

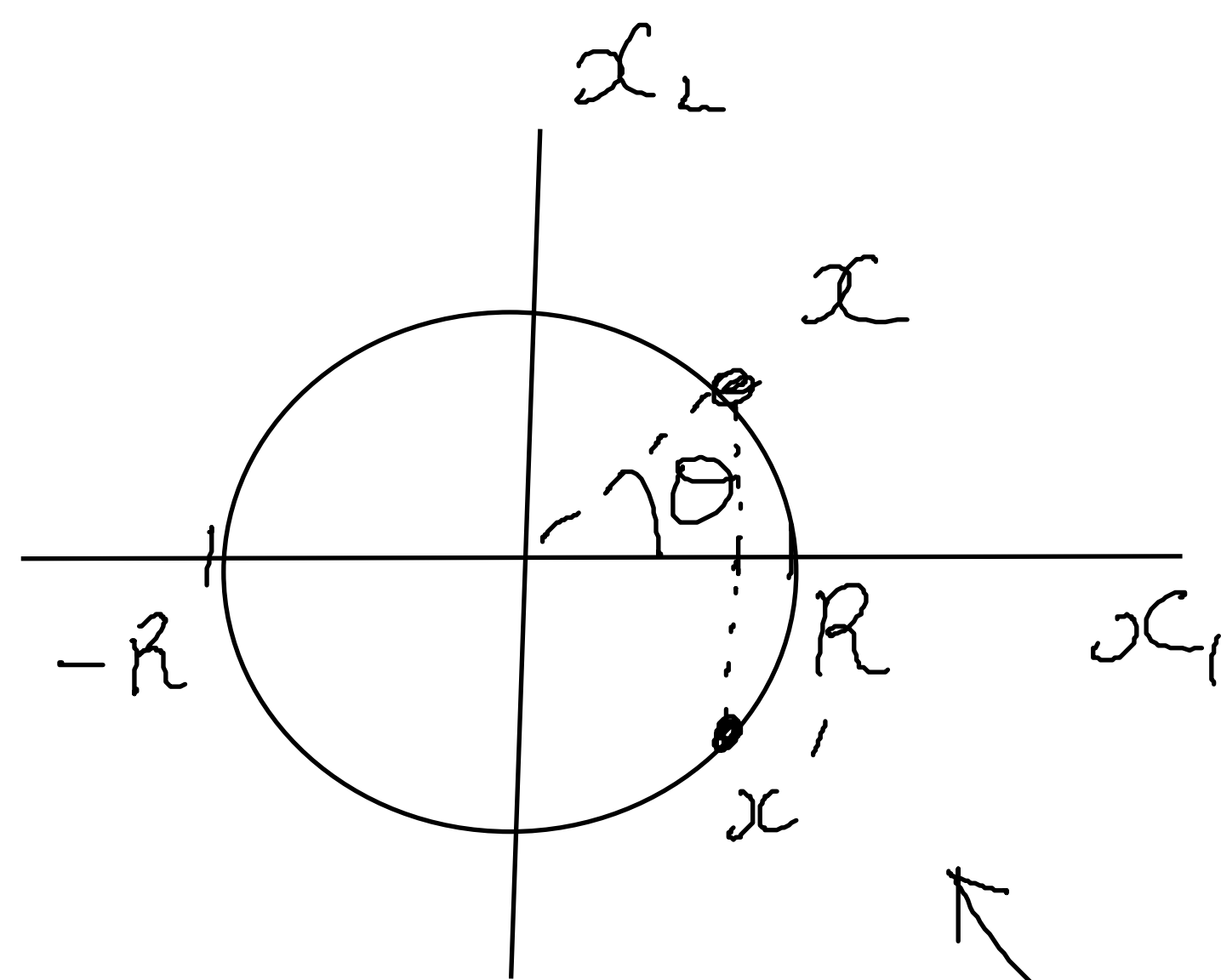
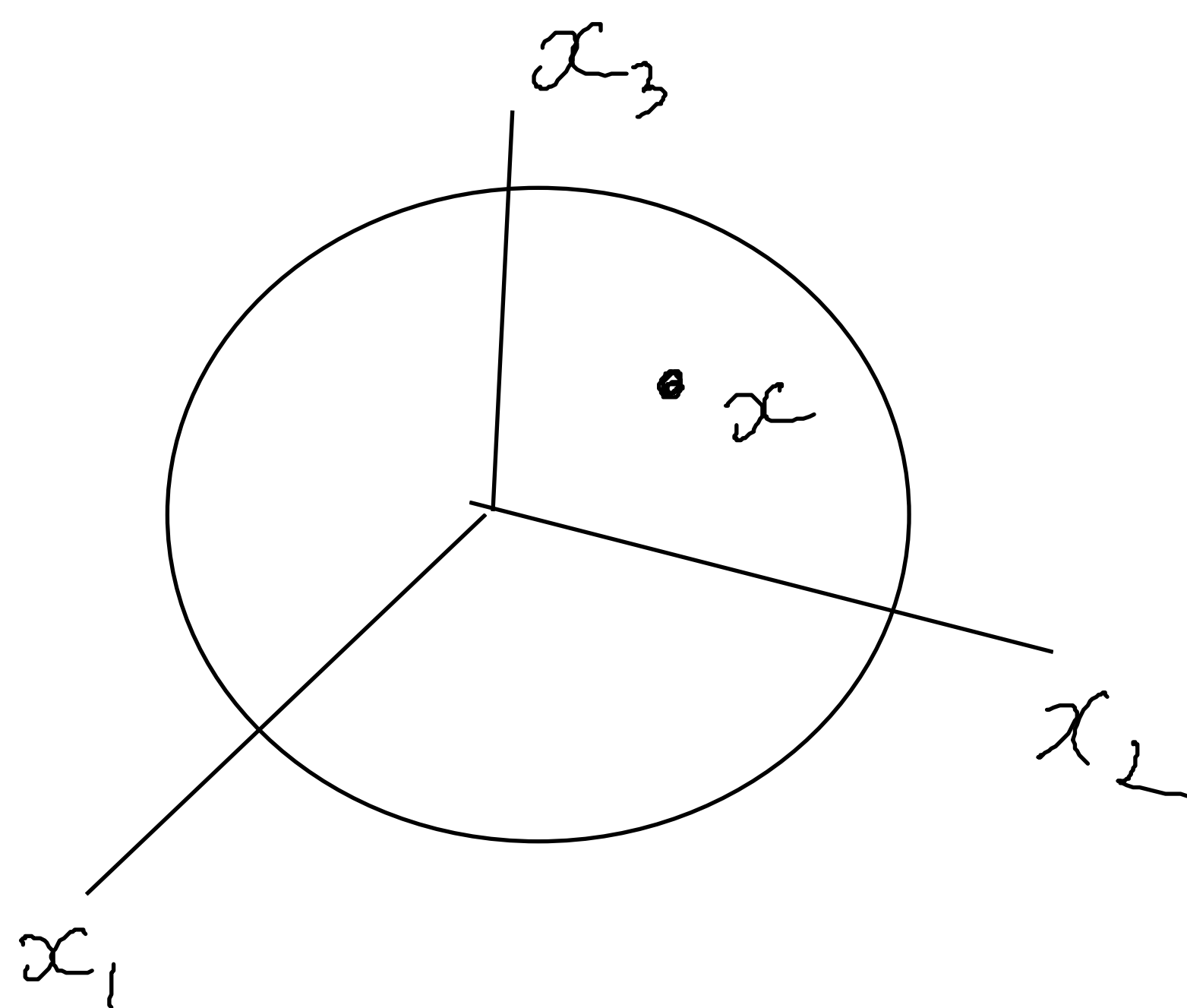
# Loi de Boltzmann dans un modèle simple

