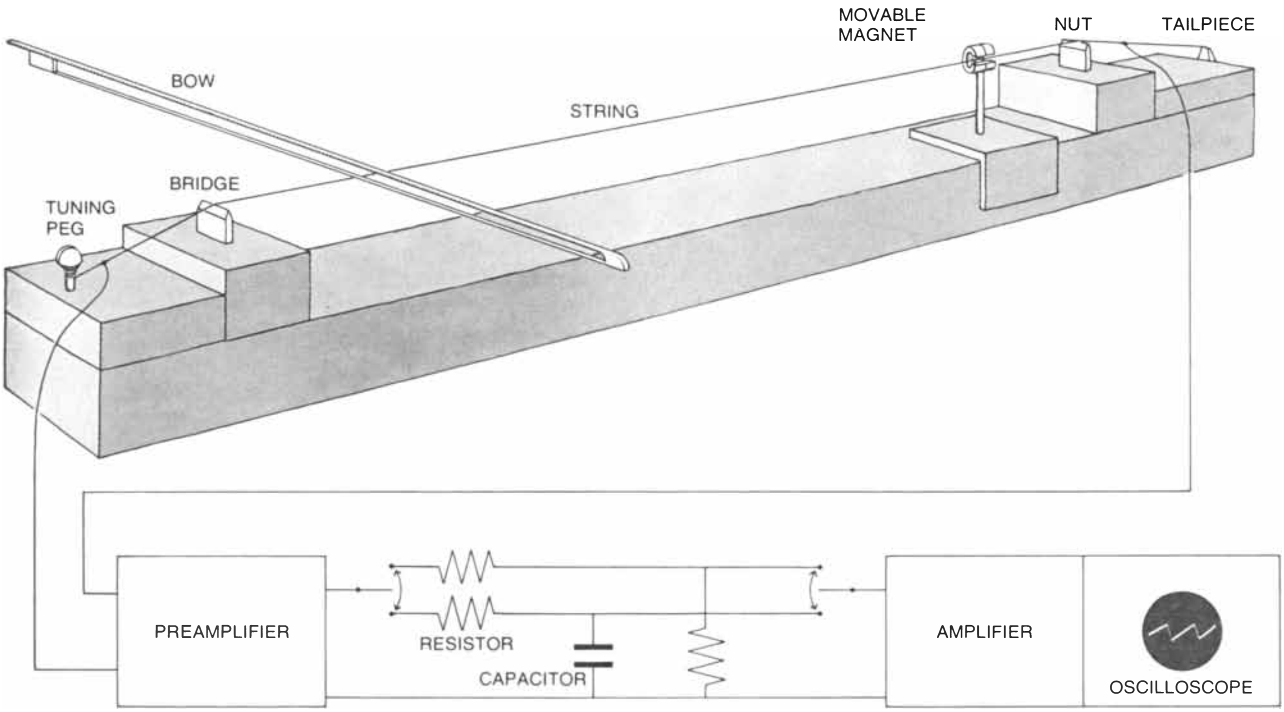
The Physics of the Bowed String

What actually happens when a violin string is bowed? Modern circuit concepts and an electromagnetic method of observing string motion have stimulated new interest in the question

by John C. Schelleng

The heart of the violin or any of its relatives, the center from which flows the acoustic pulse that is the very life of the music, is the bowed string. The string—its action under the fingers and the bow, its gratifying responsiveness and even the problems it forces the player to solve—plays a major role in establishing the musical identity of this family of instruments. Conceptually the string is the simplest of components, although its manufacture calls for meticulous care: it must be flexible, uniform and strong. In spite of this simplicity its action under the bow presents many unanswered questions. The elementary physics of its behavior can nonetheless be of considerable importance to the player.

Of the many papers published by Hermann von Helmholtz, ranging through physiology, anatomy, physics and the fine arts, there is one titled "On the Action of the Strings of a Violin" that appeared in the proceedings of the Glasgow Philosophical Society in 1860. Up to that time little was understood about what actually happens when a string is bowed. Helmholtz' procedure is a good example of how a well-conceived experiment combined with simple mathematics can illuminate a problem that could not at the time be solved with either approach alone. Today we would call his apparatus an oscilloscope; to him it was a "vibration microscope," an invention he credited to the French physicist Jules Antoine Lissajous. Through this instrument he looked at a grain of starch fastened to a black string, which he set in vibration by bowing. The objective lens of the microscope was mounted on a

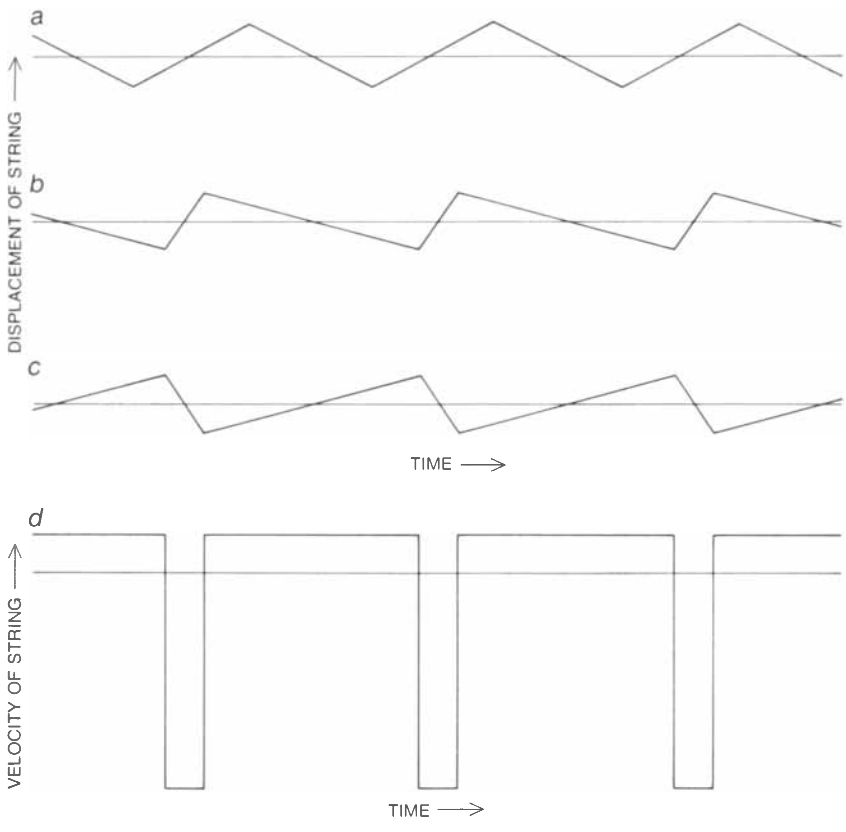


MONOCHORD, a simple experimental arrangement used by the author to study the motion of a bowed string, consists of a single electrically conducting string mounted between two massive bridges on a firm base. The movement of the string through the magnetic field set up by a small movable magnet generates an output signal that can be amplified and displayed on an oscilloscope screen (*see circuit diagram at bottom*). With the two switches in the up position the system displays string velocity; with the two switches down the system displays string displacement. The string can be bowed by hand, by a pendulum-driven bow or rotary bow.

large tuning fork so as to vibrate slowly parallel to the length of the string. When both string and fork were set in motion at suitable rates, Helmholtz saw a "Lissajous figure," a form of oscillogram that displayed the position of the starch particle as it varied within the period of vibration of the fork. By similarly examining the motion at other points he experimentally acquired the basis for a mathematical description of the motion of the string as a whole.

