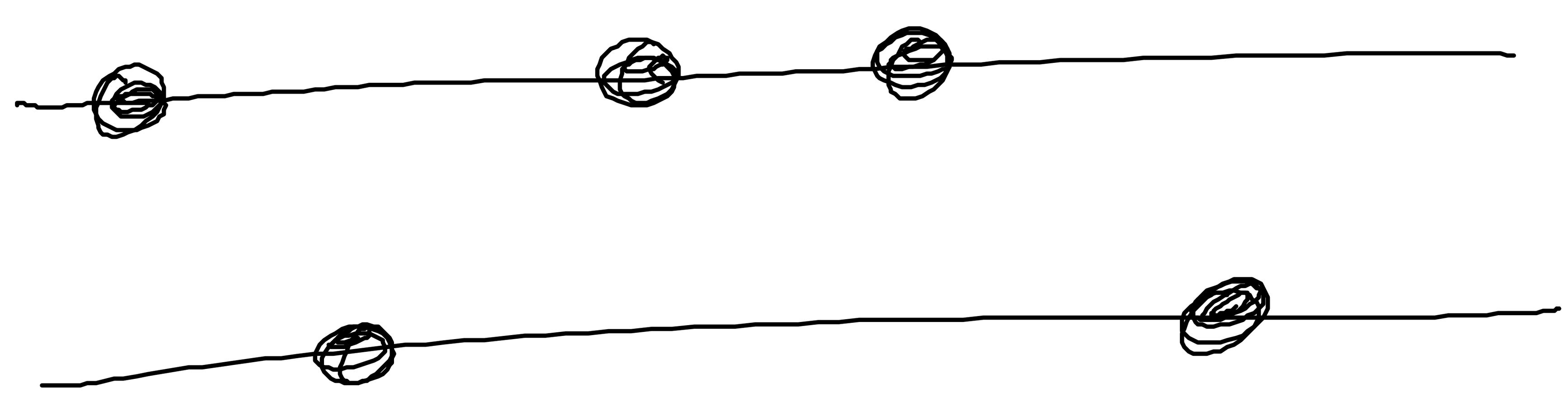
Température négative

(1)



ex :
$N = 5$
$N_1 = 3$

$$E = N_0 \underbrace{\varepsilon_0}_{=0} + N_1 \varepsilon_1 = N_1 \varepsilon_1$$

