SCHEDULE WEEK 11

Conference *Rational points on irrational varieties*

Dan Abramovich: *Resolution in characteristic 0 using weighted blowing up*

Given a variety, one wants to blow up the worst singular locus, show that it gets better, and iterate until the singularities are resolved.

Examples such as the whitney umbrella show that this iterative process cannot be done by blowing up smooth loci - it goes into a loop.

Date: Monday, June 24th–Sunday, June 30th.

We show that there is a functorial way to resolve varieties using weighted blowings up, in the stack-theoretic sense. To an embedded subvariety of a smooth variety one functorially assigns an invariant and a center whose stack-theoretic weighted blowing up has strictly smaller invariant under the lexicographic order.

This is joint work with Michael Tëmkin (Jerusalem) and Jarosław Włodarczyk (Purdue). A similar result was discovered by M. McQuillan.

Anna Cadoret: *Families of abelian varieties with a common isogeny factor*

I will discuss the following question, raised by Roessler and Szamuely. Let X be a a variety over a field k and A an abelian scheme over X. Assume there exists an abelian variety B over k such that for every closed point x in X, B is geometrically an isogeny factor of the fiber A_x. Then does this imply that the constant scheme $B \times k(\eta)$ is geometrically an isogeny factor of the generic fiber A_η? When k is not the algebraic closure of a finite field, the answer is positive and follows by standard arguments from the Tate conjectures. The interesting case is when k is finite. I will explain how, in this case, the question can be reduced to the microweight conjecture of Zarhin. This follows from a more general result, namely that specializations of motivic l-adic representation over finite fields are controlled by a "hidden motive", corresponding to the weight zero (in the sense of algebraic groups) part of the representation of the geometric monodromy.

Frédéric Campana: *"Special" manifolds: rational points and entire curves*

We prove (joint with J. Winkelmann) for any rationally connected projective manifold X analytic analogues of several conjectural properties in arithmetic geometry: the "Potential Density", the Weak Approximation Property, and the Hilbert Property (in the form stated and conjectured by Corvaja-Zannier). In particular: given any countable set N on any rationally connected X, there is a holomorphic map $\mathbf{C} \to X$ whose image contains N. The proof rests on deformation properties of rational curves.

More general conjectures claim similar (arithmetic and analytic) properties for the much larger class of "special manifolds", defined as the ones having no "$\Omega^\bullet$-big" line subbundle. They lie on the geometric spectrum at the opposite side of manifolds of "general type", for which Lang conjectured "Mordellicity". We conjecture that 'potential density' is equivalent to "specialness" for any X defined over a number field k.

These two conjectures combine to describe the structure of $X(k)$ for any X/k, by means of the "core map" $c : X \to C$, which "splits" X into its "special" part (the fibres), and "general type" part (the orbifold base).

Yang Cao: *Sous-groupe de Brauer invariant et application*

Pour étudier l'approximation faible ou forte d'un espace homogène ou, plus généralement, d'une variété munie d'une action d'un groupe algébrique connexe, la notion de sous-groupe de Brauer invariant est naturelle et compatible avec la méthode de descente. Tout d'abord, je vais introduire cette notion. Ensuite, je vais expliquer son lien avec descente. À la fin, je vais parler des applications, en particulier, la méthode de fibration équivariante.

Kęstutis Česnavičius: *Purity for the Brauer group of singular schemes*

For regular Noetherian schemes, the cohomological Brauer group is insensitive to removing a closed subscheme of codimension ≥ 2. I will discuss the corresponding statement for schemes with local complete intersection singularities, for instance, for complete intersections in projective space. Such purity phenomena turn out to be low cohomological degree cases of purity for flat cohomology. I will discuss the latter from the point of view of the perfectoid approach to such questions. The talk is based on joint work with Peter Scholze.

François Charles: *Affine and mod-affine varieties in arithmetic geometry*

We will explain how studying arithmetic versions of affine schemes and their birational modifications leads to a generalization to arbitrary schemes of both Fekete's theorem on algebraic integers, all of whose conjugates lie in a certain compact subset of C, and of classical results on approximation of holomorphic functions by polynomials with integral coefficients. We will try and introduce the relevant geometry of numbers in infinite rank as a means of studying the cohomology of coherents sheaves on these objects. This is joint work with Jean-Benoît Bost.