Helmholtz wrote that "during the greater part of each vibration the string is carried on by the bow. Then it suddenly detaches itself and rebounds, whereupon it is seized by other parts of the bow and again carried forward." Plotting the position of the bit of starch as a function of time, he found that every aspect of the picture as he found it except one could be represented by straight lines. During one period of vibration, almost regardless of where on the string he looked or where he bowed, the curve was a zigzag of two straight lines [*see illustration below*]. The two periods of time into which the vibration was broken were always in the same ratio as the two lengths into which the point of observation divided the string.

Something can be learned merely by looking at the lowest string of an instrument while it is being vigorously bowed [*see illustration on opposite page*]. In appearance it widens into a ribbon bounded end to end by two smooth curves. (Actually the position around which the string vibrates is moved a little to one side by the average force exerted by the bow in its direction of motion.) Helmholtz found mathematically that the boundaries are parabolas; because of their shallowness they are indistinguishable from arcs of a circle. It would be a mistake, however, to suppose that the string itself has this shape at any time. The string, Helmholtz found, has at any instant the shape it would have if it were pulled aside by the finger to some point on the arc: it is a straight line sharply bent at one point. The bend races around this edge once in every vibration; for the open *A* string of a violin, for example, it goes around 440 times in one second. If Helmholtz had been able to view the string with the aid of a stroboscopic lamp, the boundary would have disappeared and he would have seen the string as a sharply bent straight line. When the bow changes from "up bow" to "down bow," the motion of the bend around the edge changes from counterclockwise to clockwise.



DISPLACEMENT OF BOWED STRING from its average position is plotted as a function of time in the first three curves in this illustration. The characteristically zigzag curves were obtained by bowing near one end of the string and observing at the middle (*a*), near the bridge end (*b*) and near the nut end (*c*). In each case the two periods of time into which the vibration is broken are in the same ratio as the two lengths into which the point of observation divided the string. Rectangular string-velocity curve (*d*) corresponds to curve *c*.

The sideways velocity of the string at any point has two alternating values, unequal in magnitude and opposite in sign. As a result a typical zigzag displacement curve has a corresponding velocity curve that is rectangular in shape. The ratio of the two alternating velocities is the same as the ratio of the lengths into which the point of observation divides the string.

Two simple physical facts underlie the action of the bowed string. The first is that "sliding" friction is less than "static" friction and that change from one to the other is almost discontinuously abrupt. The second is that the flexible string in tension has a succession of natural modes of vibration whose frequencies are almost exact whole-number multiples of the lowest frequency; as a result the duration of a single vibration in the first, or lowest, mode precisely equals the duration of two vibrations in the second mode, three in the third and so on. Without outside compulsion the string is therefore by its very nature given to supporting a "periodic" wave, that is, a repetitive series of similar vibrations with a wave form dictated by the "stick-slip" process. The string allows the coexistence of a multitude of harmonics; the peculiarities in friction require it.

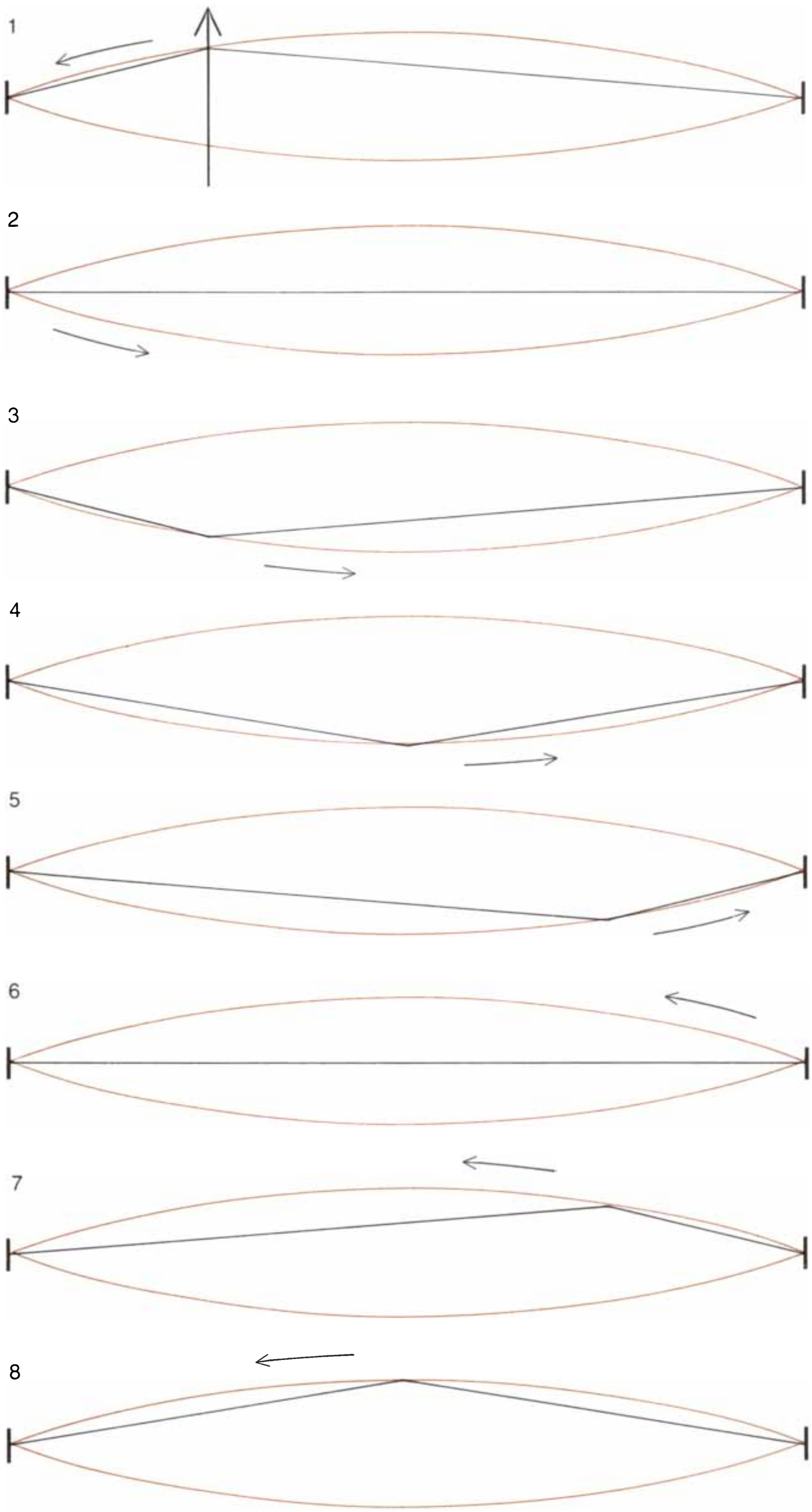
Helmholtz' shuttling discontinuity is the timekeeper that precisely triggers the capture and release of the string at the bow. There is a perennial explanation that views the string as a spring periodically pulled sideways to the breaking point of static friction. The spring recoils and is again captured. This view cannot explain the constancy in repetition rate over the wide range of bow forces, or "pressures," applied by the hand. The correct explanation must be given in dynamic terms; such an explanation is suggested by the constancy in time needed in the flexible string for the bend to travel twice its length. The timing is vividly illustrated by striking a long, taut clothesline near one end with a stick. A discontinuity is clearly seen hurtling to the far end, where it is reflected. On its return one feels through the stick (still resting on the line) an impulse much like the momentary frictional force on the string, which fails to hold at escape but succeeds at recapture.