à E fixé $\Longleftrightarrow$ N_1 fixé, le nombre de configurat° est

$$\Omega = \binom{N}{N_1} = \frac{N!}{N_1!\,(N-N_1)!}$$

donc $S(E) = k \ln \Omega$

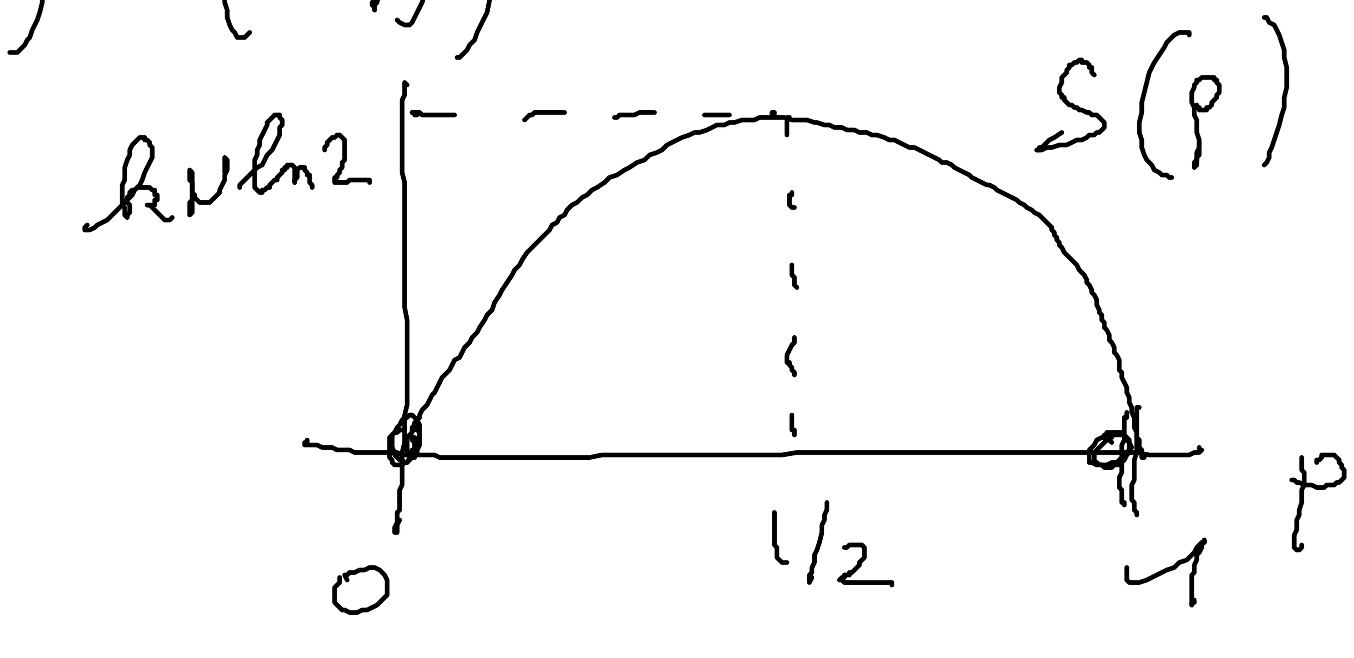
$$\sim k \left(N \ln N - N_1 \ln N_1 - (N-N_1) \ln (N-N_1) \right)$$

posons $p = \dfrac{N_1}{N} = \dfrac{E}{\varepsilon_1 N} \in [0,1]$

alors $S = k\left(N \ln N - pN \ln (pN) - N(1-p) \ln (N(1-p))\right)$

$$= k\left(\cancel{N \ln N} - pN \ln p - \cancel{pN \ln N} - \cancel{N(1-p) \ln N} - N(1-p)\ln(1-p)\right)$$

