A MATHEMATICAL MODEL OF THE GRINDING WHEEL PROFILE REQUIRED FOR A SPECIFIC TWIST DRILL FLUTE

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Abstract—Obtaining a straight cutting edge in the case of small twist drills warrants an accurate grinding of the "primary" flute. A transformed mathematical model which would yield the proper flute-grinding wheel profile for a straight cutting edge is developed from an existing orthogonal flute model. Also discussed in this paper is a mathematical model that describes the wheel profile for the "secondary" (non-cutting edge) flute as a function of parameters such as land width, web thickness and chip removal capacity of the drill. The model enables an analysis of the effects of improper wheel setting and point grinding on the straightness of the cutting edge. From this analysis, an optimum wheel profile that would grind a maximum number of acceptable flutes can be obtained.

1. INTRODUCTION

It is desirable that the cutting edge of a twist drill be straight in shape [6]. Since the cutting edge is formed by the intersection of the drill's flank and flute surfaces, its shape is governed by the configuration of these surfaces. Therefore, for a given flank configuration, to obtain a straight cutting edge requires the "primary" flute to be accurately ground. Consequently, the grinding wheel profile has to be accurately determined.

In the case of circuit board drills which are relatively small in diameter, obtaining a suitable flute-grinding wheel profile (for a straight cutting edge) can be difficult. In such cases, the wheel profile is normally obtained by a trial-and-error process which can be very tedious. A mathematical solution that is both fast and accurate is therefore needed.

Mathematical models for the wheel profile required to grind a general, helical flute were developed by Semenov [8] and Dibner et al. [2], but these models were not based upon the shape of the cutting edge. Galloway [4] (and later Tsai [10]) developed a mathematical model for the "orthogonal" flute corresponding to a straight cutting edge. However, since the flute-grinding wheel is set at an angle (the helix angle) to the drill axis, the "oblique" flute profile has to be determined (from the shape of the orthogonal flute) to obtain the required wheel profile.

Due to the helical geometry of the flute, the transformation between the orthogonal and oblique flute profiles is not simple and direct. Graphical methods such as that given by Chitale [1] are over-simplified and hence inaccurate, especially for grinding the flute of small drills.

The objectives of this study are, first, to present a mathematical approach to obtain the appropriate wheel profile (for a straight cutting edge) from transformation of the orthogonal flute profile modeled by Galloway [4]. Furthermore, factors such as land width, web-thickness, maximum chip-removing capacity and the grinding wheel thickness depend on the "secondary" flute (corresponding to the non-cutting edge). Earlier studies on the drill flute [1, 2, 3, 4, 8, 9] have not expressed the shape of the secondary flute as a function of these parameters. The second aspect of this study is a mathematical approach to determine the secondary flute shape and hence, the corresponding wheel profile, based on these parameters.

The third objective of this study is to perform a sensitivity analysis to understand how critical the effects of factors such as wheel setting and point grinding are on the cutting edge shape. Finally, from this sensitivity analysis, a mathematical technique to determine an optimum wheel profile is presented. Such a profile would enable the wheel to grind a maximum number of flutes, while yielding an acceptable (straight) cutting edge.
2. MATHEMATICAL DETERMINATION OF THE GRINDING WHEEL PROFILE

2.1. Wheel profile for primary (or cutting edge) flute

Figure 1 shows the cutting point of the drill with straight cutting edges. The primary flute is that portion of the drill flute which yields the cutting edge (through the intersection of the flute and flank surfaces, as shown in Fig. 2). Any point \( P' \) on the straight cutting edge has a corresponding point \( P_0 \) on the primary flute whose coordinates in the \( r-v \) polar reference system are given by Galloway [4] as

\[
\begin{align*}
    r &= t \csc \Psi \\
    v &= \Psi + \frac{t}{R_0} (\tan \zeta_0)(\cot \Psi)(\cot \rho)
\end{align*}
\]  

where \( t \) is the half-web thickness, \( R_0 \) is the radius of the drill, \( \zeta_0 \) is the helix angle of the drill (at the periphery), \( \rho \) is the half-point angle of the drill and \( \Psi \) is the angle between \( OP' \) and \( OX \) in Fig. 1.