A simple experiment partially confirms this picture of the action of the bowed string [*see illustration on next page*]. An instrument is mounted with the strings horizontal. A light bow, suspended at its heavy end by a long thread, rests on one of the strings at a point near the bridge. A second bow sets the string in vigorous vibration. Before the hanging bow can begin to move slipping occurs at all times except at moments when the string reverses. Since friction in slipping is nearly independent of speed of slipping, the forces in the two directions of vibration are the same but the impulses imparted to the bow are proportional to the duration. The direction of the acceleration of the hanging bow will therefore indicate the direction of string motion during the longer duration.

The experiment shows, however, that the direction in which the hanging bow moves is the same as the direction in which the driving bow is moving. Therefore it is in the longer interval that the string moves with the driving bow; relative motion between driver and string is accordingly less than in the shorter interval and sticking is presumably occurring. If the hanging bow is now placed near the opposite end of the string, it will be found to move in a direction opposite to that of the driver.

Helmholtz believed that the velocity of the string while it is snapping back is constant. A half-century later C. V. Raman found that in most cases this is only approximately true. Raman's discovery came in the course of an ingenious study of the mechanical action of the violin involving both experiment and theory. With respect to the bowed string his point of departure was to describe the motion in terms of the progressive waves of transverse velocity that make up the standing waves of the Helmholtz system. The same wave can be described in terms of its lateral displacement or its lateral velocity. One advantage in emphasizing velocity is that these waves can be represented as straight lines.

The shape of the Raman wave in those cases that are of interest in music (Raman dealt with many that are not) again is a zigzag, but it differs from the displacement curves introduced above in that although the "zigs" are slow, the "zags" are instantaneous [*see top illustration on page 91*]. When such a wave is reflected from the immovable end of the string, it looks exactly as it did before except that its direction of propaga-



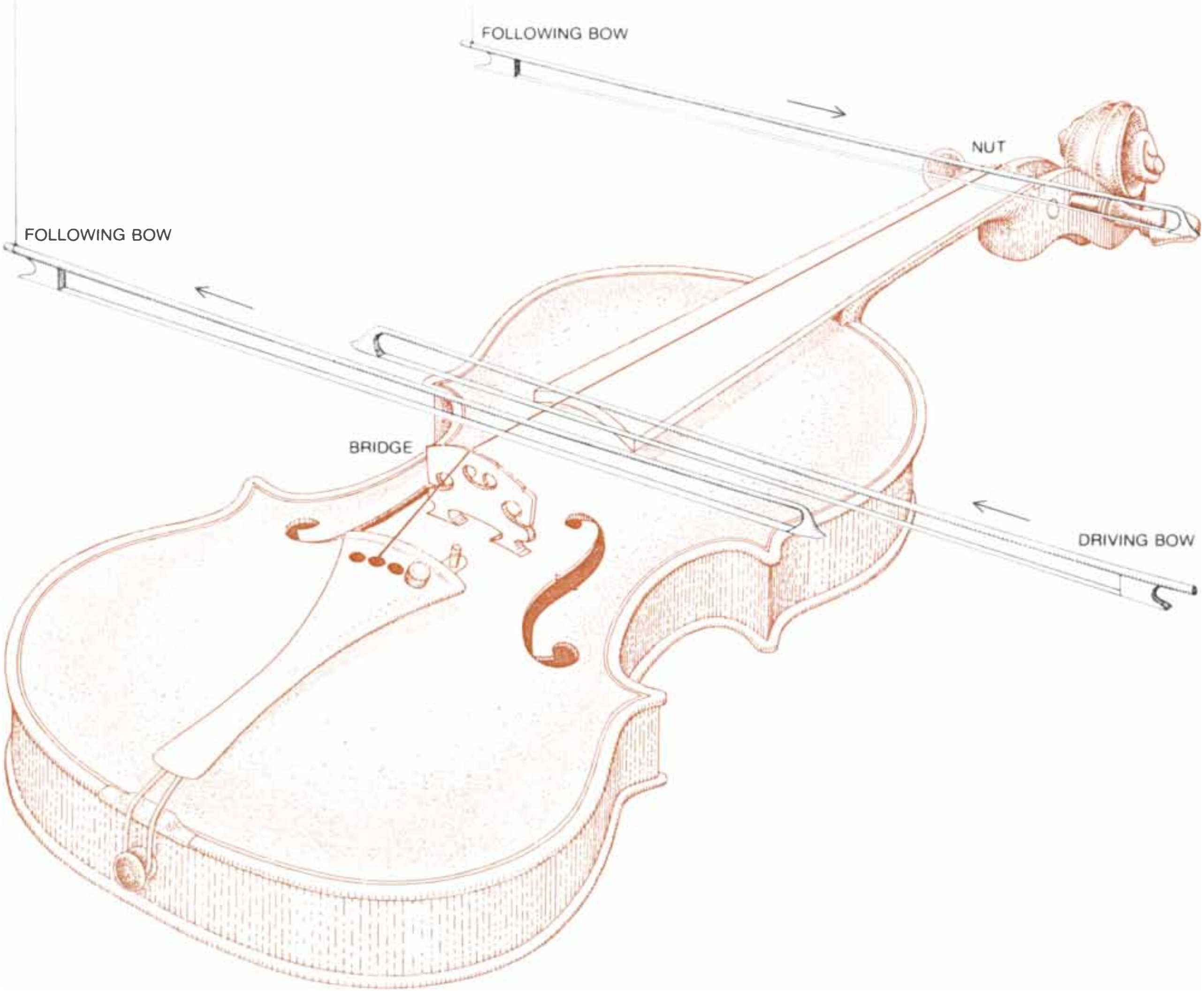
SHAPE OF BOWED STRING appears to widen into a ribbon bounded end to end by two smooth parabolic curves (*colored outlines*). As Hermann von Helmholtz found more than a century ago, however, the actual shape of the string at any instant is a straight line sharply bent at one point (*black line*). The bend races around the boundary once in every vibration. The direction of the bend's circulation in this particular series of diagrams corresponds to an upward motion of the bow; reverse the direction of the bow's motion and the bend reverses its direction of circulation. This peculiar motion is a form of standing wave.

tion is reversed. When vibration is in the fundamental mode, the length of the string is half the distance between zags.