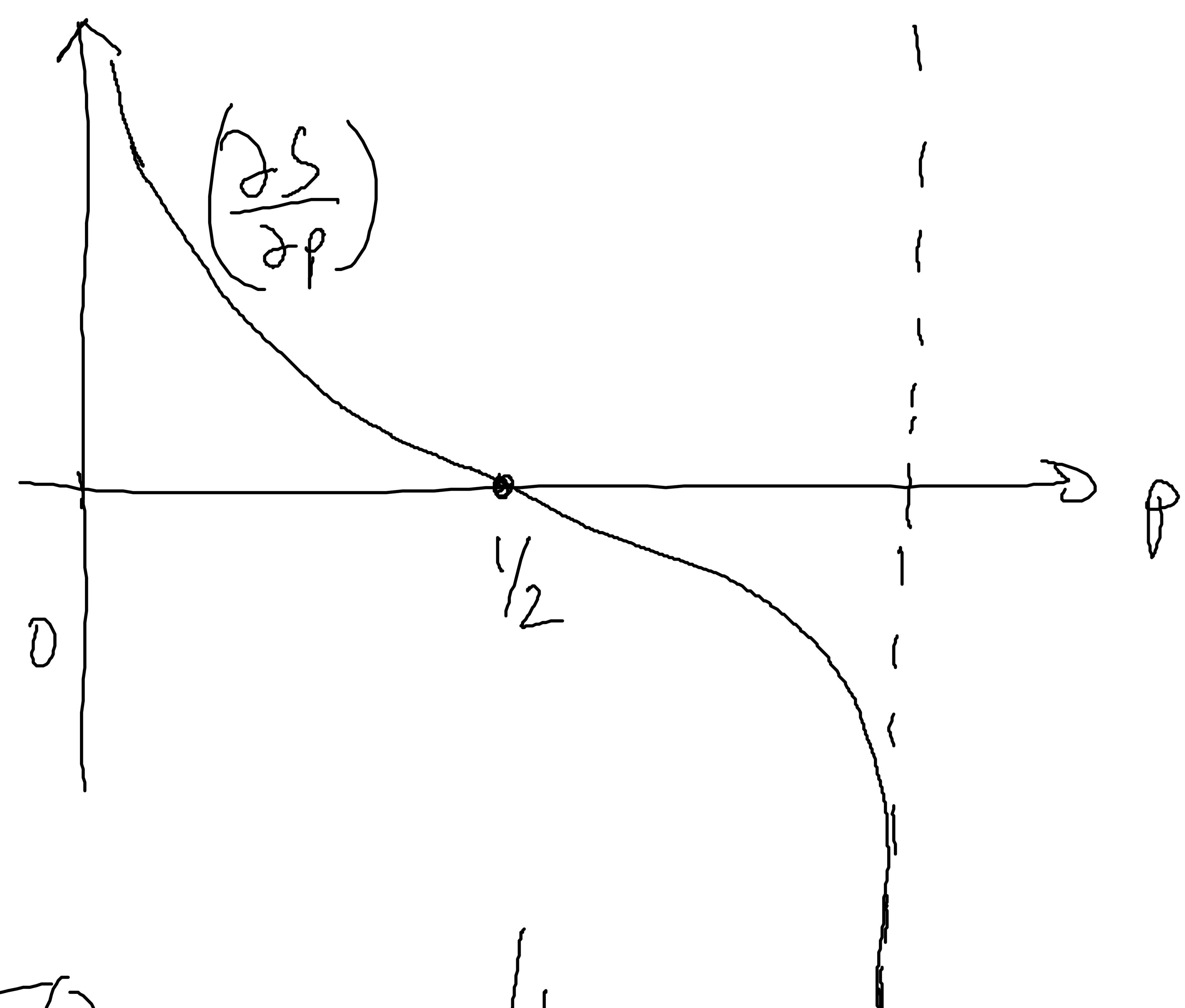
$$= kN\left(-p \ln p - (1-p) \ln (1-p)\right)$$