$$x = (x_1, \dots, x_N) \in \mathbb{R}^N$$

distribuée aléatoirement sur la sphère

$$S_R^{N-1}$$

$$x_1^2 + x_2^2 + \dots + x_N^2 = R^2$$



① si  $N=2$ ,

$$\text{on écrit } x_1 = R \cos \theta$$

$$x_2 = R \sin \theta$$

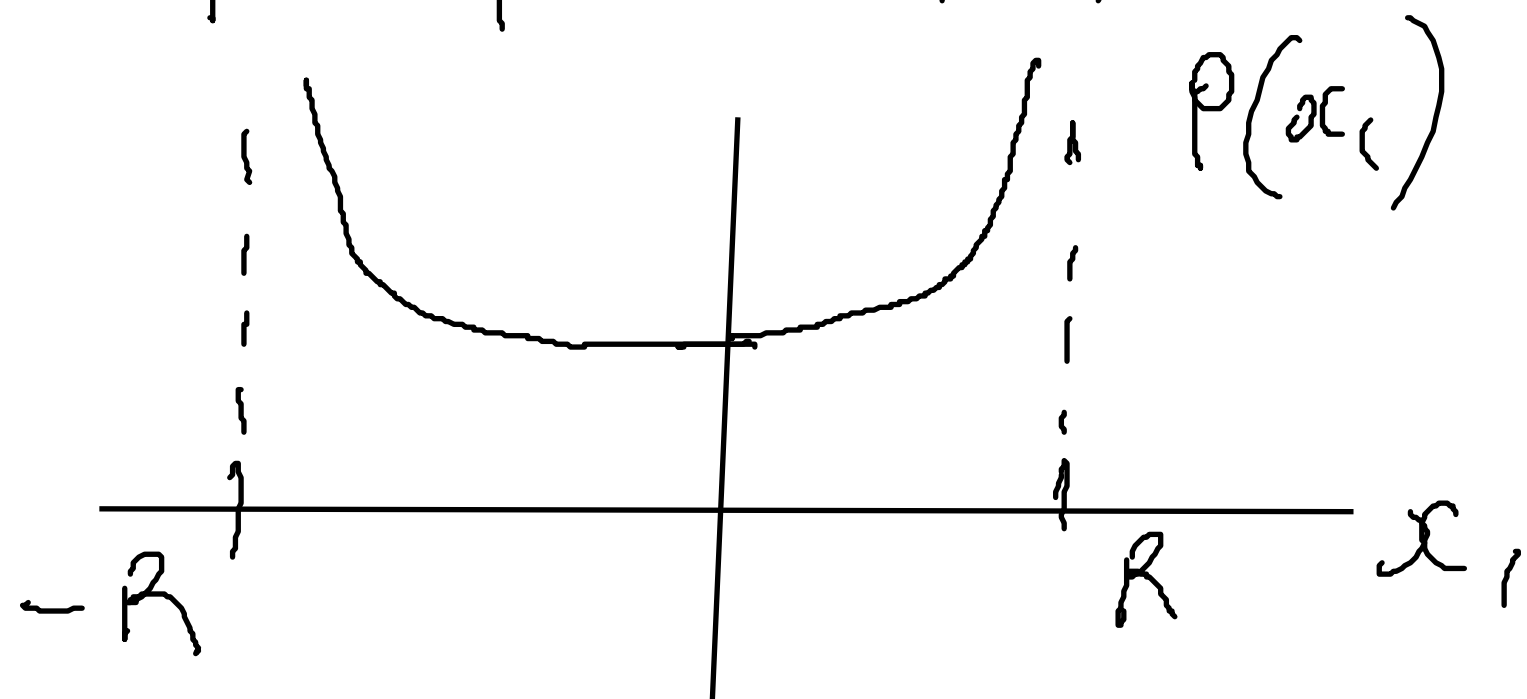
$\theta \in [0, 2\pi]$  uniforme