This progressive velocity wave is of interest because, being incident on the bridge of the violin, it exerts a vibrational force whose shape is identical with its own. Insofar as the Helmholtz approximation holds, its harmonic structure therefore describes the tone quality of the string itself at the point in the instrument where the sound spectrum has not yet been influenced by resonances in or radiation from the body. The spectrum is remarkably simple: the amplitude of the nth harmonic is $1/n$ times the amplitude of its lowest, or fundamental, frequency. This simple relation is of considerable importance in investigating the sound spectrum of the violin as a whole.

Within the past few years, stimulated by circuit concepts and an electromagnetic method of observing string motion, there has been a renewal of interest in the physics of the bowed string both in this country and in Europe. More than half of the strings currently in use in stringed instruments are electrically conductive. If a small magnet is placed close to the string, the combination of a conductor moving in a magnetic field constitutes a magneto whose output can be displayed merely by inserting the string in the input circuit of a suitable amplifier connected to an oscilloscope. The electromotive force is proportional to the velocity of the string. The string can be mounted on the instrument proper or on a monochord: an experimental arrangement consisting of two massive bridges on a firm base with some means for providing tension in the string and a mount for the magnet [*see illustration on page 87*]. In my experiments two methods of bowing were used besides bowing by hand: a rotary bow developed by F. A. Saunders in his researches on violins and string action, and an ordinary bow driven by a 50-pound pendulum.

An electrical circuit connected to the monochord (or to the instrument) makes it possible to display the velocity or displacement of the string in the form of oscillograms. The first oscillogram in the bottom illustration on the opposite page, for example, displays velocity at the bow in a very flexible string. In the long period velocity lies above the zero line; that is also the velocity of the bow (if one ignores the slight ripples). In the short period of slipping there is a high negative velocity as the string whips



"FOLLOWING BOW" EXPERIMENT, performed in the course of the author's investigation, partially confirms Helmholtz' dynamic picture of the action of a bowed string. With the instrument mounted horizontally, a light bow, suspended at its heavy end by a long thread, rests on one of the strings at a point near the bridge. A second bow sets the string in vigorous vibration. In this situation the hanging bow is found (after a short period of slipping) to move in the same direction as the driving bow. The direction in which the hanging bow moves indicates the direction of string motion during the longer interval of each vibration. When the hanging bow is placed near the opposite end of the string, one finds that the "follower" moves in a direction opposite to that of the "driver."

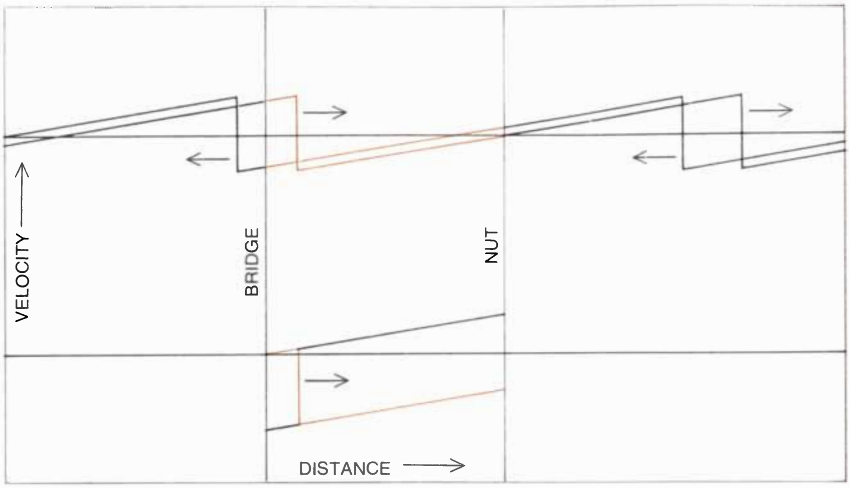
backward to take a new hold on the bow. The bow was near the bridge in this case and the shape of the curve is close to what the Helmholtz construction predicts. The zigzag in the second oscillogram shows the same vibration in terms of displacement instead of velocity.

Simplicity in instrumentation is not the only advantage in displaying velocity rather than displacement. In this way high-frequency detail that would not be suspected is brought out clearly.

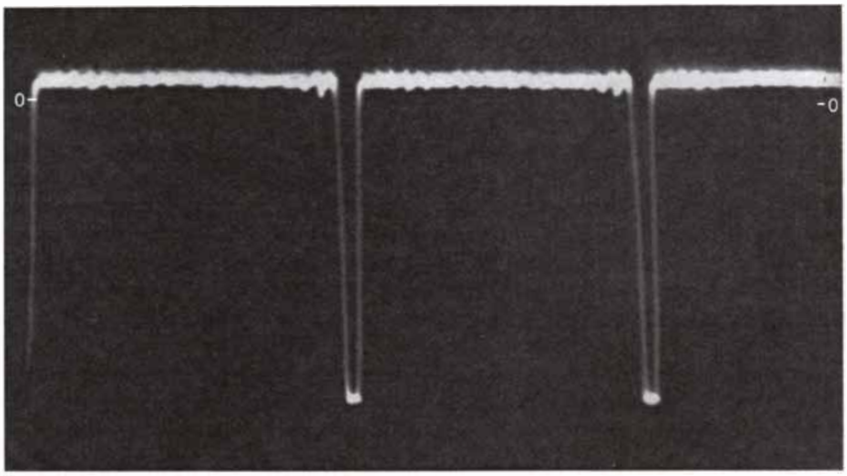
When the circuit is arranged to indicate displacement close to the bridge, it also indicates the transverse vibrational force applied by the string. The "sound of strings" alone, divorced from the modifying effect of the body of the instrument, can be produced by placing a magnet for each of the strings close to the bridge. The output from the four magnets in series is then passed through the integrator and is amplified and recorded. This arrangement sums up the forces applied by all strings. Playback therefore gives the effect of strings alone. (Recording is unnecessary if a dummy fiddle radiating no sound is used.)