![Fig. 1. The cutting point of the drill.](image)

![Fig. 2. Formation of cutting edge from intersection of flank and flute contours [9].](image)
In the X–Y coordinate system, the coordinates can be represented as
\[ x = r \cos \nu \quad \text{and} \quad y = r \sin \nu \] (2)
and for the outermost point \( B_0 \) of the flute, these coordinates are
\[ x_0 = R_0 \cos \nu_0 \quad \text{and} \quad y_0 = R_0 \sin \nu_0 \] (3)
where \( \nu_0 \) is the coordinate \( \nu \) for \( B_0 \).

By obtaining different \( x \) and \( y \) values (from equation 2), the "orthogonal" profile of the flute in the XY plane can be determined. However, as can be seen from Fig. 5 the wheel profile corresponds to the "oblique" profile of the flute in the X'Y' plane. Hence, the flute profile in the X'Y'Z' (or oblique) coordinate system, corresponding to the profile in the XYZ (or orthogonal) coordinate system, should be determined. This is done as follows.

Figure 3 shows the reference system X'Y'Z' and its relation with the reference system XYZ. The cross-section CB, of the flute helix, in the plane X'Y', corresponds to the portion of the wheel profile required to give the primary flute. This will correspond to the orthogonal flute profile CB, seen in the orthogonal plane XY.

In order to obtain the coordinates of \( B_1 \), it may be noted that \( B_1 \) lies on the orthogonal flute profile in plane PLB, at distance \( z_1 \) from the plane PLO. This profile has the same shape as the orthogonal profile in the plane PLO, but is rotated by angle \( \gamma_1 \), given by
\[ \gamma_1 = \left( \frac{z_1}{R_0} \right) \tan \lambda_0 \] (4)
since the lead of the helix, \( L = 2\pi R_0 / \tan \lambda_0 \). Through coordinate transformation, the coordinates of \( B_1 \) in the orthogonal reference system XYZ can hence be expressed in terms of the coordinates of \( B_0 \) as
\[
\begin{align*}
x_1 &= x_0 \cos \gamma_1 - y_0 \sin \gamma_1 \\
y_1 &= x_0 \sin \gamma_1 + y_0 \cos \gamma_1 \\
z_1 &= z_1.
\end{align*}
\] (5)

These can be transformed to yield the coordinates of \( B_1 \) in the oblique reference system X'Y'Z' as
\[
\begin{align*}
x_1' &= x_1 \cos \lambda_0 + z_1 \sin \lambda_0 \\
y_1' &= y_1 \\
z_1' &= 0 = -x_1 \sin \lambda_0 + z_1 \cos \lambda_0.
\end{align*}
\] (6)

Substitution of \( x_1 \) in terms of \( z_1 \) from equations 4 and 5 into the last of equation 6 yields
\[ [x_0 \cos (mz_1) - y_0 \sin (mz_1)] \sin \lambda_0 - z_1 \cos \lambda_0 = 0 \] (7)
where \( m = (\tan \lambda_0) / R_0 \) and \( x_0, y_0 \) can be determined from equation 3 for a given straight cutting edge. Using an iterative technique, \( z_1 \) can then be solved for.
The solution of \( z_1 \) yields \( x_1 \) and \( y_1 \) from equations 4 and 5 and hence \( x'_1, \ y'_1 \ (z'_1 = 0) \) from equation 6, which then specifies point \( B \) on the wheel profile. Similarly, any other point \( P \) along the profile \( C \) and in plane PLP (Fig. 3) can be determined, knowing the corresponding point \( P_0 \) on the orthogonal flute in plane PLO. Profile \( C \), which corresponds to the portion of the wheel profile required to grind the primary flute (for a straight cutting edge), is thus obtained.

2.2. Wheel profile for secondary (or non-cutting edge) flute

Here, an approximate determination of the part of the wheel profile (portion \( A_1C \) of the oblique flute in Fig. 3) required to obtain the portion \( A_0C \) of the orthogonal flute (Fig. 1) is described. Though this secondary flute is not as critical as the primary flute for the cutting action, its shape will affect (and is hence governed by) the land width, web thickness, chip removal capacity, strength of the cutting point and wheel thickness.

