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The dripping faucet revisited

Z. Nédá, B. Bakó, and E. Rees^{a)}

Department of Physics, Babeş-Bolyai University, str. Kogălniceanu 1, RO-3400 Cluj-Napoca, Romania

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High accuracy experimental results on the nonlinear dynamical behaviour of a dripping faucet are presented. The distribution functions for droplet sizes and drip intervals together with return maps are studied for various dripping rates. Increasing this control parameter, chaotic behaviour is obtained and discussed. © 1996 American Institute of Physics. [S1054-1500(95)01304-9]

I. INTRODUCTION

The dripping faucet is a well known model system, used for illustrating the appearance of chaotic behaviour in nonlinear systems.¹⁻⁹ The experiment studies the statistics of liquid drops dripping from a leaky tap when the *mean dripping rate* (control parameter of the system) is increased.

In the earlier works²⁻⁹ the measured quantity was the *drip interval* (time interval between successive liquid drop detachments). The sequence of these drip intervals ($t_1, t_2, \dots, t_n, \dots$) is usually used to describe the dynamics of the system. Plotting the results in a time-delayed coordinate system (t_{n+1} vs t_n), the experiments revealed⁹ the following results:

at low dripping rates the system is periodic;

above a critical dripping rate the system exhibits chaotic behaviour characterized by qualitatively different types of strange attractors.

For the moment there are no clear conclusions regarding the scenario for the onset of the chaotic behaviour. Evidences for the period doubling route,⁴ for the intermittent route^{5,6} and also for the gap road¹⁰ to chaos were found.⁹ Influences of the surface tension and temperature were investigated in Ref. 5. From the fluid dynamics point of view the problem was studied in two regimes.

For **large flow rates** a steady jet is formed which accelerates, gets narrow under gravity and breaks up into drops. The problem is known in hydrodynamics as the capillary instability of the jet.¹¹

For **very small flow rates** the drop diameter increases in static equilibrium until the volume reaches a critical value. After this, a part of the drop breaks away and the remaining part recovers to form a new drop. In this regime a reasonable estimate for the mean drop sizes can be given by the Rayleigh formula: $m \approx 3.8 \cdot \sigma \cdot (r/g)$ (m is the mass of a drop, σ the surface tension, r the radius of the nozzle and g the gravitation constant). If the fluid's viscosity is large enough during the rupture process a *secondary drop* forms.¹² The formation process of this secondary drop was recently briefly discussed by Kadanoff.¹³

We mention that all these fluid dynamics approaches fail in the chaotic regime, and they do not consider the problem from a nonlinear dynamical point of view.

Theoretical studies based on nonlinear dynamical equa-

tions were also considered.¹⁴ The proposed simplified models offer qualitative agreements with experimental data, and predict the period doubling route to chaos.

Although several experiments were reported on the problem of the dripping faucet there are still many open questions. In this sense, the size distribution of the drops and an accurate estimate for the distribution of the drip intervals could be of interest. Detecting and taking into account the presence of secondary drops could improve considerably the understanding of the problem. The scenario for the formation of the chaotic behaviour from the periodic one must be clarified. The present study is intended to offer results in this sense. We complete the earlier experiments by considering higher experimental resolutions, drop size detection and several well-controlled flow rates.

II. EXPERIMENTAL SETUP

Our experimental apparatus is basically the same as the one used in earlier experiments.^{4-7,9} The schematic setup is presented in Fig. 1.

The apparatus consists of a part providing the well-controlled dripping conditions and a part performing the data acquisition and its storage.

The flow debit was kept constant by using a micrometric screw-controlled tap and by keeping a fixed fluid level in the storage tank. The drops were detected with a very simple opto-electronic device (laser + resistance + phototransistor) controlled from the parallel port of a 486 DX2 IBM-PC. The short data acquisition program was written in the C programming language.

The computer registered an integer different from 0 during the interruption of the laser beam by the drop and 0 when the phototransistor was illuminated. Our arrangement allowed us to collect data with frequencies up to 100 kHz, i.e. the previously reported performances were increased almost 10 times. For each stabilized flow rate we analysed the statistics of 10 000 drops. From these results we constructed the distribution functions for drop size and drip intervals. To prove the onset of chaos we also constructed the “traditional” return maps (t_{n+1} versus t_n).

The nozzle where the drops were formed had the interior and exterior diameters of 7 and 8 mm, respectively, and the laser beam passed at 25 mm under it. The flow rate was determined by collecting the drops in a calibrated tank and

^{a)}Visiting professor in ergonomics.

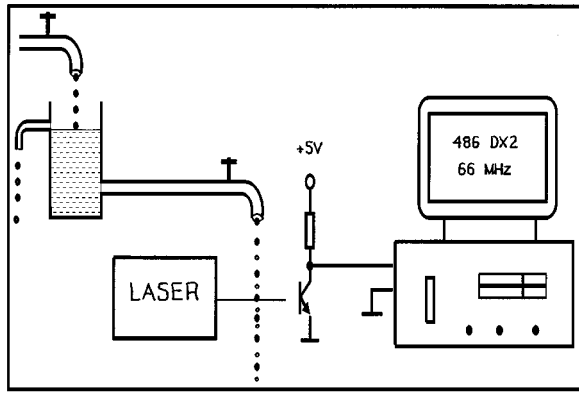


FIG. 1. Schematic setup of the apparatus.

by measuring the time. We used normal undistilled water for our experiments.

