Quasi-Geostrophic Motions in the Equatorial Area*

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Abstract

Quasi-horizontal wave motions in the equatorial area are discussed. A single layer of homogeneous incompressible fluid with free surface is treated. The Coriolis parameter is assumed to be proportional to the latitude. In general, waves of two different types are obtained as solutions, one being the inertio-gravity wave and the other Rossby wave. They are distinguished from each other by the difference of frequencies and by the relationships between pressure and velocity fields.

For the solutions of the lowest mode (waves confined near the equator), however, the distinction between the Rossby and the inertio-gravity waves is not clear. The wave moves westward and the frequency of this wave is compared to that of the gravity wave, if wave length is large. With the increase of the wave number the frequency decreases and approaches to that of the Rossby type wave. The pressure and wind fields of this wave show somewhat mixed character of the two types, and change continuously with the wave number. In this connection it seems impossible to "filter out" gravity waves from large scale motions.

Another interesting feature of the equatorial disturbances is that the low frequency waves are trapped near the equator. It is shown that the both waves of inertio-gravity type and of the Rossby type have appreciable amplitude only near the equator. The characteristic north-south extent of the waves is $(c/\beta)^{1/2}$, where c is the velocity of long gravity waves and β is the Rossby parameter. This expression is identical with that derived by Bretherton (1964) for inertio-gravity oscillations in a meridional plane.

In the later half, "forced stationary motion" in the equatorial region is treated. Based on the same model, mass sources and sinks are introduced periodically in the east-west direction. Then the motions and surface topography caused by them are calculated.

As expected, high and low pressures appear where mass source and sink are given respectively. But these high and low cells are splitted into two parts separated by troughs or ridges located along the equator. Strong east-west current was formed along the equator. The flow directs from source to sink and it is intensified by the turning of the circular flow in the higher latitudes.

1. Introduction

It is well known that quasi-horizontal motions of the atmosphere or of the ocean have two different types, the one being inertio-gravity wave and the other the so-called quasi-geostrophic wave, and that these two waves behave in quite different manner. It is an established fact that the most part of the energy of large scale motions is borne by the latter in the middle and high latitudes. Therefore, in numerical studies on large scale motions by use of the primitive hydrodynamical equations, it is an important problem to get a suitable pair of wind and pressure fields which does not include inertio-gravity oscillations of unrealistically large amplitude. Many methods have been devised for that purpose. (Charney, 1955; Phillips, 1960; Hinkelmann, 1959).

The present author proposed a scheme of finite difference of time integration, which may filter out gravity oscillations in the process of integration. (Matsuno, 1966). All these methods are based on the fact that

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there are two distinct modes of motions. Moreover in the case of mathematical filtering procedure, it was assumed that the frequency of the inertio-gravity oscillations are much higher than that of quasi-geostrophic waves of the same horizontal scale. These conditions are actually satisfied in the middle and high latitudes and these methods were used successfully.

Here arises a problem how we must modify above arguments when we treat the motions in the lower latitudes especially those in the equatorial area. Can we get two waves of different types in the equatorial area? Is there quasi-geostrophic motion even at the equator? It is possible to eliminate the gravity oscillations by use of the filtering procedures mentioned previously?

Concerning the wave motions in the equatorial area several works have been made in relation to long-term variations of sea level in the equatorial ocean. Yoshida (1959) pointed out from theoretical considerations that low frequency gravity waves may be trapped in the narrow belt along the equator. This problem was further discussed by Stern (1963) and Bretherton (1964), though their studies were confined only to the motions in the meridional plane. These authors limitted the discussion only to the inertio-gravity oscillations or pure inertia oscillations. Ichiye (1960) investigated the wave motions near the equator, and got the result that in the equatorial ocean there may exist both Rossby and gravity waves, and that the frequency of the former is much lower than the latter. However, in his treatments he made many assumptions. Some self-inconsistent result were obtained concerning the Rossby wave. The author's intention in this paper is to discuss the behaviours of the Rossby and the gravity waves in the equatorial area more precisely and to answer some of the questions mentioned earlier.

2. Model and basic equations

The simplest model suitable for the discussion of our interests is the so-called divergent barotropic model, *i. e.*, a layer of incompressible fluid of homogeneous density with a free surface under hydrostatic balance. It is known that treating such a model and applying the so-called beta-plane approximations we can get various characteristics of large scale motions in the middle and high latitudes mentioned ealier.