Referring to Fig. 4, if \( l \) is the required land width, then

\[
h_1 = l \cos \lambda_0 \quad \text{where} \quad \lambda_0 \text{ is the helix angle.}
\]

For the outermost point \( B_0 \) (or \( B_0 \)) of the primary flute, Galloway’s equations (equation 1) yields

\[
\Psi_0 = \sin^{-1} \left( \frac{t}{R_0} \right) \quad \text{and} \quad v_0 = \Psi_0 + \left( \frac{t}{R_0} \right) \left( \tan \lambda_0 \right) \left( \cot \Psi_0 \right) \left( \cot \rho \right)
\]

where the variables are as defined earlier in equation 1. Therefore, the \( y \) coordinate of the outermost point \( A_0 \) of the secondary flute \( A_0C \) is

\[
h = h_1 - R_0 \sin v_0 = l \cos \lambda_0 - R_0 \sin v_0.
\]

Also, the \( y \) coordinate of the innermost point \( C \) of the flute should be equal to the half-web thickness. These two conditions determine the end points of the secondary flute.

It may happen that the wheel width (equal to the width of the entire oblique flute profile) thus determined for a given land width, does not correspond to a standard (available) wheel thickness. In that case, a standard wheel width close to the required value may be suitably chosen so as to obtain a new land width as close as possible to the required value.

The shape of the curve between the end points of the secondary flute may depend upon factors such as the desired maximum chip removal capacity of the flute and the strength of the cutting point. It can generally be assumed to be represented by a function \( f(x, y) = 0 \).
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Involving two or more unknown constants. For a circular or elliptical curve there will be two unknown constants which can be solved for from the two end conditions. However, if a polynomial involving three or more constants is assumed, aspects such as the chip removal capacity of the flute and/or strength of the cutting point have to be considered. The Appendix shows how the chip removal capacity can be used as a condition for determining the flute shape. Once the orthogonal, secondary flute shape is determined, it can be transformed similar to the primary flute (as described in subsection 2.1), to obtain the corresponding wheel profile.

2.3. Examples

Using the approaches outlined in subsections 2.1 and 2.2, two sample wheel profiles were determined for the following set of drill design parameters:

- helix angle, \( \phi_0 = 20 \)
- point angle, \( \phi = 119 \)
- web thickness, \( 2t = 0.053 \text{ in. (1.35 mm)} \)
- drill diameter, \( 2R_0 = 0.25 \text{ in. (6.35 mm)} \)
- wheel thickness = \( \begin{cases} 0.125 \text{ in. (3.18 mm)} & \text{for example 1} \\ 0.1875 \text{ in. (4.76 mm)} & \text{for example 2.} \end{cases} \)

For simplicity, the secondary flute profile was determined by assuming it to be part of an ellipse with its center lying on the \( y' \) axis.

Computer plots of the entire flute profile for the two examples are shown in Fig. 6. It may be noted that the secondary flutes in the two examples are different in shape because of the different wheel thicknesses. Since the wheel thickness is more in the case of example 2 [Fig. 6(b)], the secondary flute is wider. (The primary flute will be the same in both cases, since the cutting edge configuration is fixed.) Consequently, the land width and chip removal capacity will also be different for the two cases.

3. Sensitivity Analysis

Though the wheel profile required for a desired flute (to provide a straight cutting edge) can be very accurately determined by the mathematical procedure outlined in section 2, there are other factors which can affect the cutting edge shape. In particular, inaccuracies in the (flute-grinding) machine setting can give rise to a helix angle different from the design value and can also set the wheel off-position. Also, an inaccurate point grinding can yield a point angle different from the designed value. All these factors will affect the shape of the flute, and
hence the cutting edge shape. In order to provide any tolerances on the wheel setting or point grinding and to develop an optimum flute grinding wheel profile, an understanding of how exactly these factors affect the cutting edge shape is required.

