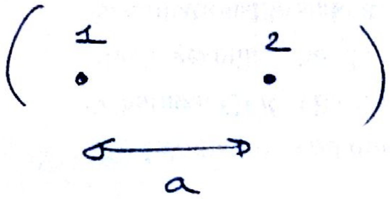
1

TP Tambour (I)

$\boxed{\text{Réseau } 2\times 1}$

Exo 4.1



• matrice $A^{-d\lambda^k}$: (2×2)

$$(A - a^2\lambda) = \begin{pmatrix} -4 - a^2\lambda & 1 \\ 1 & -4 - a^2\lambda \end{pmatrix}$$

$$D(\lambda) = (4 + a^2\lambda)^2 - 1$$
$$= a^4\lambda^2 + 8a^2\lambda + 15$$
$$= \lambda^2 + 8\lambda + 15 \qquad \text{avec } \boxed{a = 1}$$

$$\Delta = 64 - 4.15 = 4$$

$$\rightarrow \lambda = -\frac{8 \pm 2}{2} = -4 \pm 1 = \begin{cases} -3 \\ -5 \end{cases}$$

Exo 4.2

$$(A - a^2\lambda)F = 0$$

$$\rightarrow \begin{pmatrix} -4 - a^2\lambda & 1 \\ 1 & -4 - a^2\lambda \end{pmatrix} \begin{pmatrix} f \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (*)$$

on enlève dernière ligne

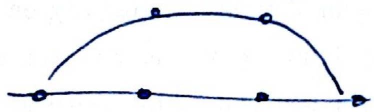
$$\rightarrow \underbrace{(-4 - a^2\lambda)}_{\tilde{A}} \underbrace{f}_{\tilde{F}} = \underbrace{(-1)}_{C}$$

$$\to \quad f = \frac{1}{4+a^2\lambda} = \frac{1}{4+\lambda} \quad \text{si } a=1$$

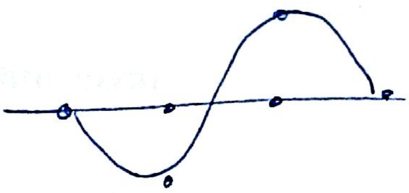
$$= \begin{cases} 1 & \text{si } \lambda = -3 \\ -1 & \text{si } \lambda = -5 \end{cases}$$

5° Dessin :

si $\lambda = -3$ (fondamental) $\to$ $F = (1, 1)$



si $\lambda = -5$ (1er excité) $\to$ $F = (-1, 1)$



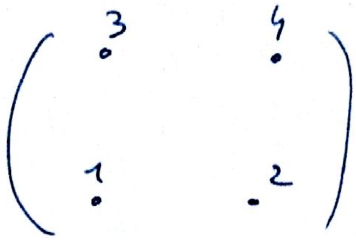
Rem : la dernière equat° de (*) est :

$$f = (4 + a^2\lambda) = \frac{1}{4+a^2\lambda} \quad ?$$

oui car $D(\lambda) = 0$

$$\Leftrightarrow \quad (4 + a^2\lambda) = 1$$

Réseau 2×2



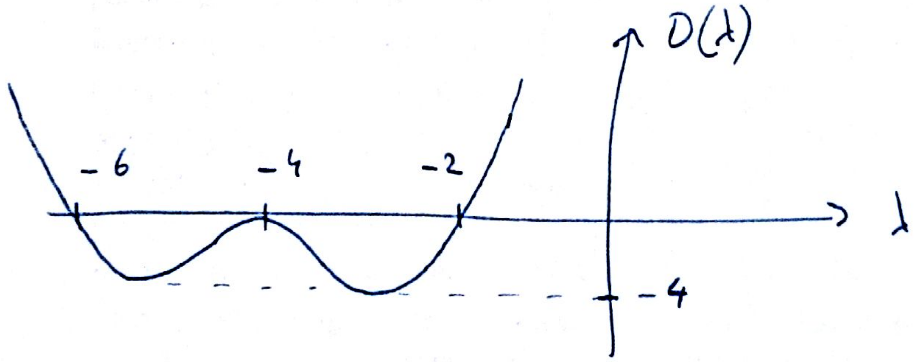
Exo 4.3

$$
(A - a^2\lambda) = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ \\ \end{array} \begin{array}{c} \begin{pmatrix} (-4-a^2\lambda) & 1 & 1 & 0 \\ 1 & (-4-a^2\lambda) & 0 & 1 \\ 1 & 0 & (-4-a^2\lambda) & 1 \\ 0 & 1 & 1 & (-4-a^2\lambda) \end{pmatrix} \\ \begin{array}{cccc} 1 & \quad 2 & \quad 3 & \quad 4 \end{array} \end{array}
$$

Exo 4.4

$$D(\lambda) = 192 + 224\, a^2 \lambda + 92\, a^4 \lambda^2 + 16\, a^6 \lambda^3 + a^8 \lambda^4$$

si $a = 1$: solutions : $-2, -4, -4, -6$



cad $D(\lambda) = (\lambda + 2)(\lambda + 4)^2(\lambda + 6)$