On a local cartesian coordinate system (Fig. 1), the equations of motion and of the mass conservation are written as;

$$\begin{aligned} &\frac{\partial u}{\partial t}-fv+g\frac{\partial h}{\partial x}=0 \\ &\frac{\partial v}{\partial t}+fu+g\frac{\partial h}{\partial y}=0 \qquad (1) \\ &\frac{\partial h}{\partial t}+H\left(\frac{\partial u}{\partial x}\right)=0 \end{aligned}$$

where u, v, are the velocities in the x and y directions respectively and h is the small deviation of the elevation of the top surface, the mean value of which is denoted by H. f is the Coriolis parameter and g the acceleration of gravity. As shown in Fig. 1 the x-axis is taken so as to coincide with the equator directing eastward, and the y-axis is taken northward. Here we shall assume that the Coriolis parameter f is linearly proportional to the latitude,

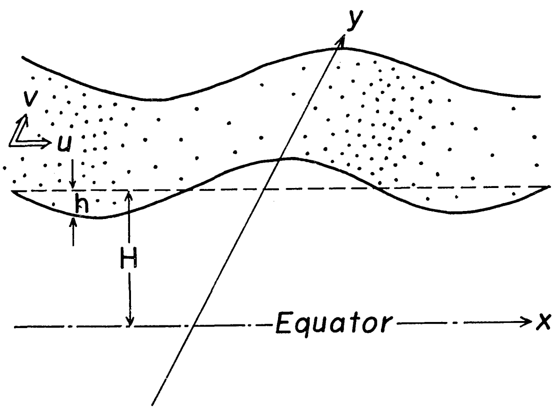


Fig. 1. Model and Coordinates.

$$f=\beta y.$$

Here β is the so-called Rossby parameter and we shall take it as a constant. In the process of mathematical analyses made in this study no further approximation will be made. It means that Coriolis parameter is treated as a variable at any occasion.

It is convenient to convert basic equations into non-dimensional form. At first we shall rewrite (1) by using the geopotential height ϕ instead of the geometrical height h.

$$\frac{\partial u}{\partial t}-\beta yv+\frac{\partial \phi}{\partial x}=0$$

$$\frac{\partial v}{\partial t}+\beta yu+\frac{\partial \phi}{\partial y}=0 \qquad (1a)$$

$$\frac{\partial \phi}{\partial t}+c^2\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=0$$

Here $c^2(=gH)$ is the square of velocity of pure gravity waves. By taking the units of time and length as following,

$$[T]=(1/c\beta)^{1/2} \qquad [L]=(c/\beta)^{1/2} \qquad (2)$$

the equations (1a) are transformed into non-dimensional form;

$$\frac{\partial u}{\partial t}-yv+\frac{\partial \phi}{\partial x}=0$$

$$\frac{\partial v}{\partial t}+yu+\frac{\partial \phi}{\partial y}=0 \qquad (3)$$

$$\frac{\partial \phi}{\partial t}+\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

Hereafter non-dimensional equations (3) will be treated and all symbols stand for dimensionless quantities. The time and length scales are shown in Fig. 2 as function of c.

Though our model is the so-called barotropic model, the equations (1) may be interpreted in some other ways. For instance, let us consider a two-layer model of the atmosphere on pressure-coordinates. Adopting the conventional notations the equations of motion and the thermodynamic equation for small perturbations are written as;

$$\frac{\partial \boldsymbol{v}_1}{\partial t}+f\boldsymbol{k}\times\boldsymbol{v}_1+\nabla\phi_1=0$$

$$\frac{\partial \boldsymbol{v}_3}{\partial t}+f\boldsymbol{k}\times\boldsymbol{v}_3+\nabla\phi_3=0 \qquad (4)$$

$$\frac{\partial}{\partial t}(\phi_3-\phi_1)+S\Delta p\omega_2=0$$

where subscripts 1, 2 and 3 stand for upper, middle and lower levels respectively, and $S\left(=\frac{\partial\bar{\phi}}{\partial p}\frac{\partial \ln\bar{\theta}}{\partial p}\right)$ is the stability factor. Taking the difference of the first and the second equations and making use of continuity relation, a closed system of equations for the differenced quantities are derived.

$$\frac{\partial \boldsymbol{v}_d}{\partial t}+f\boldsymbol{k}\times\boldsymbol{v}_d+\nabla\phi_d=0$$

$$\frac{\partial \phi_d}{\partial t}+S\Delta p^2/2\nabla\boldsymbol{v}_d=0 \qquad (4a)$$

Here symbols with subscript d are defined as follows;

$$\boldsymbol{v}_d=\boldsymbol{v}_3-\boldsymbol{v}_1, \quad \phi_d=\phi_3-\phi_1$$

This set of equations (4a) is just equivalent to (1a) and consequently (3), if we replace $(u,\ v,\ \phi)$ by $(u_d,\ v_d,\ \phi_d)$ and c^2 by c_i^2 $(=S\Delta p^2/2)$ where c_i is the velocity of the internal gravity waves. It means that the equations (1a) are valid to internal mode of motions, and in this case the velocity should be taken as the wind shear and the geopotential height should be replaced by thickness or the temperature.