The effect of the various factors can be mathematically analyzed by obtaining an equation of the flute profile for a general (not just straight) cutting edge shape. This is obtained from Galloway's equation, equation 1, by replacing the constant \( t \) by a variable \( y_p \), the ordinate of any point \( P \) on a general cutting edge in the XY plane. This yields the general equations

\[
\begin{align*}
r &= \frac{y_p}{\sin \Psi} \\
v &= \Psi + \frac{y_p}{R_0} \left( \tan \theta \right) \left( \cot \Psi \right) \left( \cot \rho \right)
\end{align*}
\]

where the quantities are as defined earlier in equation 1.

3.1. Effect of point angle variation

Referring to Fig. 7, \( CB_a, CB_b, \) and \( CB_c \) represent the orthogonal primary flute profiles as obtained from equation 1 for point angles \( 2\rho_a, 2\rho_b, \) and \( 2\rho_c \), respectively, where \( 2\rho_a > 2\rho \) and \( 2\rho_b < 2\rho \). Since the cutting edge is formed by the intersection of the flute and flank surfaces (Fig. 2), both the flute and flank configurations should be produced based upon the same point angle (design value) to obtain a straight cutting edge. But, if the point angle ground on the flank is different from the design value \( 2\rho \) for which the flute grinding wheel profile is designed, a concave or convex shape of the cutting edge is obtained, depending on whether the actual point angle is larger (\( 2\rho_a \)) or smaller (\( 2\rho_b \)) than \( 2\rho \).

To mathematically determine the effect of point angle variation, values of \( r \) and \( v \) obtained
from the flute profile corresponding to the actual point angle ground (such as curve CB₀° for angle 2ρ₀) may be substituted in equation 10. With the actual point angle value used, the equation can be solved for \( y_p \) and \( \Psi \), and hence for \( x_p \) (= \( y_p \cot \Psi \)), corresponding to different sets of \( r \) values. The coordinates \( x_p \) and \( y_p \) of points \( P' \) on the cutting edge then yield the actual cutting edge shape that would be obtained. Figure 8 shows the shape of the cutting edge for different point angles (actual), obtained through a computer, for the following set of drill design parameters:

- helix angle, \( \lambda_0 = 20 \)°
- point angle, \( 2\rho = 119 \)°
- web thickness, \( 2t = 0.053 \) in. (1.35 mm)
- Drill diameter, \( 2R_0 = 0.025 \) in. (0.64 mm)

### 3.2. Effect of helix angle variation

If the wheel profile (for the primary flute) is made for the design value (\( \lambda_0 \)) of the helix angle but the wheel set at a different angle greater (\( \lambda_b \)) or smaller (\( \lambda_s \)) than \( \lambda_0 \), the shape of the cutting edge would not be straight.

Suppose the setting angle (i.e., the actual helix angle) is \( \lambda_b < \lambda_0 \). In Fig. 9, \( \mathbf{OB}_1 \) and \( \mathbf{OB}_1 \) refer to the primary flute profiles corresponding to helix angles \( \lambda_0 \) and \( \lambda_b \) respectively. \( \mathbf{X}Y'Z' \) and \( \mathbf{X}'Y''Z'' \) are the corresponding Cartesian reference systems (with \( \mathbf{X}' \) and \( \mathbf{X}'' \) axes along \( \mathbf{OB}_1 \) and \( \mathbf{OB}_1 \) respectively). The profile \( \mathbf{CB}_1 \) in the \( \mathbf{X}'Y'Z' \) coordinate system will be the same.
as the profile CB₁ in the X"Y"Z" coordinate system since both represent the same wheel profile (for the primary flute).

To obtain the resulting orthogonal flute profile (and hence the cutting edge shape), the primary flute profiles along the Z" axis should first be determined from the profile CB₁. For this, any outermost point B₁ in a plane at z" = z₁ on the flute can be represented in the X"Y"Z" reference system by the coordinates

\[
\begin{align*}
x'₁ &= x'₁ \cos \beta_n - y'₁ \sin \beta_n + z''₁ (\sin \phi)(\cos \phi) \\
y'₁ &= x'₁ \sin \beta_n + y'₁ \cos \beta_n \\
z''₁ &= z₁
\end{align*}
\]

where \(x'₁, y'₁\) are the coordinates of point B₁ and \(\beta_n = (z''₁/R₀) \sin \phi\).