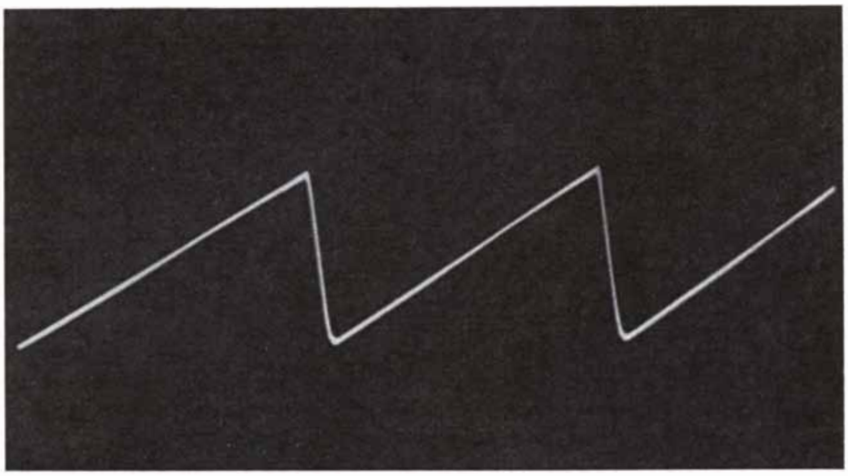
The result definitely resembles a bowed-string instrument, but an inferior one. If the system faithfully translates the force on the bridge into radiated sound pressure, the resulting sound spectrum with a given speed of bow will vary with frequency in an essentially inverse manner, the strongest effect being in the lowest tone. This "sound of strings" is not unlike the sound of the violin family in its upper octaves, but it differs drastically in the lower ones, where instead of the fiddle's achieving its strongest fundamental on the lowest tone the effect falls practically to zero because of the small size of the instrument in relation to the wavelength of such a tone in air. The harmonics then are to be credited with the fundamental tone heard subjectively. It is hardly necessary to emphasize the role that this characteristic has played in the evolution of the instrument.

In playing a bowed-string instrument there are certain limits to the speed of the bow, its distance from the bridge and the normal force applied; these limits must not be overstepped in a specific musical situation. For the experienced player this is usually a matter of choosing almost subconsciously from familiar patterns of action, but in extreme cases he is probably always aware of the limitations imposed on him. The ranges of these mechanical parameters are fortunately wide: the bow-to-bridge distance, for example, may vary from a

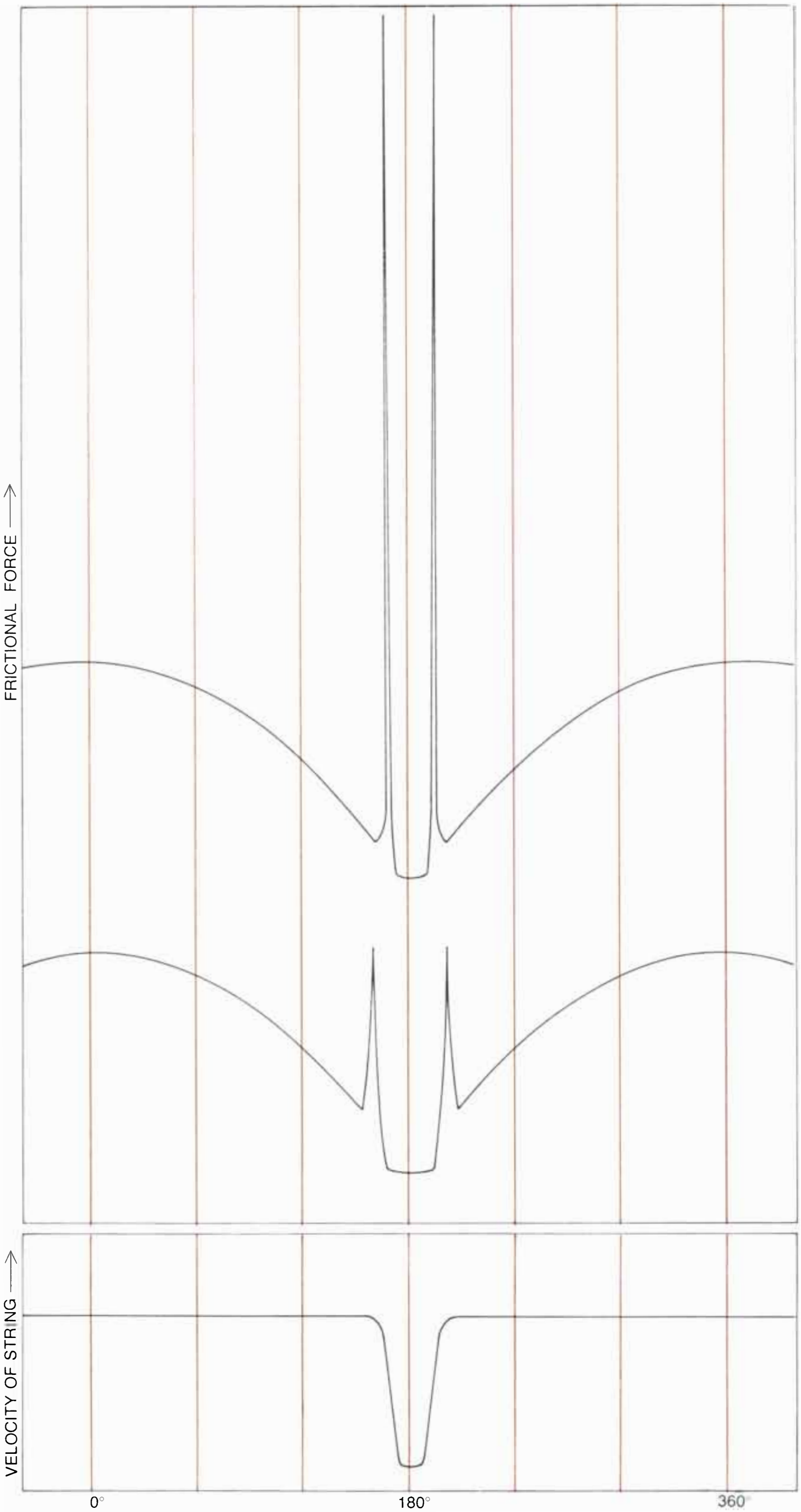


RAMAN WAVES were introduced by C. V. Raman to describe the motion of the bowed string. The shape of such a progressive transverse velocity wave differs from the corresponding Helmholtz standing wave of string displacement in that whereas the "zigs" are slow, the "zags" are instantaneous. When the oppositely moving Raman waves (*top*) are summed, the resulting wave (*bottom*) shows how the two velocities that exist alternately at any point on the string depend on the position of the discontinuity between "slipping" and "sticking."





MOTION OF VERY FLEXIBLE STRING at the bow is represented by these two oscillograms, which show string velocity (*top*) and string displacement (*bottom*) for the same vibration. The bow in this case was located about a twentieth of the length of the string away from the bridge. The shape of the curves is close to what Helmholtz picture predicts.



"RABBIT EARS" characterize the curves that represent the frictional force that is exerted by a bow in order to vibrate a string. One ear forms on the curve for frictional force when the discontinuity arrives from the nut, overcomes static friction and initiates slipping; the other ear forms when the discontinuity returns from the bridge and initiates sticking. The top curve shows the frictional force when the bow force has a typical value midway between the upper and lower allowable limits; the middle curve applies to the lower limit. For clarity ripples have been omitted from the wings of both of these curves (*left and right*). The curve at bottom is the string-velocity curve corresponding to the frictional-force curve at top.

