# SCHEDULE WEEK 12

To be completed

| Day | Morning | Afternoon | Tea | Late afternoon |
|---|---|---|---|---|
| Monday 1/7 | Poonen — Darboux (11h–12h) | Alpoge — Darboux (14h–15h) | Tea (16h) | |
| Tuesday 2/7 | Voloch — Darboux (11h–12h) | | Tea (16h) | |
| Wednesday 3/7 | Poonen — Darboux (11h–12h) | Alpoge — Darboux (14h–15h) | Tea (16h) | |
| Thursday 4/7 | Voloch — Darboux (11h–12h) | Matthiessen (14h–15h) | Tea (16h) | Paredes (16h–17h) |
| Friday 5/7 | Goncharov IHES; Čestavičius IHES | Gao IHES | Colmez IHES / tea | Mazur IHES |
| Saturday 6/7 | Week-end | | | |

**Levent Alpoge:** *Arithmetic Statistics*

I will give an introduction to the techniques that allow one to count number fields of small degree and bounded discriminant, to bound the average ranks of elliptic curves and of higher genus hyperelliptic curves, and to bound the number of integral/rational points on said curves. The methods for counting orbits in certain cases are well-enough developed that I hope to impart on the audience how to turn

the crank, so to speak. It will transpire that in these cases we will always reduce to counting integral points of bounded height in a lattice. I will indicate some first results towards counting orbits with nontrivial polynomial relations among their invariants. I will try to present the parametrizations and other techniques with as much intuition as I can give (for example: how one could have come up with these parametrizations), and the material will be accessible to all participants of the program.

**Lilian MATTHIESEN:** *Rational points in fibrations*

I will describe progress on Serre's problem on the number of varieties in a family that have a rational point. In particular, I will focus on the analytic part of this work which involves tools from additive combinatorics as well as results on multiplicative functions. This is joint work with Dan Loughran.

**PAREDES:** *Badly distributed sets lying on algebraic varieties over global fields*

Let $N$ be a natural number and let $S$ be a subset of $N^n$. A result of Walsh states that if $S$ is badly distributed at the level of residue classes modulo $p$ for various primes $p$, then $S$ is "small", or it possesses some "algebraic structure", namely a positive proportion of $S$ lies in an hypersurface defined over the rational numbers of small degree. Moreover, Walsh also proved that a positive proportion of any such $S$ always lies in an hypersurface defined over the rational numbers of small degree, but now the degree depending on $N$.

In this talk, we will show that these results can be generalized to sets $S$ lying in projective varieties defined over global fields, and that they can be used to obtain some diophantine applications. Part of this talk is work in progress with J. Menconi and R. Sasyk.

**Bjorn POONEN:** *p-adic approaches to rational points on curves*

In these four lectures, I will describe Chabauty's p-adic method for determining the rational points on a curve whose Jacobian has rank less than the genus, hint at Kim's nonabelian generalization, and finally discuss the recent paper of Lawrence and Venkatesh that uses p-adic period maps to give a new proof of Faltingsś theorem.

**References:**

- McCallum-Poonen, The method of Chabauty and Coleman
- Corwin, From Chabauty's method to Kim's non-abelian Chabauty's method
- Lawrence-Venkatesh, Diophantine problems and p-adic period mappings

**Notes:**

pdf file

**Felipe VOLOCH:** *Obstructions for rational points on curves*

This series of four lectures will discuss the various obstructions to the existence of rational points on curves over global fields, such as the Brauer-Manin obstruction, the finite descent obstruction and the obstructions coming from Grothendieck's section conjecture and the Chabauty method as well as weakenings and strengthenings of these various obstructions. We will discuss the relationship among these conjectures and some of the progress that has been made. We will look at the function field case, where more is known. We will discuss the connection with modularity, where some progress can be made sometimes contingent on some familiar conjectures.