Brendan Creutz: *Descent obstructions on constant curves over global function fields*

Let C and D be proper geometrically integral curves over a finite field and let K be the function field of D. I will discuss descent obstructions to the existence of K-rational points on C. In this context, an adelic point on C can be viewed as a Galois equivariant map between the geometric points of D and C. We show that the adelic points surviving abelian descent correspond to those maps which extend to homomorphisms of the Jacobians and use this to prove that abelian descent cuts out the set of rational points when the genus of D is less than the genus of C. I will also present a connection between the finite etale descent obstruction and anabelian geometry. This is joint work with Felipe Voloch.

Jordan Ellenberg: *Rational points and fundamental groups*

I will talk about some results old and new about the relationship between rational points on varieties and fundamental groups. In a paper with Hast, we extend the class of curves to which Kim's non-abelian Chabauty method can be applied by means of a strange and interesting result of Bogomolov and Tschinkel about covers of hyperelliptic curves. I will also talk about old unpublished work with Venkatesh about uniform bounds for points of bounded height on varieties with large fundamental groups, in the hopes that people in the audience will have some ideas for contexts in which such a result might be useful!

Yonatan Harpaz: *Squares represented by a product of three ternary quadratic forms, and a homogeneous variant of a method of Swinnerton-Dyer*

Let k be a number field. In this talk we will consider K3 surfaces over k which admit a degree 2 map to the projective plain, ramified over a union of three conics. Such surfaces always admit a fibration into curves of genus 1, which one can try to exploit in order to study their rational points. When all three conics are simultaneously diagonalizable the associated Jacobian fibration admits a particularly nice form, rendering it amenable to the descent-fibration method of Swinnerton-Dyer. Assuming finiteness of the relevant Tate-Shafarevich groups, we show that when the coefficients of the diagonal forms are sufficiently generic, the Brauer-Manin obstruction is the only one for the Hasse principle on X. Using that the singular fibers of the fibration all lie over rational points, the dependence on Schinzel's hypothesis can be removed by adapting the method to run with the homogeneous version of that hypothesis, which is known in the case of linear forms thanks to the seminal work of Green–Tao–Ziegler.

Diego Izquierdo: *Homogeneous spaces, algebraic K-theory and cohomological dimension of fields*

In 1986, Kato and Kuzumaki stated a set of conjectures which aimed at giving a Diophantine characterization of the cohomological dimension of fields in terms of Milnor K-theory and points on projective hypersurfaces of small degree. Those conjectures are known to be wrong in general. In this talk, I will prove a variant of Kato and Kuzumaki's conjectures in which projective hypersurfaces of small degree are replaced by homogeneous spaces. This isjoint work with Giancarlo Lucchini Arteche.

Zev Rosengarten: *Tamagawa Numbers of Linear Algebraic Groups over Function Fields*

In 1981, Sansuc obtained a formula for Tamagawa numbers of reductive groups over number fields, modulo some then unknown results on the arithmetic of simply connected groups which have since been proven, particularly Weil's conjecture on Tamagawa numbers over number fields. One easily deduces that this same formula holds for all linear algebraic groups (not just reductive) over number fields. Sansuc's method still works to treat reductive groups in the function field setting, thanks to the recent resolution of Weil's conjecture in the function field setting by Lurie and Gaitsgory. However, due to the imperfection of function fields, the reductive case is very far from the general one; indeed, Sansuc's formula does not hold for all linear algebraic groups over function fields. We give a modification of Sansuc's formula that recaptures it in the number field case and also gives a correct answer for pseudo-reductive groups over function fields. The commutative case (which is essential even for the general pseudo-reductive case) is a corollary of a vast generalization of the Poitou-Tate nine-term exact sequence, from finite group schemes to arbitrary affine commutative group schemes of finite type. Unfortunately, there appears to be no simple formula in general for Tamagawa numbers of linear algebraic groups over function fields beyond the commutative and pseudo-reductive cases. Time permitting, we may discuss some examples of non-commutative unipotent groups over function fields whose Tamagawa numbers (and relatedly, Tate-Shafarevich sets) exhibit various types of pathological behavior.