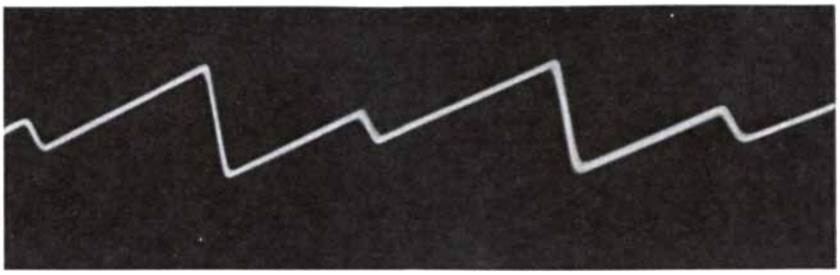
minimum value to five times that minimum; speed and force may range up to 100 times their minimum value. Given any two of these parameters, in order to assume an acceptable tone the third must fall within a range that depends on the physical constants of the string and the body of the instrument. For sustained tones these ranges are generous, although clearly all portions of a given range are not equally desirable. For example, position and speed being given, the highest permissible force may typically be 10 times the minimum. The first question is: What are the processes that determine the existence of these limits?

In order to explain the limitations on bow force it is helpful to consider how the frictional force at the point of contact of the bow and the string varies in time. Although one cannot at present display this force on an oscilloscope, it is possible to form a simplified qualitative picture on the basis of the physics involved. To do this one assumes (1) that the elementary laws of static and dynamic friction can be applied, (2) that the bridge acts as a high "mechanical" resistance (analogous to resistance in an electrical circuit) and (3) that the mass and tension of the string and its motion at the bow are known.

Assuming that the force on the string is always in the direction of the bow's motion, the points of maximum bow force occur at intervals of zero degrees, 360 degrees, 720 degrees and so on; the minimum points will then fall at 180 degrees, 540 degrees, 900 degrees and so on [*see illustration at left*]. The cyclic swinging of force between these two levels is what is needed in order to vibrate the instrument. In the small interval around 180 degrees slipping occurs. In transition from sticking to slipping there is a brief moment when the maximum of static frictional force required by bow force is exerted. Research has shown that "static friction" is really not quite static: velocity, although minuscule, is finite, and friction in reality changes continuously with velocity near zero velocity with a narrow maximum corresponding to static friction. The same curve is presumably followed in reverse in going from slipping to sticking. The "rabbit ears" evident in such frictional-force curves are the result. Such curves differ in detail in going from note to note because of complexity in the action of the body.

Consider the situation where the bow force has a typically intermediate value. From the zero-degree mark onward, with the string clinging to the bow, the force falls toward the minimum, which is dictated by dynamic friction. Then, at the moment when sticking seems most secure, the discontinuity arrives and overcomes static friction. The discontinuity needs to provide only the amount by which static friction exceeds dynamic friction. The discontinuity is capable of contributing more than is required here, perhaps much more. As the bow force is increased, however, the time will come when the discontinuity will lose out in this test of strength and the vibration will become an erratic squawk. Maximum bow force will have been exceeded.

A different kind of failure occurs when the bow force is decreased to a minimum. Here the "ears" of the frictional-force curve fall to the same level as the maximum force at the zero-degree mark, and with the lightest additional decrease static friction (as indicated by the "ears") becomes insufficient to hold the string near the zero-degree level. The result is an unstable string-displacement curve in which a new zigzag begins to form [*see top illustration on this page*]. If this new zigzag is allowed to develop, the fundamental tone will be replaced by the octave tone; in short, one will have failed to provide minimum bow force.

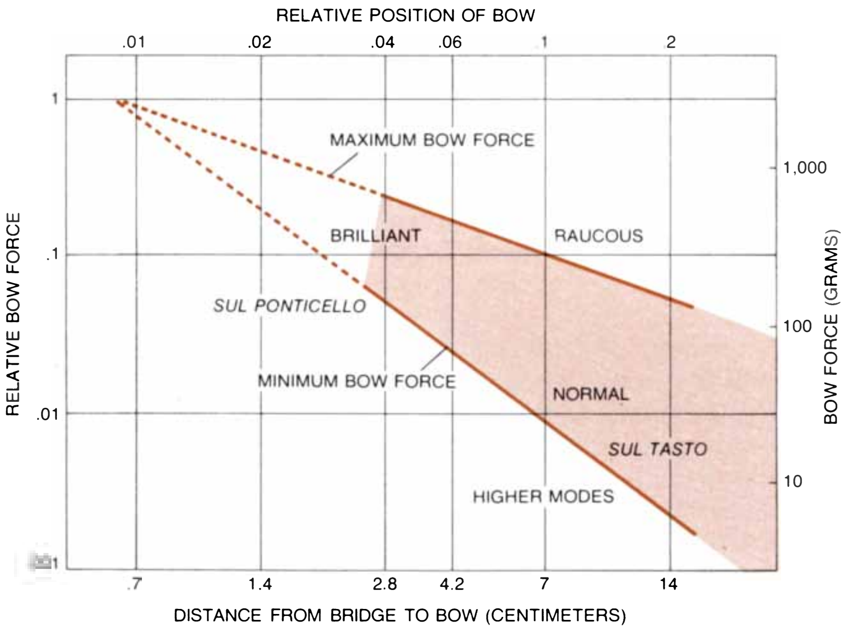


NEW ZIGZAG begins to form in the oscillographic displacement curve of a bowed string when the bow force is allowed to fall below the minimum bow force. If this unstable condition is allowed to develop, the fundamental tone will soon give way to the octave tone.

It is an important fact in the mechanics of playing that maximum bow force, which depends primarily on the string and on frictional coefficients, is inversely proportional to the first power of the distance of the bow from the bridge, whereas minimum bow force, which in addition depends on the body of the instrument, is (at least approximately) inversely proportional to the second power of the same distance. The quantities necessary for calculating these limits are known well enough to explain how the string reacts to bow forces. For sustained tones with a given bow velocity one can display the logarithmically linear trends of maximum and minimum bow force in terms of the relative distance of the bow from the bridge expressed as a fraction of the total length of string [*see bottom illustration on this page*]. The most important result is that the maximum bow force and the minimum bow force are equal when the bow is placed at a certain point very close to the bridge and that they diverge as the bow moves away from it. (Actually the curves near the intersection are to be regarded as extrapolations from the right, where normal conditions obtain.) It is this open space between the limits that gives to bow force the wide tolerance that makes fiddle-playing possible.

The forces toward the left between these lines are impractically high; normal playing is confined to the area toward the right. Farthest from the bridge the volume of the sound is least, the content of upper harmonics is minimal and the timbre has the gentle character that composers seek by designating *sul tasto:* "bow over the fingerboard." Exceed maximum bow force and the result is unmusical; fall short of the minimum and the solid fundamental tone is lost, leaving what is sometimes called a surface tone. The closer the bow is to the bridge, the less generous is the ratio between maximum and minimum bow force and the steadier is the hand that is needed. The experienced player prizes this domain for its nobility of tone; the beginner finds it prudent to play closer to the fingerboard. Closer still to the bridge bow force mounts prohibitively and the solidity of the fundamental tone disappears until little more than a swarm of high harmonics remains to suggest the fundamental tone; this is the eerie *sul ponticello* ("bow over the little bridge") of the composer. Within the normal-playing area the relative harmonic content increases—the tone becomes more brilliant—either as the bow moves toward the bridge or as the bow force increases toward its maximum.

