FAMILY NAME : **GIVEN NAME :**

GROUP :

Université Grenoble Alpes MAT334 Year 2016-2017

CC1b : Third partial examination (November 28th, 2016)

Only one handwritten sheet A4 allowed

All electronic devices are strictly forbidden

Duration 45 min

The unclear answers will be automatically excluded

Exercise 1. Let $a > 0$. The work done by the force $F : \mathbf{R}^3 \to \mathbf{R}^3$ given by

$$F(x,y,z) = (2xy, y^2, -x^2) = 2xy\vec{i} + y^2\vec{j} - x^2\vec{k}$$

to a particle moving along the curved described as the intersection of the hyperboloid $x^2 + y^2 - 2z^2 = 2a^2$ and the plane $y = x$ from the point $(a, a, 0)$ to the point $(a\sqrt{2}, a\sqrt{2}, a)$ is

☐$(2\sqrt{2} - 7/3)$; ☑ $(2\sqrt{2} - 7/3)a^3$;
☐ $-(2\sqrt{7} + 5)a^3$; ☐ $-a^3$.

Exercise 2. The integral

$$\oint_C (x+y)^2 dx - (x^2+y^2) dy$$

where C is the curve given by any counterclockwise parametrization of the triangle of vertices $(0,0)$, $(1,0)$ and $(0,1)$ is

☐ $-5/3$; ☐$5/3$; ☐1;
☑ -1; ☐$4/3$; ☐ $-4/3$.

In the following exercises complete the correct answers.

Exercise 3. Let $T : \mathbf{R}^3 \to \mathbf{R}^3$ be the unique linear transformation satisfying that

$$T(1,0,0) = (1, -\frac{3}{2}, 2)$$
$$T(0,1,0) = (-3, \frac{9}{2}, -6)$$
$$T(0,0,1) = (2, -3, 4).$$

Then $T(5, 1, -1) = (0, 0, 0)$. The kernel of T is the subspace of equation $x + ay + bz = 0$, where $a = -3$ and $b = 2$. The image of T is the subspace defined by the equations $x/2 = y/c = z/d$, where $c = -3$ and $d = 4$.

Exercise 4. Let

$$A = \begin{pmatrix} 4 & -4 & 2 \\ 2 & -2 & 2 \\ 0 & 0 & 1 \end{pmatrix}.$$

(a) The eigenvalues of A are 0, 1 and 2.

(b) The matrix A has three one-dimensional eigenspaces. They are spanned by the vectors $(1, 1, 0)$, $(2, 2, 1)$ and $(2, 1, 0)$, respectively.

Exercise 5. Consider

$$A = \begin{pmatrix} 6 & 4 \\ -6 & -4 \end{pmatrix}.$$

Then, given any $n \in \mathbf{N}$,

$$A^n = \begin{pmatrix} a & b \\ c & d \end{pmatrix},$$

with $a = 3\ 2^n$, $b = 2^{n+1}$, $c = -3\ 2^n$ and $d = -2^{n+1}$.