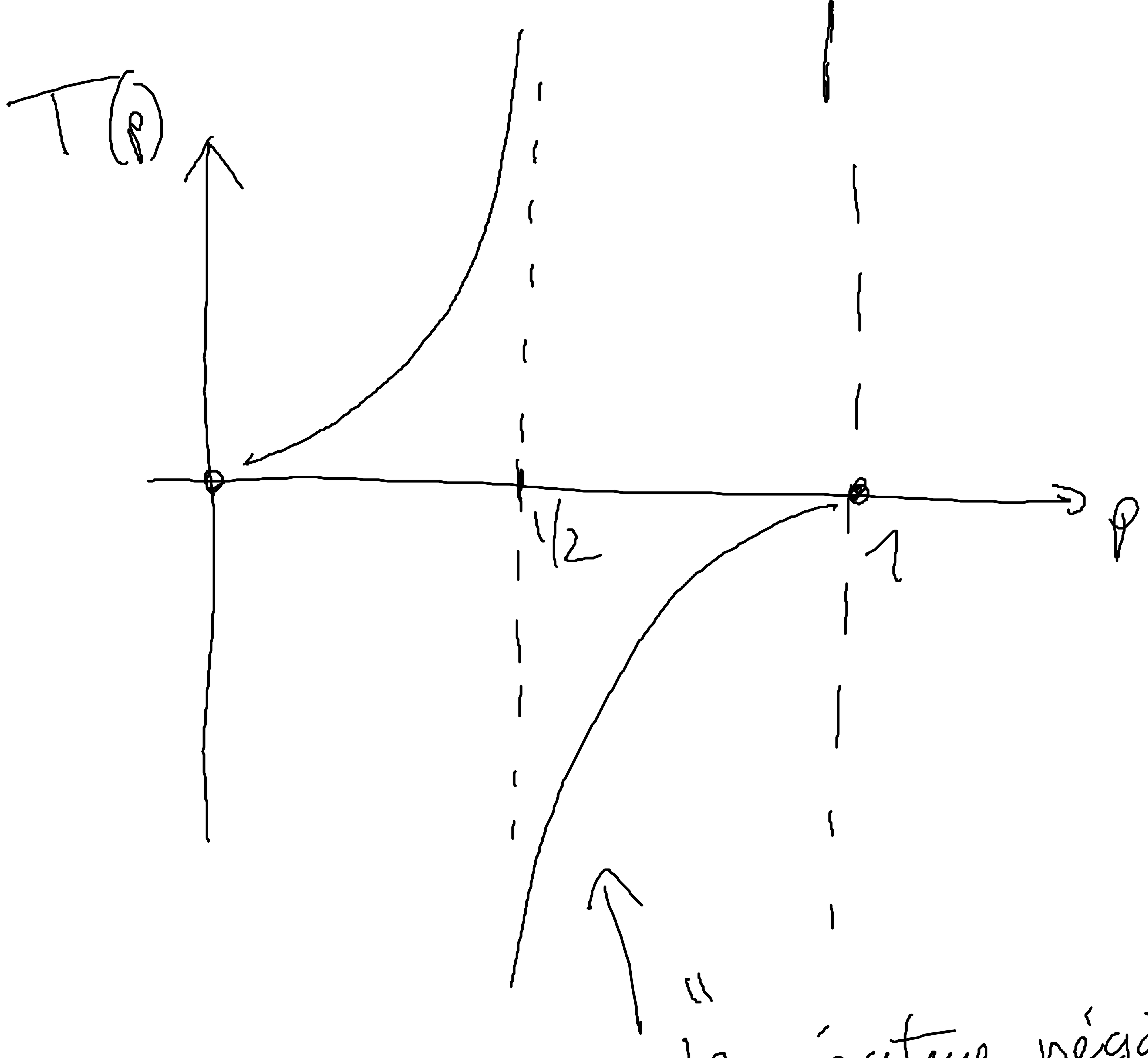


② $\frac{1}{T} = \frac{\partial S}{\partial E} = \left(\frac{\partial S}{\partial \rho}\right)\left(\frac{\partial \rho}{\partial E}\right) = \frac{1}{\varepsilon N}\left(\frac{\partial S}{\partial \rho}\right)$

or $\frac{\partial S}{\partial \rho} = kN\left(-\ln\rho - 1 + \ln(1-\rho) + 1\right)$

$= kN\ln\left(\frac{1-\rho}{\rho}\right) = kN\ln\left(\frac{1}{\rho} - 1\right)$





"température négative" car l'entropie diminue avec E.

③ A l'équilibre $T = T_{ext}$,

$$\Longleftrightarrow \left(\frac{\partial S}{\partial E}\right) = \frac{1}{T_{ext}}$$

$$\Longleftrightarrow \left(\frac{\partial S}{\partial p}\right) = \left(\frac{\partial E}{\partial p}\right) \cdot \frac{1}{T_{ext}}$$

comme : $p \in [0,1] \longrightarrow \frac{\partial S}{\partial p} \in \mathbb{R}$ est une bijection,

cela détermine p, selon $T_{ext} \in \mathbb{R}$.

$$\Longleftrightarrow k N \ln\left(\frac{1}{p} - 1\right) = \frac{\varepsilon_1 N}{T_{ext}}$$

$$\Longleftrightarrow \frac{1}{p} = \exp\left(\frac{\varepsilon_1}{k T_{ext}}\right) + 1$$

$$\Longleftrightarrow p = \frac{1}{1 + e^{\frac{\varepsilon_1}{k T_{ext}}}}$$