Full name : ....

1
2

MAT332 - Series and integration
Fall — 2022

Third written examination

The unclear answers will be automatically excluded.

Any electronic device is strictly forbidden.

12pt

1. Mark the correct answer :

(*a*) $\int_0^7 \sqrt[3]{1+x}dx$ is
☐ $\frac{112}{19}$ ☑ $\frac{45}{4}$ ☐ 12 ☐ 11 ☐ $\frac{15}{4}$ ☐ $\frac{9}{2}$;

(*b*) $\int_0^4 \frac{(x-3)^2}{x+1}dx$ is
☐ $-15+15\ln(5)$ ☐ $-15+16\ln(5)$ ☐ $-20-15\ln(5)$
☐ $-20+15\ln(5)$ ☑ $-20+16\ln(5)$ ☐ $-20-16\ln(5)$;

(*c*) $\int_0^1 4x^2e^{2x}dx$ is
☑ e^2-1 ☐ e^2 ☐ $e-1$ ☐ e ☐ $2e^2-1$ ☐ 1;

(*d*) $\int_{-\pi}^{\pi} \cos(2x)\sin(5x)dx$ is
☐ 2π ☑ 0 ☐ $\frac{10}{21}\pi$ ☐ $\frac{20}{21}\pi$ ☐ 3π ☐ π;

(*e*) $\int_2^4 \frac{8}{x\ln^3(x)}dx$ is
☐ $\frac{1}{3}\ln^2(2)$ ☑ $\frac{3}{\ln^2(2)}$ ☐ $\frac{1}{4}\ln^2(2)$
☐ $\frac{4}{\ln^2(2)}$ ☐ $\frac{13}{4\ln^2(2)}$ ☐ $\frac{4}{13}\ln^2(2)$;

(*f*) $\int_0^1 \ln^2(x)\sqrt{x}dx$ is
☑ $\frac{16}{27}$ ☐ 0 ☐ $+\infty$ ☐ $-\infty$ ☐ $\frac{8}{9}$ ☐ $\frac{4}{3}$.
Hint : Do integration by parts twice.

8pt

2. Mark the correct answer :

(*a*) if $f : \mathbb{R} \to \mathbb{R}_{\geqslant 0}$ is a C^1 function with $f(0)=0$ and $f'(0)=1$, then $\int_0^1 \frac{f(x)}{x^r}dx$ converges exactly for all
☐ $r \leqslant \frac{1}{2}$ ☐ $r < \frac{1}{2}$ ☐ $r<1$ ☐ $r \leqslant 1$ ☑ $r<2$ ☐ $r<0$;

(*b*) for $r,s>0$, the integral $\int_1^{+\infty} \frac{1}{x^r+\ln^s(x)+1}dx$ is convergent exactly for all
☐ $r,s>1$ ☐ $r,s>1/2$ ☐ $r \geqslant 1$ and $s>3$
☐ $s \geqslant 3$ ☑ $r>1$ ☐ $r \geqslant 1$ and $s \geqslant 2$;

(*c*) the integral $\int_1^{+\infty} \frac{\sin(x)}{x^r}dx$ is convergent exactly for all
☑ $r>0$ ☐ $r \geqslant 0$ ☐ $r>1$ ☐ $r \geqslant 1$ ☐ $r>2$ ☐ $r \geqslant 2$;

(*d*) the integral $\int_0^1 \frac{1}{x^r}\sin(\frac{1}{x})dx$ is convergent exactly for all
☐ $r<0$ ☐ $r<1$ ☐ $r \leqslant 1$ ☑ $r<2$ ☐ $r \leqslant 2$ ☐ $r<3$.