The Art Gallery Problem

Maé Miachon Lemeulle et Liticia Renault

Problem statement

Let's represent art galleries by polygons of a given number of vertices w, and let be G(w) the set of the numbers of guards required to surveil the w-walled galleries we can build.

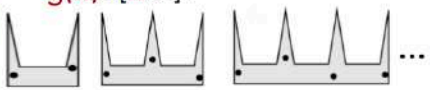
We want to find g(w):=Min(G(w)) knowing w.

Notes

[x] is the greatest integer smaller or equal to x

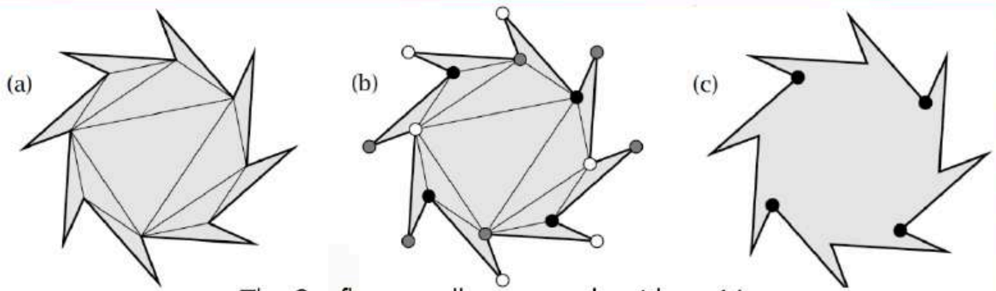
- We can notice that any triangular gallery can be guarded by only one guard, i.e. g(3)=1 :

- Crown galleries give us g(w)≤[w/3] :



6 walls 9 walls 12 walls

Proof : Fisk's colourful proof of the Art Gallery Theorem : g(w)=[w/3]



The Sunflower gallery example with w=16

We want to show that g(w)≥ [w/3] (see §Notes for ≤)

a) Triangulation of the gallery

We divide the art gallery into triangles by building noncrossing lines that join well-chosen corners.

b) Polychromatic 3-colouring

We allocate a colour (white, grey or black) to each corner of the triangles so that every triangle has one corner of each colour.

c) Placing the guards

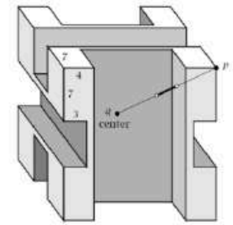
We select the smallest group of corners of the same color (in b), it's the group of the black corners). We then place the guards at these corners. In this example the number of guards required is ≤4 (3 is enough) which is less than [16/3]=5.

Thus, the number of guards g(w) is at most the integer part of w/3.

Higher dimension : 3D

Unlike in 2D, placing a guard in each corner of a polyhedron does not assure every point will be watched.

Counter-example : The Octoplex



Literature used

T.S. Michael, (2009). *How to guard an art gallery & other discrete mathematical adventure* [Chapter 3], The John Hopkins University Press (Baltimore)

Images

Les coquelicots, Claude Monet.
T.S Michael's book's illustrations.

Further information

Please see :
the ScienceDirect article _Note on an art gallery problem_ by György Csizmadia and Géza Tóth.