An individual coordinate function in the tx- or ty-plane can be graphed (simultaneously, with the *MacMath* software) at the right of Figure 6.1.1; these too are tangled. Since the coordinate functions intersect all over the place, these right-hand graphs are quite different (and less helpful) than those produced by *MacMath* for equations in $\mathbb{R}^1$. With sufficient effort you may be able to sort out which curves correspond (e.g., two of them are marked 1 and 2 respectively on all three graphs). But visualizing how in general $x(t)$ and $y(t)$ synthesize to produce the spatial motion of solutions in $\mathbb{R}^3$ is extremely difficult.

To begin to sort out some of these difficulties, to the extent that it is possible, we shall return to the important but overly simple Example 6.0.1 of the system in $\mathbb{R}^2$ resulting from $x'' = -x$: here the result is familiar enough to aid in visualization, and we can discuss the various possible representations of the solutions. Then we shall return to the problem of Example 6.1.1 and give some indication of why it is particularly difficult to represent solutions in a meaningful way.

Example 6.1.2. Consider the system

$$\begin{aligned} x' &= y \\ y' &= -x, \end{aligned} \tag{5}$$

which produces Figure 6.1.2 from a few solutions. This picture is more organized than Figure 6.1.1, but still somewhat jumbled:

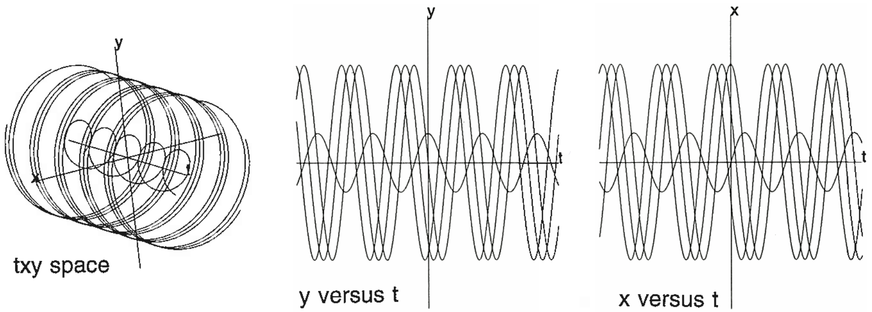

FIGURE 6.1.2. Some solutions to $x' = y$, $y' = -x$.

For the purpose of untangling the jumbled first impression of Figure 6.1.2, let us jump ahead to the fact (which you might guess from the second order equation $x'' = -x$, or from the coordinate function graphs on the right of Figure 6.1.2, or else can confirm by substitution) that some of the solutions to the system (5) are

$$x = y(t) = C \sin t$$

For each choice of the constant C, the equations (6) represent in txy-space a *circular spiral*, moving forward in time, as shown in Figures 6.1.3 and 6.1.4 for several different values of C; each solution starts at $t = 0$, with $x_0 = 0$, $y_0 = C$. (To show the coordinate functions most clearly, the time axis in Figure 6.1.3 goes only to 10 in each direction; to show the spirals most clearly, the time axis in Figure 6.1.4 (and 6.1.2) runs to 100 in each direction.)

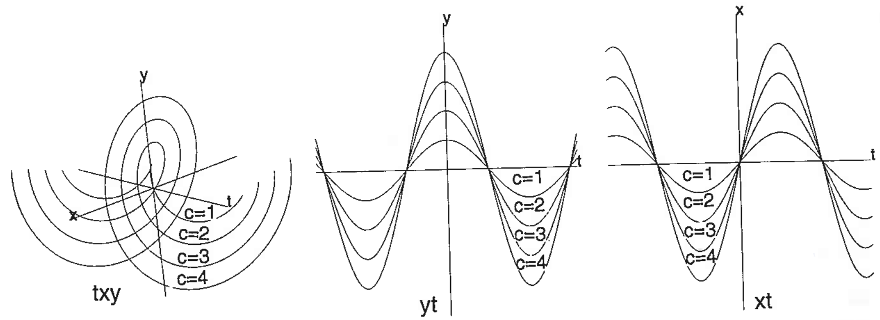

FIGURE 6.1.3. Selected solutions to $x' = y$, $y' = -x$, with $x_0 = 0$, $y_0 = C$.

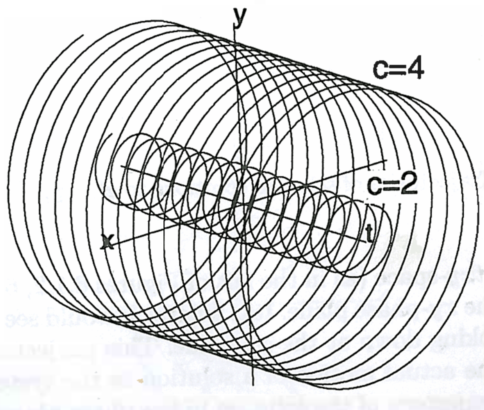

FIGURE 6.1.4. Selected solutions to $x' = y$, $y' = -x$, with $x_0 = 0$, $y_0 = C$, but with time axis running to 100 rather than 10. ▲