Lagrangien et Hamiltonien dans un référentiel tournant

(1) on a $\tilde{\theta} = \theta + \phi(t)$, $\dot{\tilde{\theta}} = \dot{\theta} + \Omega$

$$E_c = \frac{1}{2} m |V|^2$$

avec $|V|^2 = (\dot{r})^2 + (r\dot{\tilde{\theta}})^2 + \dot{z}^2$

$$= (\dot{r})^2 + r^2(\dot{\theta} + \Omega)^2 + \dot{z}^2$$

$$L(r, \theta, z, \dot{r}, \dot{\theta}, \dot{z}) = E_c - U$$

$$= \frac{1}{2} m \left[(\dot{r})^2 + r^2 (\dot{\theta} + \Omega)^2 + \dot{z}^2 \right] - U(r, \theta, z)$$

(2) $p_r = \frac{\partial L}{\partial \dot{r}} = m\dot{r}$

$p_\theta = \frac{\partial L}{\partial \dot{\theta}} = m r^2 (\dot{\theta} + \Omega) \quad \Longleftrightarrow \quad \dot{\theta} = \frac{p_\theta}{m r^2} - \Omega$

$p_z = \frac{\partial L}{\partial \dot{z}} = m\dot{z}$

$$H(r, \theta, z, p_r, p_\theta, p_z) = p_r \cdot \dot{r} + p_\theta \dot{\theta} + p_z \dot{z} - L$$

$$= m\dot{r}^2 + m r^2 (\dot{\theta} + \Omega)\dot{\theta} + m\dot{z}^2 - L$$

$$= \frac{1}{2} m \dot{r}^2 + \frac{1}{2} m r^2 \dot{\theta}^2 - \frac{1}{2} m r^2 \Omega^2 + \frac{1}{2} m \dot{z}^2 + U(r, \theta, z)$$

$$= \frac{p_r^2}{2m} + \frac{p_\theta^2}{2mr^2} + \frac{p_z^2}{2m} - \Omega p_\theta + U(r, \theta, z)$$