$$P(x_1) dx_1 = 2 \cdot P(\theta) d\theta = \frac{2}{2\pi} d\theta$$

rem : facteur 2  
car  $x_1$  a deux images  $x, x'$

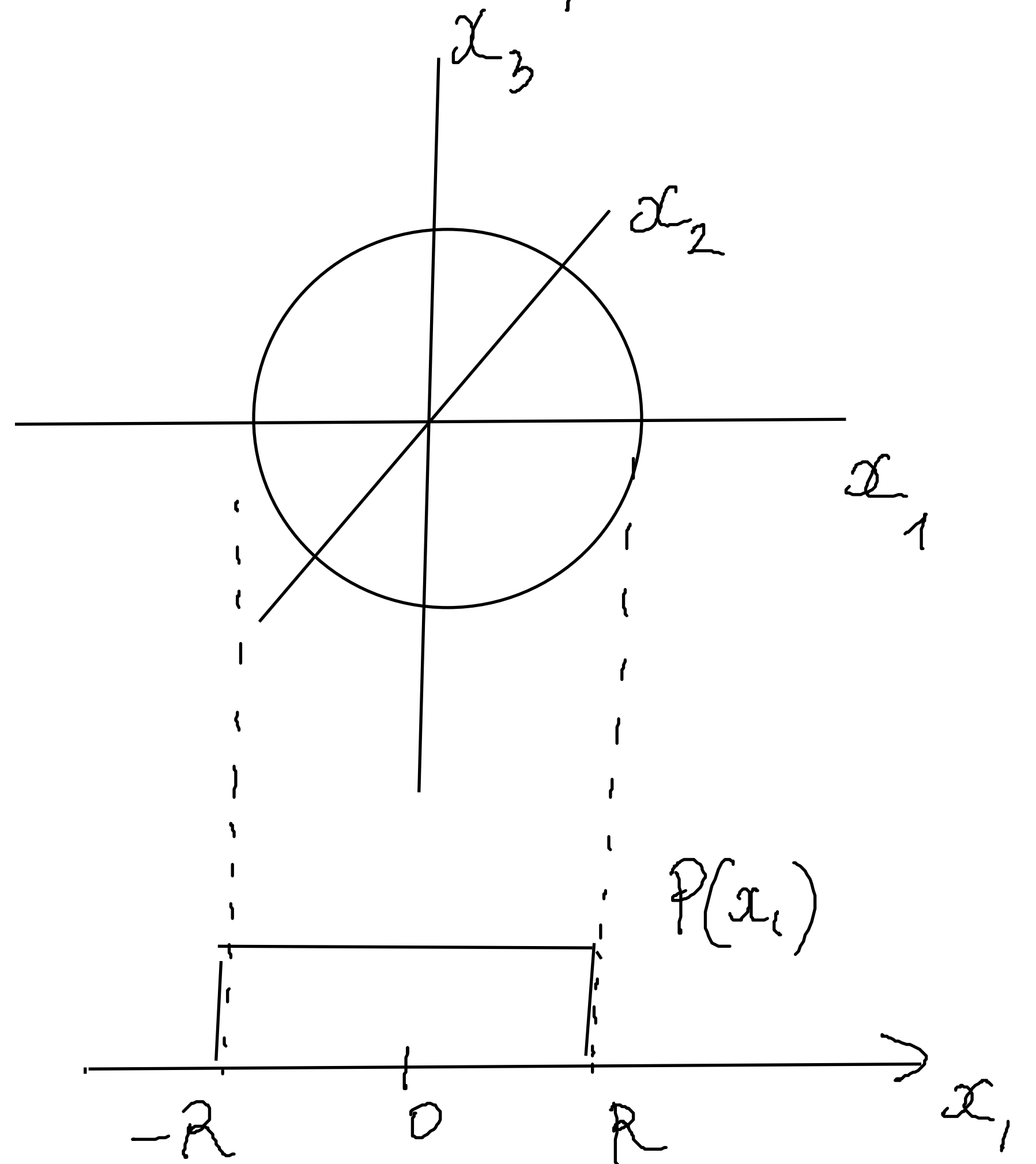
$$\Leftrightarrow P(x_1) = \frac{1}{\pi} \left| \left( \frac{dx_1}{d\theta} \right)^{-1} \right| = \frac{1}{\pi} \frac{1}{R |\sin \theta|} = \frac{1}{\pi |x_2|}$$