In the XYZ reference system the coordinates can be represented by

\[
\begin{align*}
x_n &= x''₁ \cos \phi - z₁ \sin \phi \\
y_n &= y''₁ \\
z_n &= x''₁ \sin \phi + z₁ \cos \phi
\end{align*}
\]

Next, to obtain the outermost point B₀ of the corresponding orthogonal flute (in plane XY where \(z = z₁ = 0\)), the expression for \(z_n\) in equation 12 is equated to 0 (after substituting for \(x''₁\) in terms of \(z₁\) from equation 11. This then yields a value of \(z₀\) for \(z₁\) the \(z"\) coordinate of \(B₀\). From this and equation 11, the remaining coordinates \(x₀\) and \(y₀\) of the point \(B₀\) are obtained. The corresponding polar coordinates are

\[
r₀ = \sqrt{(x₀² + y₀²)} \quad \text{and} \quad \phi₀ = \tan^{-1}(y₀/x₀).
\]

Similarly, the coordinates \((r, \phi)\) of other points on the orthogonal flute profile can be determined. Substituting these coordinate values in equation 10, values of \(y_p\) and \(\Psi\) (and hence \(x_p\)) can be determined from which the corresponding cutting edge shape can then be obtained.

Figure 10 shows the effect of the helix angle variation on the cutting edge shape, obtained through a computer, for the set of drill design parameters given for the example in subsection 3.1. Compared with Fig. 8, it can be seen that the helix and point angle variations have opposing effects on the cutting edge shape. That is, an increase in the convexity of the cutting edge is seen with a decreasing point angle or an increasing helix angle (and vice versa for concavity).

3.3. Effect of wheel offset

Another important condition for proper flute grinding is that the wheel should be in the
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correct position with respect to the drill axis, as shown in Fig. 11. That is, at any instant, the lowest point on the wheel (point C in Fig. 5, corresponding to the maximum wheel radius) should be directly above the drill axis (above point O). But in practice, inaccurate settings of the wheel may cause it to be offset from the true position. This will produce a distorted flute and hence a non-straight cutting edge. The amount and nature of the distortion will depend upon the magnitude and direction of the offset. Only small offsets (between $OO_1$ and $OO_2$ along the $Z'$ axis in Fig. 11) and their effect on the primary flute will be discussed in this section.

For a small offset $\varepsilon$ in the $+z'$ direction (Fig. 11), the flute profile will be cut in the plane $PLA_{\text{act}}$, instead of plane $PLA_{\text{theo}}$ (the cutting plane for no offset). For small offsets, the intersection of the drill and wheel configurations will yield, in plane $PLA_{\text{act}}$, a flute profile as shown in Fig. 12. The straight portion of the flute profile would result due to part of the side of the wheel cutting into the drill. The end points $A_0'$ and $B_0'$ of the entire flute profile $A_0'B_0'$ will be the points of intersection of the wheel and drill cross-sections in plane $PLA_{\text{act}}$. To obtain these points the equation for the elliptical boundary of the drill cross-section can be represented by

$$\frac{(x' - \varepsilon \tan \lambda_0)^2 \cos^2 \lambda_0}{R_0^2} + \frac{y'^2}{R_0^2} = 1 \quad (13)$$

while the equation of the wheel profile in the $X'Y'$ plane is given in section 2. The profile of the flute between these endpoints will be the same as the known wheel profile $A_0'B_0'$, as can be seen from Fig. 12(a).

Once the flute profile $A_0'B_0'$ is obtained, the profile in other oblique planes parallel to $PLA_{\text{act}}$, can be obtained as discussed in subsection 3.2 (for the effect of helix angle variation).
Then, using an iterative procedure, the points of intersection of the XY plane (where \( z = 0 \)) and the oblique planes can be determined; this yields the orthogonal flute profile \( A_0B_0 \) [Fig. 12(b)]. Substituting the values of \( r \) and \( v \) of the orthogonal profile in equation 10, the corresponding cutting edge shape can be determined. A similar analysis can be performed for offsets in the \( -z' \) direction, whose effect on the oblique flute will yield a flute profile that is the lateral inversion of that obtained for the \( +z' \) offset. Figure 13 shows the quantitative effects of \( \pm z' \) offsets for the set of drill design parameters given for the example in subsection 3.1. In general, in such cases, a convexity is obtained for the cutting edge, with a sharp concavity at the outer edge. It may also be noted (from Fig. 13) that the amount of concavity increases as the offset \( \varepsilon \) increases.

