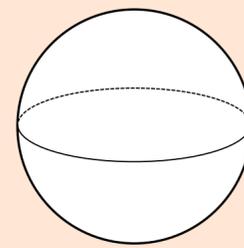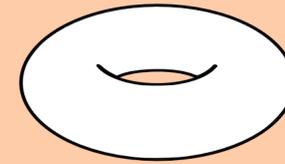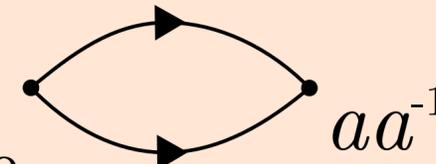Surface :
connected manifold
of dimension 2
that admits
a countable
topological base

a polygone
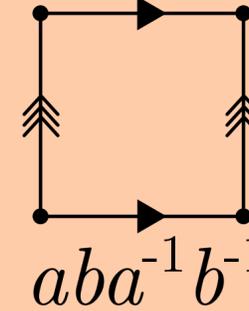whose edges are pairwise
identified

a word

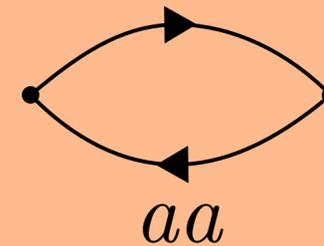at least four letters :
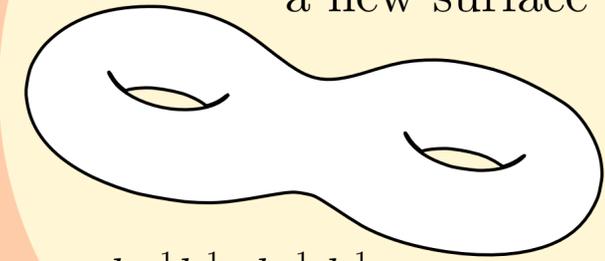pairs of letters aa^{-1} can
be deleted

Sphere

aa^{-1}

Torus

$aba^{-1}b^{-1}$

Projective
plane

aa

Glue surfaces together
in order to obtain
a new surface :

$aba^{-1}b^{-1}cdc^{-1}d^{-1}$
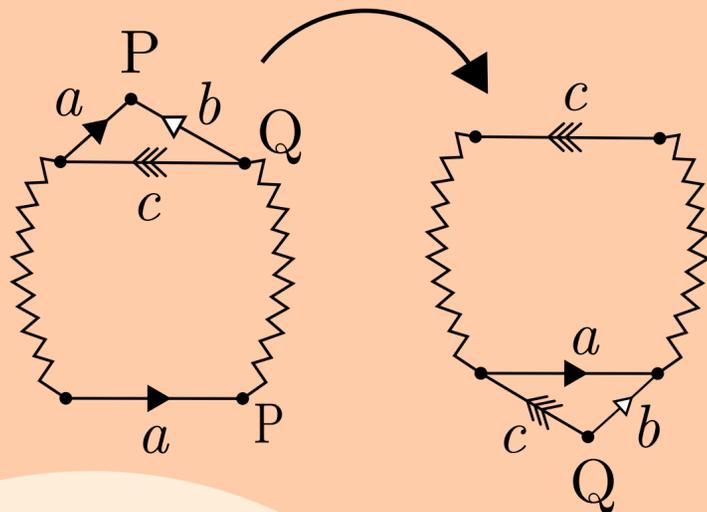
words
concatenation

Classification
of Compact Surfaces

Any surface is either
- a sphere
- a connected sum of tori
- or a connected sum of projective planes
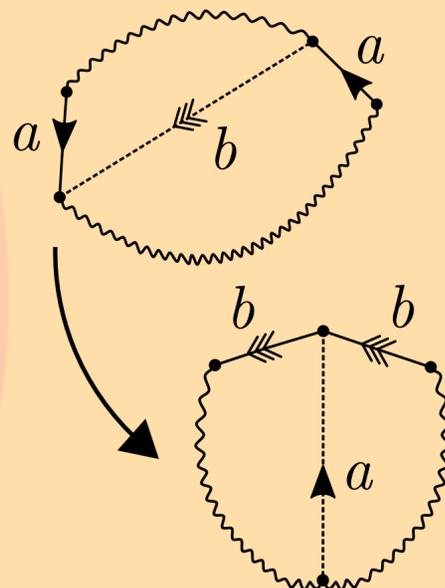
1 *Cut-and-paste* method

The polygone is
transformed such as all
vertices are identified together

2 If a letter a occurs twice, the word is
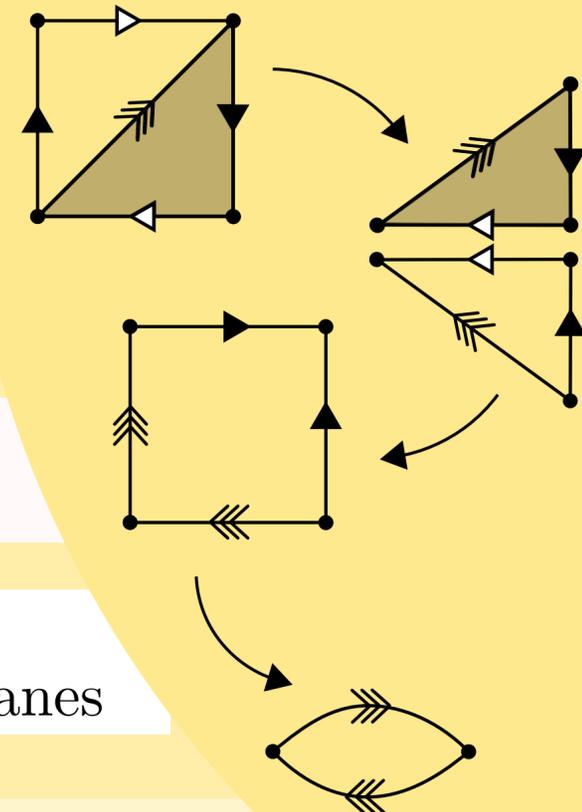equivalent to an other with the two
letters adjacent aa.

both a and a^{-1} in the word
$\Rightarrow$ it contains a subword $aba^{-1}b^{-1}$

no subword aa
$\Rightarrow$ connected sum of tori

otherwise :
$\Rightarrow$ connected sum of projective planes

3

Example

Bertrand Cédric
Dépernet Sarah

Bibliography :   W. S. Massey Algebraic Topology, an introduction