III. EXPERIMENTAL RESULTS

We present results for 9 different flow rates (q), all below the continuous regime.

For each stabilized flow rate, we first constructed the distribution function for drop sizes. The results are given in Fig. 2. In these figures the “detected time” refers to the time when the laser beam falls on the drop (no light on the phototransistor), this time being proportional with the drop diameter. On the vertical axis we have the number of detected drops (dN) with detection times between t and $t+dt$ ($dt \approx 0.016$ ms). The statistics was smoothed by averaging for 5 neighbouring intervals. For small flow rates we note the existence of two separate peaks. The peak corresponding to smaller drops is due to the secondary drops, formed during the rupture process. The peak corresponding to the primary drops is higher and much narrower. One can observe that as the debit increases both of these peaks get smaller and wider. Increasing the flow rate further the two peaks become con-

nected, and for $q \approx 1$ cm³/s debit, there is in principle no clear separation between primary and secondary drops. For this flow rate the size distribution function suggests chaotic behaviour, a very wide peak being observed. Analysing the sequence of drop sizes, periodicity is no longer found. For even higher flow rates the peak corresponding to the secondary drops disappears and the peak corresponding to the primary drops splits in two. By increasing the debit, one of these two peaks vanishes, and just below the continuous regime we have a simple one-peak distribution. In our experiments the continuous flow regime was observed for $q \approx 1.3$ cm³/s flow rate.

The distribution function for the drip intervals was constructed considering only the primary droplets (droplets with “detected time” larger than 0.7 ms). In this way we eliminated the double period effect introduced by the formation of the secondary drops. The results (for the same flow rates as in Fig. 2) are presented in Fig. 3. On the horizontal axis we represented the time interval t (“detected time”) between two consecutive primary drops, and on the vertical axis the number of detected intervals (dN) with sizes between the values of t and $t+dt$ ($dt \approx 0.016$ ms). We smoothed our statistics again by averaging for 5 neighbouring intervals. For the values of $q = 0.447$ cm³/s and $q = 0.633$ cm³/s we have only one peak in the statistics. This peak gets wider and is shifted in the direction of smaller drip intervals as the debit increases. For the $q = 0.764$ cm³/s debit we observe the formation of two peaks in the distribution function, suggesting a period doubling in the evolution of the system. Increasing further the flow rate we obtain evidences for a period three ($q = 0.882$ cm³/s) and for a period four ($q = 0.911$ cm³/s) dynamics. For the $q = 0.993$ cm³/s debit all the drip intervals become probable suggesting a chaotic regime. For even higher flow rates this plateau in the drip interval distribution function splits in two peaks. These peaks separate more and more, and the peak for larger drip intervals disappear as the flow rate approaches the continuous flow regime.

The obtained return maps (Fig. 4) are in perfect agree-

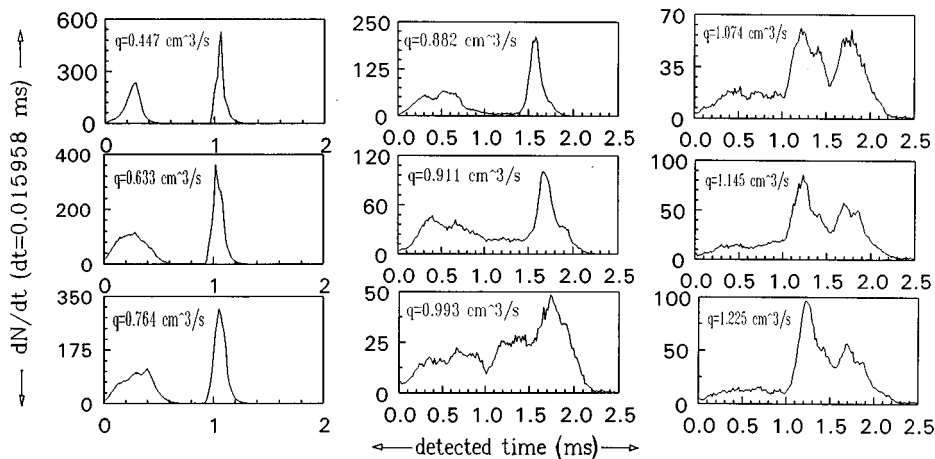


FIG. 2. Distribution functions of the droplet sizes for various values of the flow rates (q). On the horizontal axes we have the “detected time,” which is proportional with droplet size. On the vertical axes we represented the number of droplets (dN) with detected time between t and $t+dt$.

