

SCHEDULE WEEK 3



Julie DESJARDINS: *Geometry of an "isotrivial" del Pezzo surface of degree 1*

The elliptic surface $E : y^2 = x^3 + AT^6 + B$ (A, B rational) is a canonical example where both (1) the density isn't yet proven with geometric means, (2) there are many cases where no (even conditional) result can be deduced from the study of the root number of the fibers ("= parity of the rank" under a weaker Birch Swinnerton-Dyer conjecture). Moreover, the blow down of the section at infinity of this surface gives a del Pezzo surface of degree 1. In this talk, I will speak about

Date: Monday, April 29th–Sunday, May 5th.

a joint work with Bartosz Naskręcki in which we describe the beautiful geometry under this particular type of surface : by describing a base of the Mordell-Weil group, we give an algorithm (with input A and B) allowing to determine the generic rank of the elliptic surface (between 0 and 3). When the rank is non-zero, this proves the Zariski-density of rational points of del Pezzo surface of degree 1 : $y^2 = x^3 + Am^6 + Bn^6$.

Jack PETOK: *Kodaira dimension of some moduli spaces of special hyperkahler fourfolds*

Let \mathcal{M}_n , with $n > 0$ even, denote the moduli of polarized degree n smooth complex hyperkahler fourfolds of K3^[2] type. For example, when $n = 2$, a general point corresponds to a smooth EPW double sextic; when $n = 6$, a general point represents the Fano variety of lines of a cubic fourfold. In each of these moduli spaces, one can study the Noether-Lefschetz divisors, also known as special divisors. The geometry of these divisors for $n = 6$ was initiated by Hassett. The Kodaira dimensions for all but finitely many special divisors in \mathcal{M}_6 were computed by Tanimoto and Várilly-Alvarado.

We study the special locus in \mathcal{M}_2 . We review how the special locus may be a countable union of divisors D_d corresponding to fourfolds X polarized by a rank 2 lattice whose orthogonal complement in $H^2(X, \mathbf{Z})$ has discriminant d . The divisor D_d is nonempty if and only if the discriminant d is 0, 2 or 4 mod 8 (and d is not 2 or 8). Using the “low weight cusp form trick” of Gritsenko, Hulek, and Sankaran, we compute the Kodaira dimensions of the (components of the) divisors D_d for all but finitely many discriminants d . We also discuss important connections to the Brauer groups of very general K3 surfaces.

Samir SIKSEK: *Explicit arithmetic of modular curves*

The arithmetic of modular curves is the key to many celebrated theorems in arithmetic geometry, such as Mazur’s isogeny theorem, Merel’s uniform boundedness theorem, and the split Cartan case of Serre’s uniformity conjecture due to Bilu, Parent and Rebolledo. In the last few years the subject has caught the imagination of computational number theorists who can prove concrete theorems about torsion subgroups of elliptic curves through calculations. The purpose of this minicourse is to convey the ideas and methods used in making the arithmetic and geometry of modular curves explicit. We will start by sketching the background on modular curves, their Jacobians, and their moduli interpretation. We will survey explicit methods for writing down equations for low genus modular curves, their degeneracy maps and j -maps, computing Mordell-Weil groups of modular Jacobians, and enumerating the rational points and low degree points in certain simple cases.