Such a diagram is to be taken as quali-



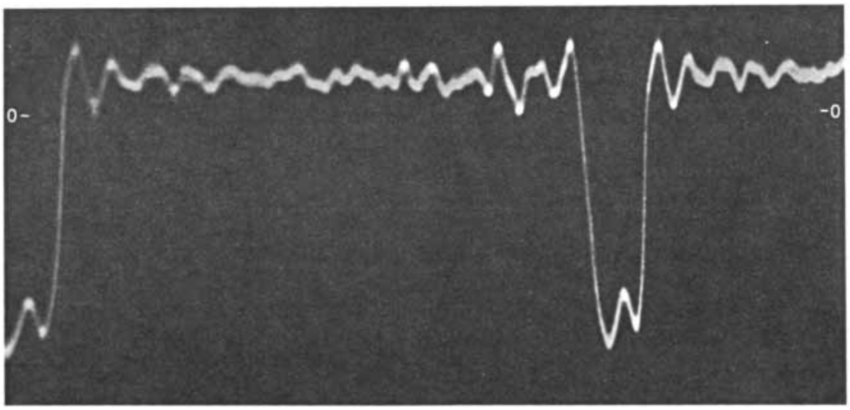
NORMAL-PLAYING RANGE for a bowed-string musical instrument is depicted for sustained tones at a constant bow velocity in this graph, which indicates the logarithmically linear trends of maximum and minimum bow force in terms of the relative distance of the bow from the bridge expressed as a fraction of the total length of the string. As the graph indicates, the maximum bow force and the minimum bow force tend toward equality (*upper left*) when the bow is placed at a certain point very close to the bridge and they diverge as the bow moves away from the bridge (*lower right*). The open space between these two limits, shown in color, accounts for the wide tolerance in permissible bow force. *Sul tasto* means "bow over the fingerboard"; *sul ponticello* means "bow over the little bridge." The second set of coordinates (*bottom scale; right scale*) suggests the normal-playing conditions for a typical *A* string of a cello bowed at a sustained velocity of 20 centimeters per second.

tative, in particular the curve for minimum force, which varies greatly from note to note because of the complexity in the response of the body of the instrument. Although the Helmholtz idealization is close enough to fact to provide a useful basis for many first-order calculations such as those described above, it is not completely trustworthy in other respects. In contradiction to what it implies, harmonic content increases with bow force, changing timbre and loudness. If loudness depended only on the "root mean square" vibrational force on the bridge, the effect would not be of much consequence, but when harmonics are radiated more efficiently than the fundamental tone or perceived more sensitively by the ear, the effect is of some importance. The fact remains that the player's major resource in controlling volume lies in the speed and placement of the bow. The implication that sound pressure is directly proportional to bow speed and inversely proportional to bow-to-bridge distance is not far wrong.

Bowed tones start in different ways, but perhaps most frequently the bow pulls the string sideways until its displacement can no longer be supported by static friction. Failure in friction, like release in plucking, sets up two oppositely traveling Helmholtz discontinuities, only one of which, the one toward the bridge, can become sustained. Until bow speed matches bow force, however, the condition is described as "raucous," and there may be many false starts before balance is realized. The art in such beginnings is to achieve the match in such a short time or at such a low sound level that an unpleasant effect is avoided. A noiseless beginning can be made by allowing the bow already in motion to make a "soft landing" on the string, thus entering the normal-playing zone by way of the zone labeled "Higher modes." Theoretically at least, bow forces and velocity can be balanced from the beginning.

In the foregoing discussion of the frictional force between the bow and the string a phenomenon of some interest was left unmentioned in the interest of simplicity, namely the role of the "ears" of the frictional-force curve in setting up reverberations between the bow and the ends of the string, some of which may persist for many periods of vibration. These effects are ignored in the classical discussion of the action of the bow but are prominent in oscillograms of string velocity. Consider a curve showing the motion of the string under the bow, where during the long interval of sticking the string might be expected to follow the unchanging speed of the bow [*see illustration below*]. It is in fact true that there is no slipping, but the string can nevertheless move by rolling on the bow except as prevented by the string's resistance to twist. The ripple in rolling implies a corresponding ripple in force exerted on the string.

A more comprehensive frictional-force curve would therefore show sharp fluctuations in force superposed on the smooth sections. One effect is to raise the minimum bow force somewhat. Away from the bow the ripples in velocity are much more pronounced. The motion would be completely suppressed at the bow if the string allowed no twisting, but it would still exist away from the bow.



ROLLING OF STRING on the bow during a long interval of sticking produces characteristic "ripples" in the string's velocity curve. To obtain this oscillogram an *A* string of a cello was bowed with 4.5 times the minimum bow force. The period immediately following the capture of the string by the bow (that is, the section of curve just to the right of each main pulse) shows mainly the decay of the pulse formed at that capture as it reverberates in the short section of string. The period before release shows mainly long-delayed reverberations in the long section, not only from the most recent release but also from earlier ones.

The term wolf note is commonly used to describe an unpleasant sound that appears consistently at an isolated frequency in a bowed-string instrument. Often its origin is obscure. There are many varieties of wolves but the most vicious of the species has its habitat in the cello (and sometimes in the violin or the viola) one octave and a few semitones above the lowest note. There is no mystery about its cause. The body of each instrument has a multitude of resonances, and the wolf tone (if one exists) arises at the most prominent of them. For the bowed string to behave properly its ends must be given a support whose rigidity is in keeping with the mass of the string. Fingering one of the heavier strings to the same frequency as the resonant frequency of the body therefore invites trouble. This nuisance manifests itself in different ways but the most characteristic way is the generation of two tones, both forced vibrations, close enough together to produce a harsh beating. Since the two tones straddle the resonance peak, they require less bow force than one tone would alone at the resonant frequency [*see top illustration on opposite page*].

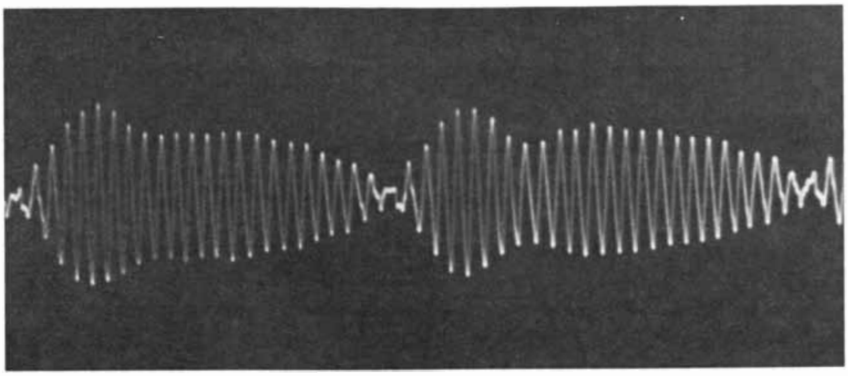
In the stiff strings of the piano, consisting as they do of thick steel wire, the frequencies of higher modes of vibration are not whole-number multiples of the frequency of the lowest mode but are somewhat sharp. That is not a defect: the "tang" it gives to the sound of the struck string is highly valued. What effect does stiffness produce in a bowed string? Clearly it is different from that in the piano. The mechanism of bowing produces a succession of almost identical vibrations. From a mathematical point of view this is another way of saying that the vibration is made up of harmonic components whose frequencies are exact whole-number multiples of the lowest frequency.

