

## SCHEDULE WEEK 6

### CONFERENCE *RATIONAL POINTS ON FANO AND SIMILAR VARIETIES*

	Monday 20/5	Tuesday 21/5	Wednesday 22/5	Thursday 23/5	Friday 24/5	Saturday 25/5
8h						Week-end
9h	Welcome					
10h	Collier-Tyrlène Hennite	Savin Hennite	Poonen Hennite	Salberger Hennite	Chambert-Loir Hennite	
11h	Coffee break	Coffee break	Coffee break	Coffee break	Coffee break	
12h	Blin Hennite	Le Boudec Hennite	Smeets Hennite	Wittenberg Hennite	Brandes Hennite	
13h						
14h						
15h	Tanimoto Hennite	Salgado Hennite	Hartmann Hennite	Loughran Hennite		
16h	Coffee break	Coffee break	Coffee break	Coffee break		
17h	Harari Hennite	Denarič Hennite	Sofoš Hennite	Xu Hennite		
18h						
19h		Party Ground floor				
20h						

#### **Margaret BILU:** *Motivic Euler products*

The Grothendieck group of varieties over a field  $k$  is the quotient of the free abelian group of isomorphism classes of varieties over  $k$  by the so-called cut-and-paste relations. It moreover has a ring structure coming from the product of varieties over  $k$ . Many problems in number theory have a natural, more geometric

---

*Date:* Monday, May 20th–Sunday, May 26th.

counterpart involving elements of this ring. I will start by explaining how one can make sense of a notion of Euler product for some power series with coefficients in this ring. A few applications to motivic stabilisation results will be mentioned, after which I will show how this notion may be used to prove a motivic analogue of Manin's conjecture for equivariant compactifications of vector groups.

**Julia BRANDES:** *Rational lines on cubic hypersurfaces*

One of the most intensely studied question in the intersection of analytic number theory and algebraic geometry concerns the existence and distribution of rational points on cubic hypersurfaces, but the analogous question regarding lines or higher-dimensional linear spaces is far less understood. In this talk, we will show that every smooth cubic hypersurface of projective dimension at least 29 contains a rational line, superseding earlier bounds due to Dietmann and Wooley. This is joint work with Rainer Dietmann.

**Antoine CHAMBERT-LOIR:** *Integral points of bounded height on toric varieties — reloaded*

We study the analogue for integral points of the Manin-Peyre conjecture about the number and distribution of rational points of bounded height. In view of the successes of the Fourier-theoretic method in the years 1995-2000, the case of equivariant (partial) compactifications of algebraic groups is particularly promising. Ten years ago, after Yuri Tschinkel and I had treated the case of vector groups, we released a preprint that claims to tackle the case of tori. Unfortunately, the proof there is incomplete, and a counterexample of Florian Wilsch demonstrates that the situation is more complicated. The goal of the lecture is to present the Fourier-theoretic method, the kind of output it leads to, and to present the new obstruction that arises, with the hope that it will suffice to describe the asymptotic number and distribution of integral points of bounded height on toric varieties.

**Jean-Louis COLLIOT-THÉLÈNE:** *Obstruction de Brauer-Manin pour les surfaces de Markoff*

C'est un problème difficile de décider quand une surface cubique affine a un point entier, et si les points entiers sont denses (en divers sens). Ghosh et Sarnak ont étudié la famille des surfaces de Markoff, données par une équation  $x^2 + y^2 + z^2 - xyz = m$ , où  $m$  est entier. Ces surfaces admettent un groupe discret d'automorphismes, avec un domaine fondamental borné pour les solutions entières. Ghosh et Sarnak montrent que le principe de Hasse entier est souvent en défaut. Certains de leurs arguments utilisent la loi de réciprocité quadratique. Dans un travail commun avec Dasheng Wei et Fei Xu, nous examinons dans quelle mesure l'obstruction de Brauer-Manin entière rend compte de leurs résultats. Un travail similaire au nôtre a été effectué par Loughran et Mitankin.

