

# Raphaël Achet

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*Ph.D. student in algebraic geometry*

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## Ph. D. Thesis

Title *Picard group of the unipotent algebraic groups on any field*

Supervisors Michel Brion

Abstract Let  $k$  be any field. We study the Picard group of the (smooth connected) unipotent  $k$ -algebraic groups.

As every unipotent algebraic group is an iterated extension of forms of the additive group  $\mathbb{G}_{a,k}$ , we study the Picard group of the forms of  $\mathbb{G}_{a,k}$ . In fact we study the Picard group of forms of  $\mathbb{G}_{a,k}$  and the affine line  $\mathbb{A}_k^1$  simultaneously using a geometric method. We obtain an explicit upper bound on the torsion of the Picard group of the forms of  $\mathbb{A}_k^1$  and their regular completion, and a sufficient condition for the Picard group of a form of  $\mathbb{A}_k^1$  to be nontrivial. We also give examples of nontrivial forms of  $\mathbb{A}_k^1$  with trivial Picard groups.

In general, a unipotent  $k$ -algebraic group is a form of the affine  $n$ -space  $\mathbb{A}_k^n$ . In order to study the Picard group of a form  $X$  of  $\mathbb{A}_k^n$  with a geometric method, we define a “restricted” Picard functor; we show that if  $X$  admits a regular completion then the “restricted” Picard functor is representable by a unipotent  $k$ -algebraic group (smooth, not necessarily connected). With this “restricted” Picard functor and geometric arguments we show that the Picard group of a unirational form of  $\mathbb{A}_k^n$  is finite. Moreover we generalise a result of B. Totaro: if  $k$  is separably closed and if the Picard group of a unipotent  $k$ -algebraic group is nontrivial then it admit a nontrivial extension by the multiplicative group.

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## Education/Experience

2014–Now **Ph.D.**, *Fourier Institute*, University of Grenoble.

2013–2014 **Master 2 of Mathematics**, *E.N.S. Lyon*, speciality algebraic groups and representations.

Research stage: *unipotent algebraic group* with Bertrand RÉMY.

2012–2013 **Preparation for the agrégation**, *E.N.S. Lyon*.

Option algebra and computer algebra. Agrégation obtained with rank 56.

2011–2012 **Master 1 of Mathematics**, *E.N.S. Lyon*, speciality algebraic geometry and number theory.

Research stage: *some characters of the idele class group of a number field* with Vincent PILLONI.

2010–2011 **Licence 3 of Mathematics**, *E.N.S. Lyon*.

Research stage: *L-functions on real quadratic field* with Pierre CHAROLLOIS

2007–2010 **Preparatory classes (competitive school)**, *Lycée Joffre*, Montpellier, MPSI–MP\*.

Admit to *E.N.S. Lyon* with rank 118

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## Publications/Preprints

- 2017 R. Achet. Picard group of the forms of the affine line and of the additive group. *Journal of Pure and Applied Algebra* 221(11): 2838 – 2860, 2017.

## Oral presentations

- 2017
- University of Lille, seminar algebraic geometry.
  - Mathematics research institute of Rennes, seminar algebraic geometry.
  - Bourgogne Mathematical Institute (Dijon), Mini-Workshop Algebraic Geometry.
  - E.S.I. (Vienna), winter school “Geometry and Representation Theory”.
- 2016
- Fourier Institute (Grenoble), seminar algebra and geometry.
  - E.N.S. (Paris), seminar rational varieties.

## Conferences

- 2017
- Summer school "Current Topics in the Theory of Algebraic Group", *Dijon*.
  - Workshop Algebraic Groups, *Oberwolfach*.
  - Geometry and Representation Theory, *E.S.I. Vienne*.
  - Colloque Tournant 2017 du GDR Théorie de Lie Algébrique et Géométrie, *Amiens*.
- 2016
- Geometric and algebraic Lie theory, *Montpellier*.
  - 5th Swiss-French workshop on algebraic geometry, *Charmey*.
  - Inter'Actions 16, *E.N.S. Lyon*.
  - Mini Course 2016, Commutative Algebraic Groups, Hermitian K-Theory and Quadratic Forms, *Lens*.
  - Representation Theory of Finite and p-adic Groups of Lie Type, *CIRM*.
- 2015
- Inter'Actions 15, *Grenoble*.
  - Gael XXIII, *KU Leuven*.
  - Mini Course 2015, Linear algebraic groups, vector bundle classification and division rings, *Lens*.
  - Summer School Reductive Groups, *Schloß Schney*.
  - Singularity and tropical geometry, *Paris 7*.
- 2014
- Inter'Actions 14, *Lyon 1*.
  - Algebraic Groups and Representations, *Lyon 1*.
- 2012
- Young researchers in number theory, *E.N.S. Lyon*.

## Teaching/Popularisation of mathematics

- 2016
- Lecture Mat103, mathematics for biologist at *Grenoble Alpes* university.
  - Lecture Mat307, Parametric curves and differential equations at *Grenoble Alpes* university.
  - Popularisation of mathematics at school *Stendhal*, Grenoble.
- 2015
- Lecture Mat11a, mathematics for biologist at *Joseph-Fourier* university.
- 2014
- Lecture Mat11a, mathematics for biologist at *Joseph-Fourier* university.
- 2011–2012
- Oral exam in mathematics at the preparatory classes *Jean Perrin*, Lyon.
- 2010
- Tutoring in mathematics at school *Robert Doisneau*, Vaulx-en-Velin.

———— Languages

French **Mothertongue**

English **Advanced**

Spanish **Basic**

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