4. OPTIMIZATION OF THE WHEEL PROFILE FOR MAXIMUM WHEEL LIFE

In the practical situation, a tolerance is always specified for the straightness of the cutting edge, in terms of the concavity ("hook") or convexity ("layback") allowed, as shown in Fig. 14. Referring to Fig. 15, if \( C_{B_0} \) is the orthogonal, primary flute profile to give a straight cutting edge, then as the wheel wears, this profile in general takes the shape \( C_0B_0 \). Since maximum material removal occurs at the maximum radius of the wheel (at point \( C \)), the wheel wear would be more at this point than at the outer edge (at point \( B_0 \)). Hence, curve \( C_0B_0 \) would be shallower than curve \( CB_0 \). As observed in the effects of helix or point angle variations, such a curve would produce a convex shape for the cutting edge (besides increasing the web thickness) as shown by the profile \( C_0B_0 \) in Fig. 15(b). The wheel would have to be redressed or replaced when the convexity reaches the allowable, limiting value.

Therefore, in order to maximize the wheel life, a wheel profile that gives an initial concavity for the cutting edge shape (equal to the maximum allowed tolerance value) should be designed. As the wheel wears, the cutting edge shape becomes straight and finally convex, thus increasing the number of acceptable flute profiles that can be ground before redressing or replacing the wheel. Such an initial wheel profile will correspond to some point angle \( (2\rho_2) \) less than that required for the drill design value \( (2\rho_1) \), (as can be seen from the effect of point angle variation discussed in subsection 3.1). As shown in Fig. 16, the outermost point on the
cutting edge obtained with the concavity would be \( K' \) instead of \( K \) for the straight cutting edge. The angle \( \psi_0 \) corresponding to both these points can be obtained from the equation

\[
v_0 = \Psi_0 + \frac{\bar{y}}{R_0} (\tan \delta_0)(\cot \Psi_0)(\cot \rho_2)
\]

where \( \Psi_0 = \sin^{-1} (y/R_0) \) and \( \bar{y} = \text{web thickness} + \text{maximum concavity allowed} \). By using this value of \( v_0 \) for \( v \) and by changing \( t \) to \( \bar{y} \) and \( \rho_2 \) in equation 1, \( \rho_2 \) can be determined. Then, substituting the value of \( \rho_2 \) in equation 1 the other points (\( r, v \)) along the (optimum) orthogonal flute profile can be determined (Fig. 16), from which the corresponding optimum wheel profile is obtained (through transformations, given by the procedure outlined in section 2).

5. CONCLUSIONS

The part of the grinding wheel profile required to generate the primary flute that would yield a straight cutting edge can be mathematically described for grinding the flute with high accuracy.

The part of the wheel profile required to produce the secondary flute (corresponding to the non-cutting edge) can be mathematically defined (with some approximation), based on the required land width, web thickness and chip removal capacity of the drill helix.

The effect of helix and point angle variations and small wheel offsets on the cutting edge shape can be quantitatively analyzed using the mathematical representation of the wheel profile. Such an analysis indicated that the point and helix angle variation have opposing effects on the shape (convexity or concavity) of the cutting edge. It was also observed that an increasing wheel offset produced an increasing "hook" at the outer end of the cutting edge.

The wheel profile can be optimized for a longer life (maximum flutes ground), for a given tolerance on the straightness of the cutting edge.
REFERENCES


APPENDIX

DERIVATION FOR THE CHIP REMOVAL CAPACITY OF THE DRILL FLUTE

The oblique cross-sectional area above the oblique flute profile $A_1CB_1$ and bounded by the drill periphery (Fig. 5) provides for half the maximum (theoretical) amount of chip removal desired. This area, which may be denoted by $(\Delta 2)$, will be equal to half the volume of chips to be removed per unit length of the flute $(Q_o 2)$; i.e., $A_o = Q_o$. Also, this area $(\Delta 2)$ can be approximately related to the orthogonal cross-sectional area $A_0CB_0E$ in Fig. 4 [which may be denoted by $(\Delta 2)$] as $(\Delta 2) = (A_2) \cos \theta_o$ where $\theta_o$ is the helix angle.

