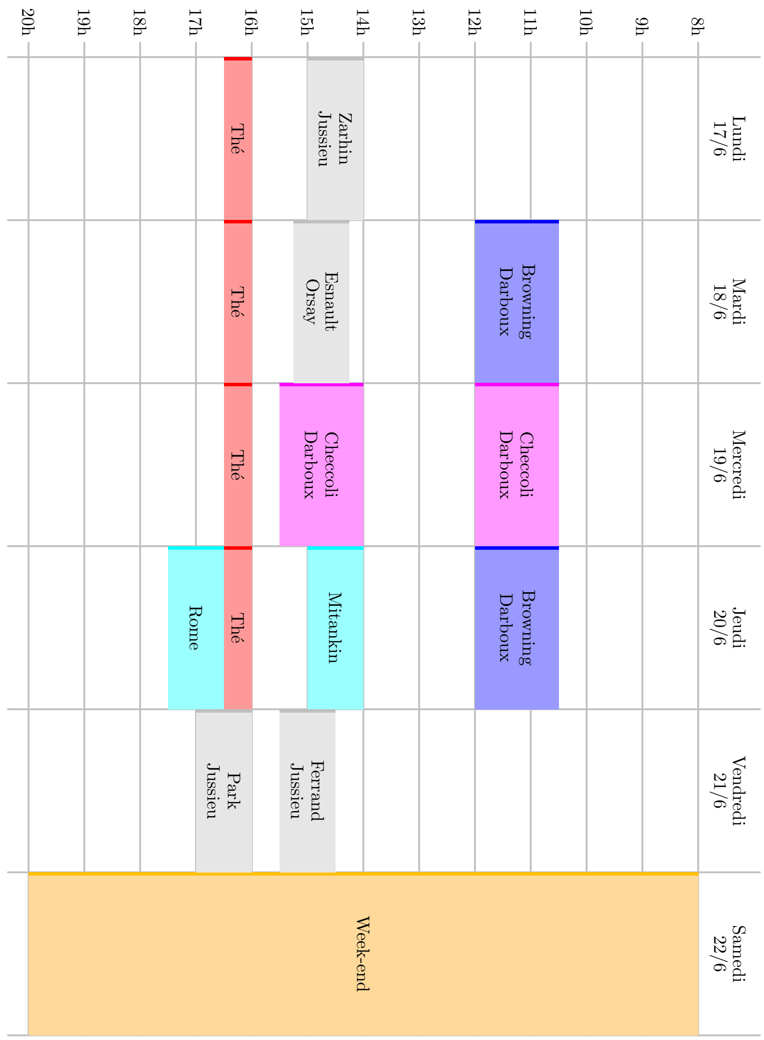
HORAIRES SEMAINE 10



Tim Browning: *Rational points via the circle method*

I will survey recent highlights around applications of the Hardy-Littlewood circle method to rational points, before focusing on what a version of this method over global fields of positive characteristic has to say about the geometry of rational curves on smooth hypersurfaces of low degree. Inspired by recent ideas of Peyre, moreover, I will also describe the parallel situation over the rational numbers, in

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which one counts points of bounded height satisfying the additional constraint that an associated tangent lattice is not too lopsided.

Sara Checcoli: *Petite hauteur dans des grands corps : sur les propriétés de Northcott et Bogomolov et la conjecture de Lehmer*

In this course we will study certain problems concerning algebraic numbers of small height.

The (logarithmic Weil) height of an algebraic number α is a non-negative real number that measures the "arithmetic complexity" of α. By Kronecker's theorem the algebraic numbers of height zero are precisely zero and the roots of unity, but what about numbers of non-zero "small" height?

There are two important statements in this context:

- The first, Northcott's theorem, ensures that a set of algebraic numbers whose all elements have both their height and degree "small" (i.e. bounded) is finite.
- This makes the height (and its variants/generalisations) a very important tool in diophantine geometry: to show the finiteness of a certain set of points (e.g. the rationals points on a variety), one tries usually to bound their height and their degree.
- The second is a famous conjecture of Lehmer, which states that for every algebraic number the product of its height and its degree is either 0 or always bigger than an absolute positive constant. This conjecture was proved for many classes of algebraic numbers, but it is still open in general.

Now, one could ask: in which cases the above statements are still true if one "forgets the degree"? More precisely, following Bombieri and Zannier, we say that a set K of algebraic numbers has the Northcott property (N) if it contains only finitely many points of bounded height and we say that K has the Bogomolov property (B) if 0 is not an accumulation point for the values of the height of the elements in K.

It is easy to see that property (N) implies property (B) and that they both hold when K is a number field. However deciding the validity of these properties for an infinite algebraic extension K of $\mathbb{Q}$ is in general a difficult problem, which has been studied by many authors.

The goal of this mini-course is to give an overview of the known results on this subject and some open problems. If time permits, we will discuss some recent works of Breuillard and Varjù, which show the equivalence between the Lehmer's conjecture and the growth conjecture in geometric group theory.

Vladimir Mitankin: *Variation of the Brauer group and frequency of the Brauer-Manin obstruction for del Pezzo surfaces of degree four with a conic bundle structure*

It is well known that for a del Pezzo surface X of degree four over the rational numbers there are only three possibilities for the quotient of the Brauer group of X modulo constants. In this talk we will explain how often each of them appears when we range across a family of del Pezzo surfaces of degree four equipped with a conic bundle structure. We will also give an explicit description of the generators of this quotient which allows us to calculate the frequency of such surfaces with a Brauer-Manin obstruction to the existence of rational points. This talk is based on a joint work with Cecília Salgado.

Rome: *Weak Approximation: Two Ways*

The aim of this talk is to convince you that weak approximation holds for a particular fibration into quadric surfaces. I will convince the geometers by a fibration argument and studying the Brauer group. The novel approach is to convince the number theorists which involves a counting argument and explicit detector functions for local solubility of quadric surfaces.