«Harmonics» project: Mathematics, music perception, improvisations and compositions.

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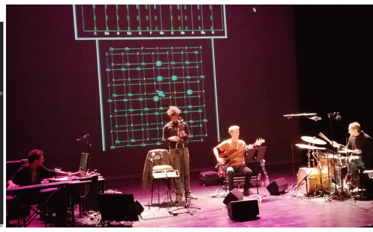
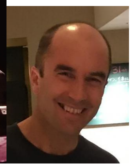
Abstract

This is a **research project** supported by University Grenoble Alpes (UGA) and IDEX in 2023-2024. This project is a collaboration between two musicians Malik Mezzadri, Bo VanDerWerf, a computer scientist Alexandre Ratchov and a mathematician/physicist Frederic Faure. This project is based on the **exploration and use of just intonation in music** and aims to develop new approaches to **composition, improvisation and musical analysis** as well as to **live performance**. In addition to performances in the form of **open rehearsals, concerts, albums and research papers**, there is also a lecture (explanations) and workshops (master classes) with students.

Contents

Remark 0.1. While reading this pdf document, you can follow the colored web links.

1 Participants



The richness of our collaboration comes from the diversity of the participants (artists and scientists):

- Malik Mezzadri (Magic Malik), internationally renowned jazz flutist (Paris). Laureate of the Prix de Rome of the Villa Medicis 2010.
- Bo VanderWerf, internationally renowned jazz saxophonist (Brussels).
- Alexandre Ratchov, computer scientist, Grenoble, France, specialist in real time computer assisted music, developer of sndio for OpenBSD and of a software reverb effect ArVerb.
- Frédéric Faure, Institut Fourier (mathematics), UFR phitem of physics, UGA, specialized in dynamical systems theory, chaos, quantum and classical waves, amateur musician. Teacher-researcher in physics/mathematics, teaches in particular musical acoustics in the musicology department for students of double Licence Physique-Musicologie. Co-leader of the Physics-Musicology double degree course.

The participants know each other well and have already collaborated together:

- Numerous musical collaborations between Bo VanDerWerf and Malik Mezzadri (albums, concerts, residencies and research...)
- Numerous science-music collaborations between Alexandre Ratchov, Malik Mezzadri and Frédéric Faure (article, résidences and concerts,, numerous meetings)
- recent collaboration between Bo VanderWerf and Frédéric Faure, around Bo VanDerWerf's PhD subject: *"from closed circuit to open circuit: adequation, integration and redeployment of harmonic, rhythmic and melodic organization models developed by Messiaen, in 'jazz' improvisation contexts"*.

2 Project research theme: musical just intonation on the tonnetz

Based on **psycho-acoustic and mathematical considerations**, we propose to re-consider the **musical universe in a new way** without any preconceptions. The aim is to discover new "musical colours and perceptions" and also to rediscover the musical rules of various cultures but from a new and enlightening angle of life.

The main research theme of this project is Just intonation which means the musical intervals present in the human voice (i.e. between the harmonics of a vowel sound) and therefore naturally perceived and analyzed by the human cognitive system (in an unconscious way). We then wish to transform the results of our research into artistic performances that can vary from a musical concert, a conference with musical demonstrations, or educational workshops with students or other audiences.

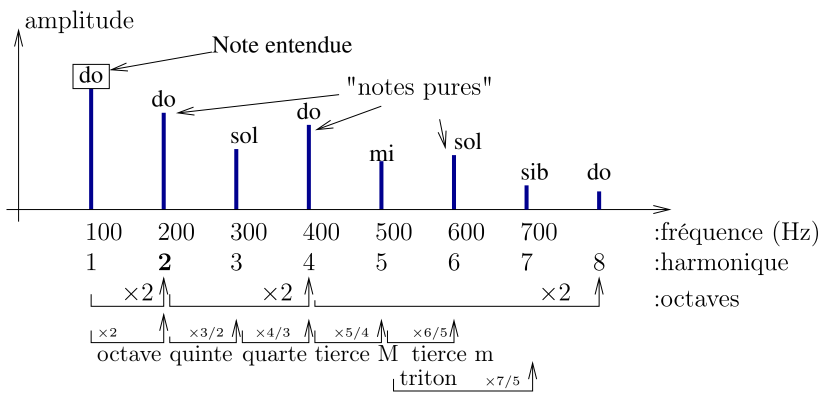
2.1 Just intonation on the tonnetz

The approach is to identify the **mechanisms of auditory perception** of voice and music, to explore harmonic musical paths based on **the natural movements of attraction between musical intervals** (just or tempered), and to develop the most natural visual representations possible. This research was initiated a few years ago, in a project called "flute what equation?", which gave rise toperformances. See the talks of Malik Mezzadri at IRCAM and talk of Frédéric Faure and this other talk. However, the research and the achievements in return are still in their infancy.