There can indeed be an effect in the bowed string. Although inharmonicity is perforce held at bay, freedom in vibration is restricted. One expects deterioration in tone quality through reduction in higher harmonics, difficulties in intonation and the need for abnormal bow forces. Before 1700, when wound strings became available, all violin strings consisted of gut, but the gut *G* string (the lowest) was unsatisfactory. The reason is not hard to find. With bowing the fundamental frequency is close to that of the lowest natural mode of vibration; in the gut *G*, however, seven times that frequency falls midway between the sixth and seventh vibrational modes and so is completely without the support of resonance. Regardless of the bow force, the seventh harmonic must be negligible. This difficulty in producing harmonics can be illustrated by a curve that shows velocity at the bow in a stiff string [*see bottom illustration at right*]. When such a string is bowed with minimum bow force, its behavior bears almost no resemblance to that of a flexible string.

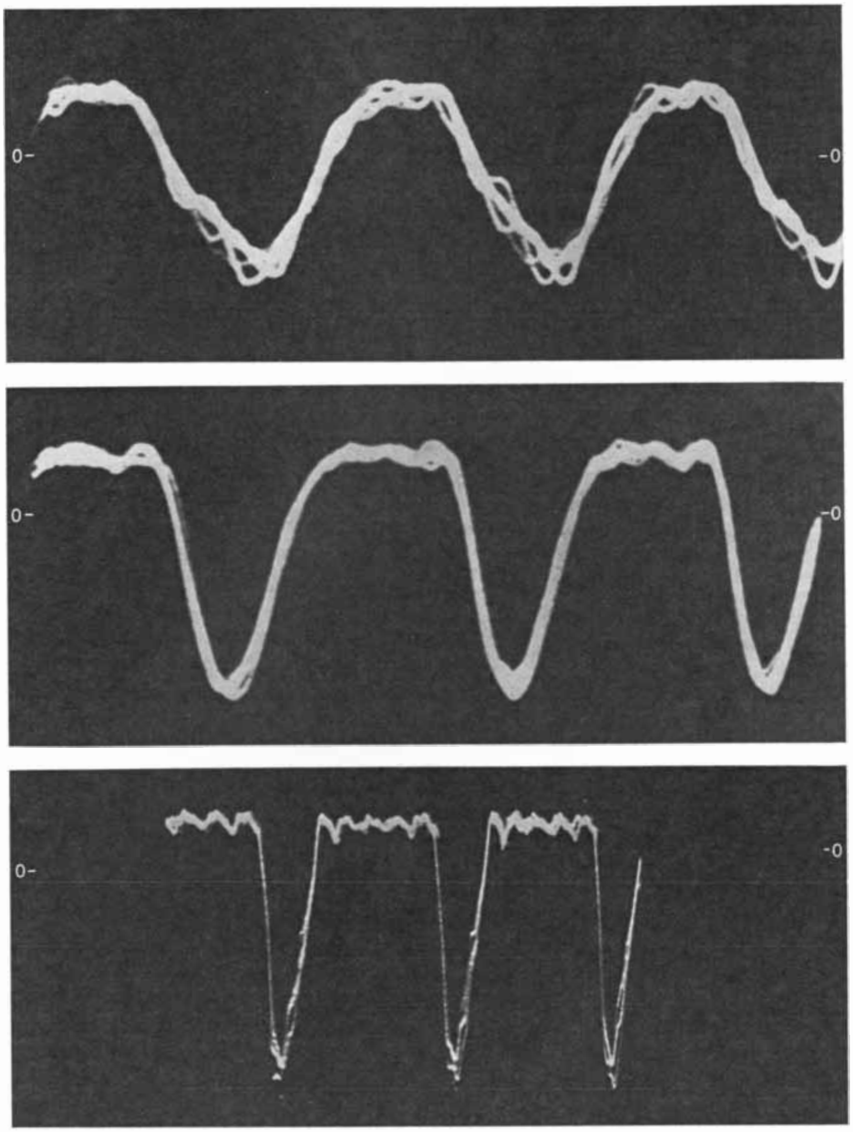
The sharpening of the *n*th mode caused by stiffness is directly proportional to the square of *n* and to a quantity called the coefficient of inharmonicity. If one changes a string mounted at a given position on a given instrument, keeping length and frequency unchanged, interchangeability requires that the same tension be maintained as well. Considering now a series of homogeneous strings of diverse materials, the coefficient of inharmonicity turns out to be proportional to the modulus of elasticity divided by the square of the density. For steel this ratio is about 50 percent greater than it is for gut; for aluminum the ratio is nearly five times greater than it is for gut. For silver, on the other hand, the ratio is about a third of the value for gut. A steel piano string that has the same pitch as the steel *E* string of the violin has an inharmonicity coefficient 20 times greater. If one ignores differences in tension in the four strings of a violin (actually it is considerably more in the highest string than in the others), inharmonicity for homogeneous strings of the same material, being inversely proportional to the fourth power of frequency, increases by a factor greater than 100 in going from the highest string to the lowest.

From calculations and measurements for several strings on the market, including some that are wound, it appears that when the coefficient of inharmonicity is equal to or less than .1, stiffness offers no disadvantage in bowing. In the steel *E* string of the violin the value is about .04. For one very good cello *C* string with metal winding on steel cable it is about the same.

As with many seemingly simple things, there is much about the bowed string that remains open to speculation. Does twisting lead to any important acoustic effect? How important is "negative resistance" throughout slipping and exactly how does rosin behave? How much do successive periods of vibration differ and is this slight wandering of any musical significance? The answers to such questions may be of little interest to the player, but the student of the bowed string would still like to know.



"WOLF NOTE," an unpleasant sound that may appear consistently at an isolated frequency in a bowed-string instrument (particularly in the cello), is produced by the "beating" of two or more tones generated by "forced" vibrations of a string clustering around the natural resonant frequencies of the body of the instrument. This oscillogram, which shows the string motion for a complicated wolf tone obtained from the *C* string of a cello, was supplied by Ian M. Firth and J. Michael Buchanan of the University of St. Andrews in Scotland.



EFFECT OF STIFFNESS in a bowed string is a deterioration in tone quality attributable in part to an increased difficulty in producing higher harmonics. This problem is demonstrated by these three oscillograms, which represent the velocity of a stiff string at the bow for three levels of bow force: minimum (*top*), intermediate (*middle*) and high (*bottom*).