**Cyril DEMARCHE:** *Structure of homogeneous spaces and applications to local-global principles*

(joint work with Giancarlo Lucchini-Arteche) We study the structure of homogeneous spaces of linear algebraic groups over perfect fields and prove a result reducing certain nice properties or conjectures about homogeneous spaces to the crucial case of homogeneous spaces of the special linear group with finite stabilizers. In particular, one can apply this reduction to the Brauer-Manin obstruction to the Hasse principle and weak approximation for rational points or zero-cycles of degree one over number fields (as in a recent work by Harpaz and Wittenberg).

**David HARARI:** *Complexes of tori and rational points on homogeneous spaces over a function field*

We explain new arithmetic duality theorems for finite group schemes and 2-term complexes of tori defined over a global field of positive characteristic. We also apply these results to prove local global principles for homogeneous spaces of linear algebraic groups (joint work with Cyril Demarche).

**Julia HARTMANN:** *Local-Global principles for tori over arithmetic surfaces*

Given a field  $F$  and a collection of overfields  $F_i$  ( $i \in I$ ), we say that the local global principle holds for an  $F$ -variety  $Z$  if the existence of a rational point over each  $F_i$  implies the existence of an  $F$ -rational point.

In this talk, we study this question when  $F$  is a semi-global field, i.e., the function field of a curve  $X$  over a complete discretely valued field, and  $Z$  is a principal homogeneous space under a torus. It is known that a local-global principle need not hold in general. We give a formula which often leads to an explicit description of the obstruction set in the case when the torus is defined over  $X$ .

This is joint work with J.L. Colliot-Thélène, D. Harbater, D. Krashen, R. Parimala and V. Suresh.

**Pierre LE BOUDEC:** *On a conjecture of Poonen and Voloch II: Lattice point counting and the variance of the number of rational points on Fano hypersurfaces*

This talk is the second part of two talks on joint work with T. Browning and W. Sawin. The aim of this second talk will be to describe in more details some of the most interesting aspects of our proof of the Poonen–Voloch conjecture.

**Daniel LOUGHRAN:** *A quantitative version of the fibration method*

Harpaz and Wittenberg have made spectacular progress on the fibration method, with regards to the study of the Brauer-Manin obstruction to the Hasse principle on rationally connected varieties. In this talk I present a quantitative version of their method, which allows one to count the number of rationally connected varieties in certain families with a rational point, even if the Brauer group is non-constant. This is joint work with Lilian Matthiesen.

**Bjorn POONEN:** *The local-global principle for stacky curves*

For smooth projective curves of genus  $g$  over a number field, the local-global principle holds when  $g = 0$  and can fail for  $g = 1$ , as has been known since the 1940s. Stacky curves, however, can have fractional genus. We construct stacky curves of genus  $1/2$  that violate the local-global principle, and show that  $1/2$  cannot be reduced. This is joint work with Manjul Bhargava.

**Per SALBERGER:** *Counting rational points of cubic hypersurfaces*

Let  $N(X; B)$  be the number of rational points of height at most  $B$  on an integral cubic hypersurface  $X$  over  $\mathbf{Q}$ . It is then a central problem in Diophantine geometry to study the asymptotic behavior of  $N(X; B)$  when  $B$  grows. We present some recent results on this for various classes of cubic hypersurfaces.

**Cecília SALGADO:** *Mordell Weil rank jumps and the Hilbert property*

We study rank jumps of the Mordell-Weil groups of the fibres of elliptic surfaces. I will discuss the cases for which we show that the set for which the rank jumps is not thin. This is a work in progress with Dan Loughran.

**Will SAWIN:** *On a conjecture of Poonen and Voloch I: Probabilistic models for counting rational points on random Fano hypersurfaces*

Poonen and Voloch have conjectured that almost every degree  $d$  Fano hypersurface in  $\mathbf{P}^n$  defined over the field of rational numbers satisfies the Hasse principle. In joint work with T. Browning and P. Le Boudec, we establish this conjecture under the mild assumption that  $n > d + 1$ . This talk is the first of two talks. Our goal in this first talk will be to introduce our main results and to present the strategy of the proof.