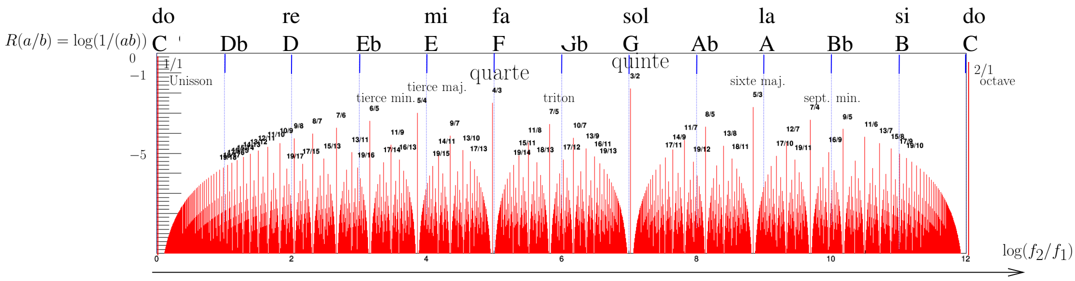
Here are some more details on this process. As a reminder, the notes of the gamme are C,D,E,F,G,A,B.

- **Questions:** Why do certain notes played together sound harmonious and the chords formed have such varied colours? What are the frequencies of these notes that resonate? Are there any notes missing from the equal-temperament scale (used in Western music only since the late 19th century [1, p.197] and absent in other cultures)? Could new notes have an interesting role in a musical creation? How to play with them? This is how we propose to approach these questions [2].

- The sound of a **sung voice** or a musical instrument is a periodic signal and is therefore a superposition of pure notes (or sine waves) called **harmonics**, located at integer multiples of the note we hear. All this richness contributes to the **timbre of the voice** or instrument.

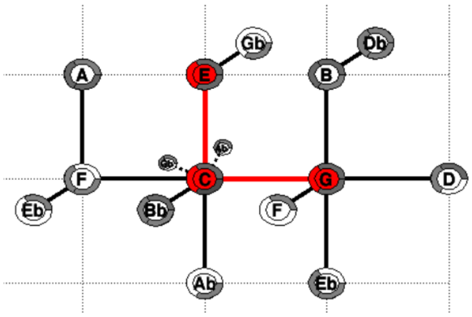


- We are very sensitive to this structure (by unconscious mechanisms that are still mysterious [3]), in particular to the first frequency ratios that appear and are universal pillars of music: $\times 2$: **octave**. $\times 3/2$: **fifth**. $\times 4/3$: **fourth**. $\times 5/4$: **major third**. $\times 6/5$: **minor third** $\times 7/5$: **triton, etc.** We can thus observe that "resonant" or "just" musical intervals are ratios of frequencies which are "simple" fractions. Generally speaking, for any just interval which is a frequency ratio a/b (irreducible Rational number) we associate its "resonance" which is $R = \log(1/(ab))$: thus the smaller a, b are, the greater the resonance is. Here are the resonances of all the right intervals $f_2/f_1 = a/b$ within an octave. They form a fractal (infinitely fine structure in red):



- In comparison, the Western equal temperament (represented by the twelve blue lines) is well suited for fifths and fourths but not for the other just intervals (red lines).

- So how do you play with these other notes? According to the Fundamental theorem of arithmetic, each fraction a/b breaks down into thePrime numbers P: 2 (octave), 3 (fifth), 5 (major third 1/5), 7 (minor seventh $-1/3$ tone, used in the blues), 11 (fourth $+1/4$ tone, used in Arabic music), 13, etc. For example the minor third (C-Eb) is $6/5 = 2 \times 3/5$. The notes are therefore represented by points on a multidimensional integer lattice $\mathbb{Z}^P$ generated by these prime numbers (or musical intervals), initially described by Euler (1739) and called **Tonnetz** or P-adic number or Adele representation. A resonant set of several notes, in other words a **just musical chord, will thus be like a compact molecule whose atoms are the notes**. The harmonic colour of the chord is determined by the shape of this "molecule". Ex: the minor chord C-Eb-G is a triangle. Its resonance is the mutual sum of the resonances of the intervals, a bit like the binding energy of a molecule. We propose a real time visualisation of the music on this Tonnetz network (forgetting the $\times 2$ dimension of the octaves). In the following figure (extracted from an instrument software working in real time and under development), the axes correspond to the factors $3, 5, 7$:



We want to explore the possibilities of this representation of notes, their rational relationships and geometric properties and make music through compositions and improvisations by Bo VanderWerf, Magic Malik and other musicians. **"Let's listen to the music of prime numbers..."**

2.2 Software development for research, composition and live concerts.

We want to **make the musical theoretical aspects presented in the previous section practical and usable**. To do this, we want to **develop a software** (free software as well as articles and tutorials), which would allow in concert and improvisation situations to have the following characteristics:

- This software would act **as a musician and also as a composer in real time: it would hear and understand** what is going on harmonically (in relation to the rules presented in the section 2.1) and **would propose different new tracks in real time**, and would be attentive to the choices that the musicians make. The real-time composition would follow dynamic and probabilistic rules predefined according to Markov graphs for example.

- During the performance this software could **project on a large screen real-time images that represent and explain the harmonic/rhythmic situation** of the moment.

- Development of just music instruments (slide flute for Mr Malik, EWI, ...), finding out how to play controlled just music with a set of instruments and **generate auditory illusions of voices**.

For this we need to develop software that works in real time and implement powerful algorithms for audio analysis and audio synthesis. We use the C++ language and appropriate audio libraries in parts developed by Alexandre Ratchov.

Concerning **lutherie, i.e. the making of new instruments** or the modification of existing instruments allowing to play on arbitrary and time-dependent temperaments, we develop the following tracks:

- Use of midi instruments like the EWI or keyboards or controllers MIDI, allowing a direct connection with the program and the synthesis of microtonal notes (i.e. at the pitch calculated and desired by the software in real time).

- Have the program interact in real time with the musicians (acoustic flute, saxophone, voice, guitars etc), detect their note and suggest small changes in pitch and volume to perfect the resonance.

- Develop a **"slide flute"** for Magic Malik, whose slide would be controlled in real time by the computer and would allow precise adjustment of the pitch.

3 Objectives

- **Using mathematics and computer science**, **develop new musical frameworks for composing and improvising**, possibly **analyzing existing music** with this new perspective. Development and publication of software and midi/audio VST3, AU, AAX, or other plugins for various computer music environments (Mac, Windows, Linux, Android, and for various host software: Ableton, Logic Pro, Protools, Ardour etc) that would be distributed

- **Conferences and musical performances** on campus open to all. **Educational workshops for students**, proposed to the different musicology and science departments and other interested public, based on the use of the software plugin and scientific and artistic demonstrations. Communication plan: via the cultural service EST and MACI of UGA.

- Research **articles** (musicology and science journals).

- An **audio/visual documentary** on the different stages of the development of the project: the development of the ideas, the rehearsals, the interactions between artist-musicians and scientists, the compositional processes and up to the production of the recorded music works.

- **Concerts** with original compositions, partnership with the hexagone de Meylan for the Biennale art-science Expérimenta and possibly other events.

- **Studio recordings, albums.** Supported by the label 11h11.

- **Dissemination of new concepts** to musicians and composers.

4 Calendar of working meetings and concerts

- **May 2023:** an artistic residency in Grenoble with Bo, Malik, Alexandre, Frédéric.

- **December 2023:** an artistic residency in Grenoble with Bo, Malik, Alexandre, Frédéric.
- **February 2024:** a 4-day artistic residency in Grenoble with Bo, Malik, Alexandre, Frédéric and 3 other musicians (rhythm section piano, bass, drums), ending with a **rehearsal open to all, a concert and a performance** at «Expérimenta» in partnership with the Hexagone de Meylan

References

[1] DJ Benson. Music: a mathematical offering. *pdf version*.

[2] F Faure, M. Mezzadri, and F. Ratchov. Analyse et jeu musical en tempérament juste adaptatif. *link*, 2015.

[3] Jan Schnupp, Israel Nelken, and Andrew King. *Auditory neuroscience: Making sense of sound*. MIT Press, webpage, 2011.