Statistiques quantiques

(1) $$p(a_j) = \sum_i p_i \cdot P_i(a_j | \psi_i)$$

$$= \sum_i p_i \frac{\langle \psi_i | P_{\varphi_j} \psi_i \rangle}{\langle \psi_i | \psi_i \rangle}$$

$$= \sum_i \mathrm{Tr}\left(p_i \frac{|\psi_i\rangle\langle\psi_i|}{\langle\psi_i|\psi_i\rangle} P_{\varphi_j} \right)$$

$$= \mathrm{Tr}\left(\hat{\rho}\, P_{\varphi_j} \right)$$

donc

$$\langle A \rangle := \sum_j p(a_j)\, a_j$$: moyenne statistique

$$= \sum_j \mathrm{Tr}(\hat{\rho}\, P_{\varphi_j})\, a_j = \mathrm{Tr}\left(\hat{\rho} \sum_j P_{\varphi_j} a_j\right)$$

$$= \mathrm{Tr}\left(\hat{\rho}\, \hat{A}\right)$$

car $\hat{A} = \sum_j a_j P_{\varphi_j}$

(2) entropie :

$$S(\hat{\rho}, \hat{A}) := -\sum_j p(a_j) \log p(a_j)$$

si $\hat{A} = \hat{\rho}$, alors,

$$S(\hat{\rho}) := S(\hat{\rho}, \hat{\rho}) = -\sum p_i \log p_i$$

$$=: -\mathrm{Tr}\left(\hat{\rho} \log \hat{\rho}\right)$$

Si $\hat{\rho} = |\psi\rangle\langle\psi|$, alors $(p_i)_i = (1, 0, \cdots 0)$

$$S(\hat{\rho}) = 0.$$