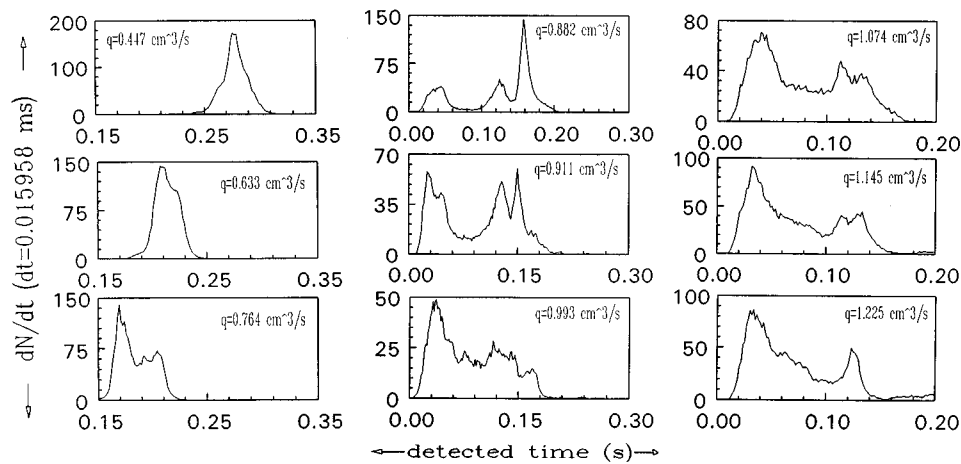


FIG. 3. Distribution functions of the drip intervals for various values of the flow rates (q). On the horizontal axes we have the time intervals between the primary drops, and on the vertical axes the number of drip intervals (dN) with detected time between t and $t + dt$.

ment with the earlier studies,²⁻⁹ and support the statements concerning the period increasing scenario and the onset of chaos. As it was suggested by the drop size and drip interval distribution functions in the limit of $q \approx 1$ ml/s chaotic dynamics appear. One can observe in this sense the formation of a strange attractor. Below this limit, in Fig. 4 we also illustrated some observed periodic dynamics.

It is also instructive for the reader to follow the presented scenarios simultaneously for the drop size, drip interval statistics and return maps (Figs. 2, 3 and 4 respectively).

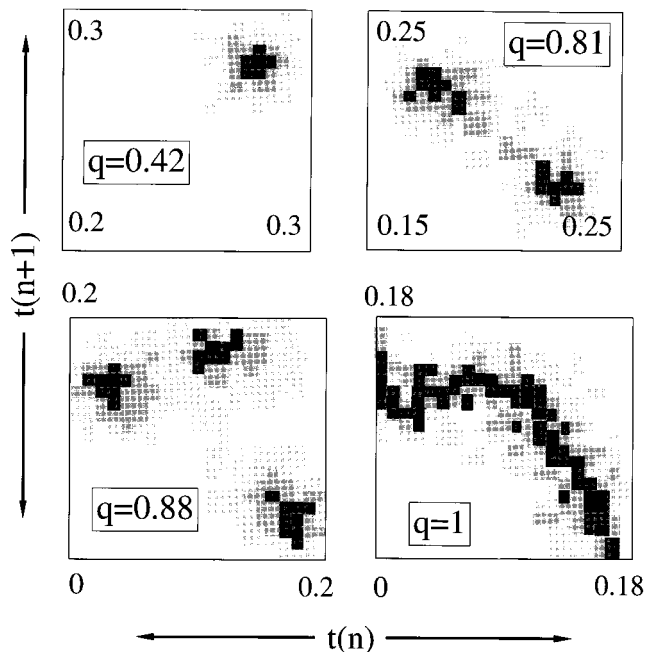


FIG. 4. Return maps (t_{n+1} vs t_n) for the dynamics of the system. The darkness is proportional with the density of detected points. (The q debits are given in ml/s, and the t period in s.)

IV. DISCUSSION

The distribution functions for drop sizes and drip intervals together with the return maps present clear indications for chaotic dynamics near $q = 1$ cm³/s flow rates. The scenario to chaos is different if we consider the statistics for particle sizes or for drip intervals.

For the drip interval distribution functions and return maps we found a period-increasing sequence for the route to chaos ($q = 0.447$ cm³/s and $q = 0.633$ cm³/s period one, $q = 0.764$ cm³/s period two, $q = 0.882$ cm³/s period three and $q = 0.911$ cm³/s period four). All these results are in agreement with the earlier obtained²⁻⁹ ones. The existence of the period three dynamics suggest that a period doubling scenario is not followed.

The drop size distribution function below the chaotic regime exhibits two characteristic peaks (for secondary and primary drops), the clear separation between these being destroyed as the chaotic regime is approached.

After the chaos appears ($q \approx 1$ cm³/s) and until the continuous flow forms ($q \approx 1.3$ cm³/s) both of the discussed statistics exhibit qualitative changes, briefly described in the previous section. An interesting feature is that at these flow rates (1 cm³/s $< q < 1.3$ cm³/s) the distribution functions become simpler, and they have basically one peak in the limit of the continuous flow.

The disappearance of the secondary drops after the onset of the chaotic dynamics, is a challenging problem for fluid dynamics studies.

In **conclusion**, our experiments confirmed the richness of dynamical behaviour in the apparently simple device of the dripping faucet. The presented drop size and drip interval statistics will be useful for theoretical modeling of the system. Our return maps confirm the earlier obtained ones. The results could be also interesting for fluid dynamical studies.

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