$$P(x_1) = \frac{1}{\pi \sqrt{R^2 - x_1^2}}$$

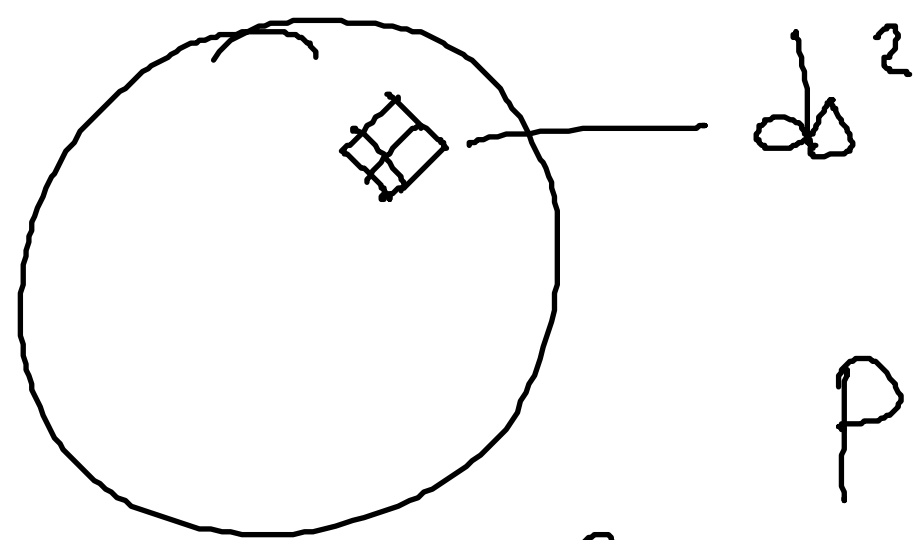


• si  $N=3$ , on écrit en coordonnées sphériques,

$$\begin{cases} x_1 = R \cos \theta \\ x_2 = R \sin \theta \cos \varphi \\ x_3 = R \sin \theta \sin \varphi \end{cases}$$



La mesure uniforme sur la  
 $\left( \frac{1}{4\pi} \sin \theta d\theta d\varphi \right)$



$$P = c ds^2$$

$$1 = \int P = c \int R^2 \sin \theta d\theta d\varphi = c R^2 4\pi \Rightarrow P =$$

donc

$$P(x_1) dx_1 = \frac{1}{4\pi} \int_{\varphi=0}^{2\pi} \sin \theta d\varphi d\theta$$

$$= \frac{1}{2} \sin \theta d\theta$$

$$\Leftrightarrow P(x_1) = \frac{\sin \theta}{2 \left| \frac{dx_1}{d\theta} \right|} = \frac{\sin \theta}{2R \sin \theta} = \frac{1}{2R}$$

$$P(x_1) = \frac{1}{2R} : \text{loi uniforme}$$