Or.

$$A_o = A_1 \cos \theta_o = Q_o \cos \theta_o. \quad (A1)$$

From Fig. 4, the area $A_{12}$

- area 1 under curve $GA_1$,
- area 2 under curve $A_1C$ – area 3 under curve $CB_1$,
- area 4 under curve $B_1G$.

Here area of semi-circle $GGE = \pi R_o^2 2$, where $R_o$ is the drill radius.

area 1 = \int_{R_o}^{R_o} \sqrt{(R_o^2 - x^2)} \, dx

area 2 = \frac{1}{2} \left[ -h \sqrt{(R_o^2 - h^2)} + R_o \sin^{-1} \left( \frac{-\sqrt{(R_o^2 - h^2)}}{R_o} \right) \right]

+ \frac{1}{2} R_o^2 \pi.

area 3 = \int_{0}^{\theta_o} \int_{0}^{\cos \theta_o} \cos \Theta \, d\Theta \, d\psi

area 4 = \int_{0}^{\theta_o} \int_{0}^{\cos \theta_o} \cos \Theta \, d\Theta \, d\psi

\frac{R_o^2 \pi}{2} - \sin \theta_o \cos \theta_o - \sin^{-1} \left( \cos \theta_o \right).

area 3 = \int_{0}^{\theta_o} \int_{1}^{1} \cos \Theta \, d\Theta \, d\psi

area 3 = \int_{0}^{\theta_o} \int_{1}^{1} \cos \Theta \, d\Theta \, d\psi

area 3 = \int_{0}^{\theta_o} \int_{1}^{1} \cos \Theta \, d\Theta \, d\psi

\frac{R_o^2 \pi}{2} - \sin \theta_o \cos \theta_o - \sin^{-1} \left( \cos \theta_o \right).

where

$$v = \tan \Psi \left[ \cos \Psi + \cos \Psi \left( \sin \Psi \right) \right], \csc \Psi \cos \Psi + \cos \Psi \cos \Psi \csc \Psi, \cosec \Psi

and

$$K = \frac{1}{R_o} \left( \tan \theta_o \cot \theta_o \right)

or.

area 3 = \frac{1}{2} \int_{0}^{\theta_o} \left( I_1 + I_2 + I_3 + I_4 + I_5 \right) d\psi

where

$$I_1 = \cos^2 K \cot \Psi + \frac{1}{2} \cot \Psi \left[ \sin 2K \cot \Psi \right];$$

$$I_2 = -\cos \Psi / \sin (2K \cot \Psi);$$
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\[ I_1 = K (\cot \Psi \cosec^2 \Psi \left[ \sin (2K \cot \Psi) \right] ) \]

\[ I_2 = K (\cot \Psi \cosec^2 \Psi \left[ \sin^2 (K \cot \Psi) \right] ) \]

\[ I_3 = - \frac{1}{2} (\cot \Psi \left[ \cos (2K \cot \Psi) \right] ) \]

\[ I_4 = - (\cot \Psi \left[ \sin^2 (K \cot \Psi) \right] ) \]

\[ I_5 = - \frac{1}{2} (\cot \Psi \left[ \sin (2K \cot \Psi) \right] ) \]

For area 2, assuming the shape of the curve A, C (secondary flute in orthogonal plane) to be of the form

\[ y = y_1 + y_2 \]

\[ y = b_1 x^2 + b_2 x + b_3 \]

\[ \text{area } 2 = \int_{x_1}^{x_2} (b_1 x^2 + b_2 x + b_3) \, dx \]

\[ = b_1 \frac{q^3}{3} - b_2 \frac{q^2}{2} + b_3 q \]

where

\[ q = \sqrt{(R_1 - h_1)} \]

Hence, equation A1 can be used as a third condition (along with the two end conditions) to solve for the unknown constants \( b_1 \), \( b_2 \) and \( b_3 \).