It could be shown that our model are applicable not only to such simplified models but to some particular cases of the stratified fluid, if we formulate the problem in a suitable manner. It will be discussed in the Section 7.

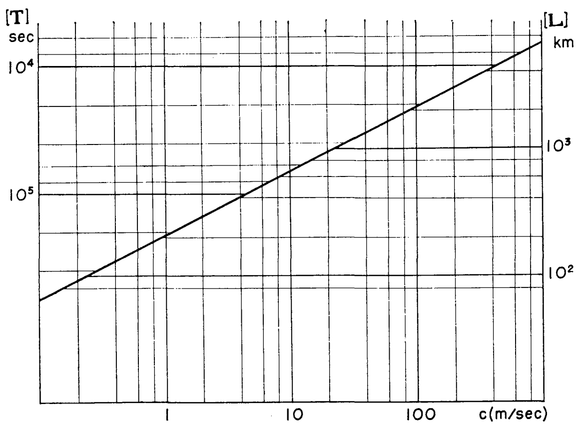


Fig. 2. Units of time (left scale) and length (right scale) as functions of velocity of pure gravity wave.

3. The frequency equation

We shall consider wave motions propagating in the east-west direction. Assuming that all quantities have the factor $e^{i\omega t+ikx}$, the equations (3) turn to;

$$
\begin{aligned}
& i\omega u - yv + ik\phi = 0 \\
& i\omega v + yu + \frac{d\phi}{dy} = 0 \\
& i\omega\phi + iku + \frac{dv}{dy} = 0
\end{aligned} \tag{5}
$$

Here the same symbols u, v, and ϕ are used to denote the y-dependent part of the corresponding quantities in (3). Eliminating u and ϕ we get the equation to v as follows;

$$\frac{d^2v}{dy^2} + \left(\omega^2 - k^2 + \frac{k}{\omega} - y^2\right)v = 0 \tag{6}$$

This equation reduces to the equation treated by Bretherton when $k \to 0$. Since we are considering wave motions near the equator or $y \approx 0$ the boundary condition;

$$v \to 0; \quad \text{when} \quad y \to \pm\infty \tag{7}$$

may be adequate. In the actual atmospheric situations there is upper limit to $|y|$, the position of the pole and boundary conditions should be different ones.

However, approximations in the boundary conditions have little effect on the solutions of lower modes, as described later.

The equation (6) with boundary conditions (7) poses an eigen-value problem, just the same as Shrödinger equation for a simple harmonic oscillator. The conditions (7) are satisfied only when the constant $\left(\omega^2 - k^2 + \frac{k}{\omega}\right)$ is equal to an odd integer;

$$\omega^2 - k^2 + \frac{k}{\omega} = 2n+1 \quad (n = 0,\ 1,\ 2,\ \dots) \tag{8}$$

Then the solution of (6) is given as;

$$v(y) = Ce^{-\frac{1}{2}y^2} H_n(y) \tag{9}$$

where $H_n(y)$ is the Hermite polynomial of the n'th order.

The equation (8) gives a relation between the frequency and the longitudinal wave number for some difinite meridional mode n. Since (8) is a cubic equation to ω, we have three roots when n and k are specified.

It is expected that two of the three roots correspond to the two inertio-gravity waves, one of which is propagating eastward and the other westward, and the last one to Rossby wave. In fact, the approximate values of three roots of ω for very large k, are given as;

$$
\begin{aligned}
& \omega_{1,2} \fallingdotseq \mp \sqrt{k^2 + 2n + 1} \\
& \omega_3 \fallingdotseq k/(k^2 + 2n + 1)
\end{aligned} \tag{10}
$$

The upper two roots are identified as the frequencies of inertio-gravity waves and the lower, ω_3 is that of the Rossby wave. This point is confirmed if we express the above relations in terms of phase velocity of waves using original dimensioned parameters;

$$
\begin{aligned}
& c_{1,2} \fallingdotseq \pm c_g \sqrt{1 + \frac{1}{k^2}\frac{\beta}{c_g}(2n+1)} \\
& c_3 \fallingdotseq -\beta \Big/ \left[k^2 + \frac{\beta}{c_g}(2n+1)\right]
\end{aligned} \tag{11}
$$

Here the velocity of pure gravity wave is denoted by c_g instead of c, in order to avoid confusion. It is noted that effects of the rotation and meridional mode are condensed in a single term including the factor

$$\frac{\beta}{c_g}(2n+1)$$

For arbitrary values of k, we can get three frequencies, by solving the cubic equation (8). They are shown in Fig. 3a in linear scale. Fig. 3b shows the same relations in logarithmic scale. For $n \geq 1$, these three frequencies are completely separated from each other over the whole range of k. Each of them are identified with the frequency of the waves of the following three kinds.