Damian Rössler: *Perfect points on abelian varieties in positive characteristic*

Let K be the function field over a smooth curve over a perfect field of characteristic $p > 0$. Let K_{perf} be the maximal purely inseparable extension of K. Let A be an abelian variety over K. We shall discuss the properties of the group $A(K_{\text{perf}})$ and some of its subgroups. Most of the results we shall present rely on a simple theory relating the Harder-Narasimhan filtration of the Lie algebra of a finite group scheme to its subgroups.

Alexander Smith: *2^k-Selmer groups and Goldfeld's conjecture.*

Take E to be an elliptic curve over a number field whose four torsion obeys certain technical conditions. In this talk, we will outline a proof that 100% of the quadratic twists of E have rank at most one. To do this, we will find the distribution of 2^k-Selmer ranks in this family for every $k > 1$. Using this framework, we will also find the distribution of the 2^k-class ranks of the imaginary quadratic fields for all $k > 1$.

Michael Stoll: *Minimization and reduction of plane curves*

When given a plane curve over $\mathbf{Q}$, it is usually desirable (for computational purposes, for example) to have an equation for it with integral coefficients that is 'small' in a suitable sense. There are two aspects to this. One is to find an integral model with invariants that are as small as possible, or equivalently, a model that has the best possible reduction properties modulo primes. This is called minimization. The other, called reduction, is to find a unimodular transformation of the coordinates that makes the coefficients small. I will present a new algorithm that performs minimization for plane curves of any degree (this is joint work with Stephan Elsenhans) and also explain how one can perform reduction (based on some older work of mine).

Bianca Viray: *Persistance of the Brauer-Manin obstruction under field extension*
We consider the question of when an empty Brauer set over the ground field gives rise to an empty Brauer set over an extension. We first consider the case of quartic del Pezzo surfaces where a conjecture of Colliot-Thélène and Sansuc predicts that the Brauer-Manin obstruction should persist under any odd degree extension. Our results corroborate their prediction, under mild assumptions on the base field. Our proof also gives strong conditions on the reduction type of quartic del Pezzo surfaces that have a Brauer-Manin obstruction to the Hasse principle.

Isabel Vogt: *Low degree points on curves*
In this talk we will discuss an arithmetic analogue of the gonality of a curve over a number field: the smallest positive integer e such that the points of residue degree bounded by e are infinite. By work of Faltings, Harris–Silverman and Abramovich–Harris, it is well-understood when this invariant is 1, 2, or 3; by work of Debarre–Fahlaoui these criteria do not generalize to e at least 4. We will study this invariant using the auxiliary geometry of a surface containing the curve and devote particular attention to scenarios under which we can guarantee that this invariant is actually equal to the gonality. This is joint work with Geoffrey Smith.

Anthony Várilly-Alvarado: *Quasi-hyperbolicity via explicit symmetric differentials*
We report on a joint project with Nils Bruin. A surface X is algebraically quasi-hyperbolic if it contains finitely many curves of genus 0 or 1. In 2006, Bogomolov and de Oliveira used asymptotic computations to show that sufficiently nodal surfaces of high degree in projective three-space carry symmetric differentials, and they used this to prove quasi-hyperbolicity of these surfaces. We explain how a granular analysis of their ideas, combined with computational tools and insights, yield explicit results for the existence of symmetric differentials, and we show how these results can be used to give constraints on the locus of rational curves on surfaces like the Barth Decic, Buechi's surface, and certain complete intersections of general type.

Yuri Zarhin: *Endomorphisms of certain superelliptic jacobians and l-adic Lie algebras*
The subject of this talk is a certain explicitly constructed class of superelliptic jacobians defined over global fields with small endomorphism rings. We also discuss l-adic Lie algebras that correspond to the Galois actions on Tate modules of these jacobians.

David Zureick-Brown: *Mazur's program B*

I'll discuss recent progress on Mazur's "Program B" – the problem of classifying all possibilities for the "image of Galois" for an elliptic curve over $\mathbf{Q}$ (equivalently, classification of all rational points on certain modular curves X_H).

This will including my own recent work with Jeremy Rouse which completely classifies the possibilities for the 2-adic image of Galois associated to an elliptic curve over the rationals, and work in progress for other prime powers. I will also survey other very recent results by many authors.