

SCHEDULE WEEK 5

Time	Monday 13/5	Tuesday 14/5	Wednesday 15/5	Thursday 16/5	Friday 17/5	Saturday 18/5
8h						Week-end 18/5
9h						
10h						
11h	Loughran Darboux	Harpaз Darboux	Harpaз Darboux	Voisin College de France		
12h						
13h						
14h	Newton IMI-PRG			Le Boudec	Tian Jussieu	
15h		Sarti Orsay	Loughran Darboux	Valloni	Sturmfelzer Jussieu	
16h	Tea	Tea	Tea	Tea		
17h						
18h						
19h						
20h						

Yonatan HARPAZ: *Rational points on elliptic fibrations (after Swinnerton-Dyer)*

In this series of four lectures we will describe an approach, originally due to Swinnerton-Dyer and further developed by several authors, to the study of rational points on surfaces fibred into curves of genus 1, and more generally, on pencils of torsors under abelian varieties. Under suitable hypotheses (and possibly assuming powerful number theoretical conjectures), one can use this method to show that if the Brauer-Manin obstruction controls the Hasse principle for most of the fibers

Date: Monday, May 13th–Sunday, May 19th.

in the fibration, then the same holds for the total space. Results obtained in this manner are currently some of the only evidence for whether the Brauer-Manin obstruction controls the Hasse principle for K3 surfaces, for example.

Pierre LE BOUDEC: *A statistical version of a conjecture of Lang*

A conjecture of Lang predicts a lower bound for the canonical height of non-torsion rational points on elliptic curves defined over a fixed number field. In this talk we will adopt a statistical point of view on this conjecture for elliptic curves defined over the field of rational numbers.

Dan LOUGHRAN: *Rational points in families of varieties*

The first couple of lectures will consist of a general introduction to some algebro-geometric techniques in arithmetic geometry, such as the Lang-Weil estimates, Hensel's lemma and the Chebotarev density theorem for finitely generated extensions of the rationals, with the common theme being the importance of models and schemes. We will also cover basic terminology and techniques from the fibration method (split/non-split fibres and local solubility).

We will then use these techniques to prove the main theorem from the paper: Loughran, Smeets - *Fibrations with few rational points*.

This theorem says that for families of varieties satisfying certain geometric conditions, 100% of the varieties in the family have no rational point.

The intended audience is PhD students in arithmetic geometry.

Domenico VALLONI: *The arithmetic of K3 surfaces with complex multiplication.*

In the realm of K3 surfaces, the ones with complex multiplications (e.g. K3 surfaces with maximal Picard rank or Kummer surfaces associated to an Abelian surface with CM) play a very special role, due to their explicit arithmetic and Hodge-theoretic properties.

For example, if X is a complex K3 surface with CM, then it is automatically defined over a number field. This was originally proved by Pyatetskii-Shapiro and Shafarevich, and it follows from the interpretation of the moduli spaces of K3 surfaces as Shimura varieties. In this talk, we will show how the CM theory for K3 surfaces is a natural tool to perform concrete computations on their Brauer groups and fields of definition. In particular, we will present a general algorithm to classify all Brauer groups that can appear as $\text{Br}(\overline{X})^{G_K}$, where X/K is a K3 with CM and G_K is the absolute Galois group of K . Moreover, we will show how many complex K3 surface with CM admit a “canonical model” over an explicit abelian extension of the CM field. This will allow us to extend the classification provided by Schütt and Elkies of singular K3 surfaces that can be defined over \mathbb{Q} , and to prove a finiteness theorem due to Shafarevich and later generalised by Orr and Skorobogatov.