**Arne SMEETS:** *Campana's orbifolds, points of bounded height and fibrations*

I will give a gentle introduction to the theory of Campana's orbifold pairs, with an eye towards arithmetic aspects, in particular the study of points of bounded height on such pairs. In particular, I will highlight recent work of Browning-Yamagishi and Pieropan-Tanimoto-Varilly-Alvarado-Smeets on the topic, and I will discuss the relation with the behaviour of rational points in fibrations.

**Efthymios SOFOS:** *Serre's problem for diagonal conics*

Assume that  $B$  is a large real number and let  $c_1, c_2, c_3$  be three randomly chosen integers in the box  $[-B, B]^3$ . Consider the probability that the "random" curve

$$c_1X^2 + c_2Y^2 + c_3Z^2 = 0$$

has a non-zero solution  $(X, Y, Z)$  in the integers. Serre showed in the 90s that this probability is  $\ll (\log B)^{-3/2}$  while Hooley and Guo later proved that it is  $\gg (\log B)^{-3/2}$ . In joint work with Nick Rome we prove an asymptotic  $\sim c(\log B)^{-3/2}$ , where  $c$  is a positive absolute constant.

**Sho TANIMOTO:** *Sections of del Pezzo fibrations over  $\mathbf{P}^1$*

Mori's Bend and Break shows that if we deform a rational curve while fixing two points, then it breaks into the union of rational curves. However, in general it is difficult to control the number of components of a breaking curve and their properties. In this talk I will talk about our recent result for sections of del Pezzo fibrations over  $\mathbf{P}^1$  which we call as Movable Bend and Break, i.e., one can break a free section of high height to the union of one free section and one vertical free curve. Then we discuss several applications of this result, Batyrev's conjecture on the number of components of the space of sections, the irreducibility of the space of sections for certain del Pezzo fibrations, and Batyrev's heuristic for del Pezzo fibrations. This is joint work with Brian Lehmann.

**Olivier WITTENBERG:** *Approximation fine pour les points rationnels sur les corps de fonctions*

(Travail en commun avec Olivier Benoist.) La notion d'approximation faible joue un rôle central dans l'étude des points rationnels des variétés rationnellement connexes, que le corps de base soit un corps de nombres ou le corps des fonctions d'une courbe complexe ou réelle. Dans ce dernier cadre, nous introduisons une notion plus forte, l'approximation fine, et établissons des théorèmes de descente et de fibration pour cette dernière. Nous en tirons des conséquences d'une part sur l'approximation faible et d'autre part sur les classes algébriques dans l'homologie du lieu réel de certaines variétés réelles.

**Fei XU:** *Strong approximation for a family of norm varieties*

Let  $L/k$  be a finite extension of number fields and  $q(t)$  be a polynomial over  $k$ . It is a classical problem to study weak approximation for the family of norm varieties defined by

$$N_{L/k}(x) = q(t)$$

by various methods. The first non-trivial example for strong approximation with Brauer-Manin obstruction of the above equation was given by Derenthal and Wei, where  $[L : k] = 4$  and  $q(t)$  is an irreducible quadratic polynomial which has a root in  $K$ .

In this talk, I will explain that the above equation satisfies strong approximation with Brauer-Manin obstruction under Schinzel's hypothesis, where  $L/k$  is cyclic,  $q(t)$  is a product of distinct irreducible polynomials  $q_i(t)$  with  $1 \leq i \leq n$  and one of the splitting fields  $M_i$  of  $q_i(t)$  having  $M_i \cap L_{\text{ab}} \subset L$  with the maximal abelian extension  $L_{\text{ab}}$  of  $L$ . This is a part of joint work with Cao and Wei.