1. The inertio-gravity wave which propagates eastward (indicated by thin solid line in Fig. 3)
2. The inertio-gravity wave which propagates westward (indicated by thin dashed line)
3. The Rossby wave that propagates westward with slow velocity (indicated by

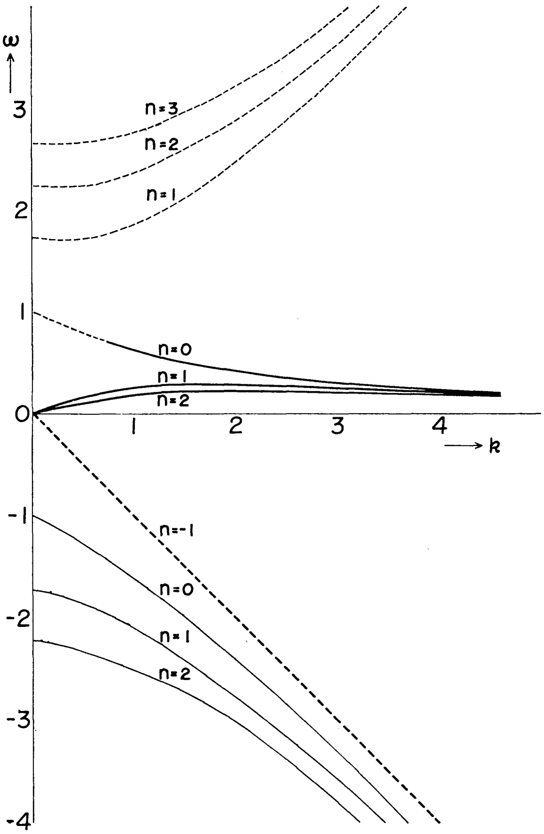


Fig. 3a. Frequencies as functions of wave number.
Thin solid line: eastward propating inertio-gravity waves.
Thin dashed line: westward propagating inertio-gravity waves.
Thick solid line: Rossby (quasi-geostrophic) waves.
Thick dashed line: The Kelvine wave like wave.

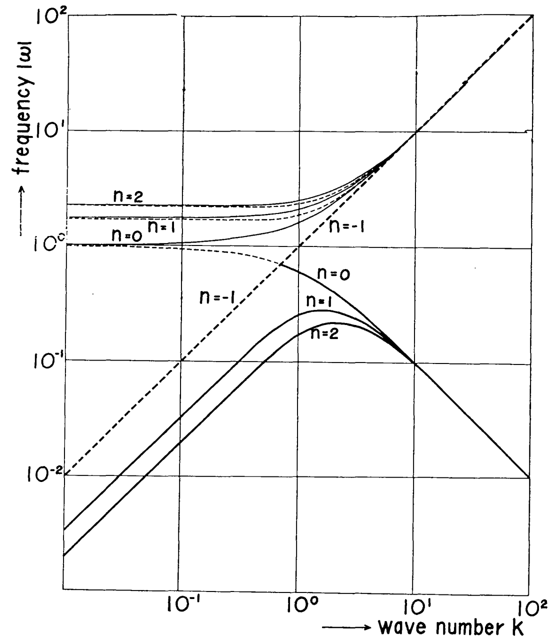


Fig. 3b. Same as Fig. 3a but both the frequency and the wave number in logarithmic scale.

thick solid line)

Conventionally the subscripts 1, 2 and 3 will be used to denote the three kinds in the above order.

Special treatments for $n=0$

Putting $n=0$ in the equation (8) we can also get three roots for ω. In this case the equation (8) is factorized as following;

$$(\omega-k)(\omega^2+k\omega-1)=0 \qquad (12)$$

From (12) we have simple expressions for the three roots of ω, but they do not correspond one to one with the frequencies of three different waves. The classification of the roots is made from their behaviors as functions of n. Namely, if we consider n as a continuous parameter, the frequencies of the three different wave for $n=0$ is obtained by;

$$\omega_l(k;\ 0)=\lim_{n\to 0}\omega_l(k;\ n)$$

where the subscript l denotes the three types of waves.

From the above considerations the three roots of (12) are classified as follows,

(east, gravity)

$$\omega_1=-\frac{k}{2}-\sqrt{\left(\frac{k}{2}\right)^2+1}$$

(west, gravity)

$$\omega_2=\begin{cases}\sqrt{\left(\frac{k}{2}\right)^2+1}-\frac{k}{2} & (\text{for } k\leq 1/\sqrt{2})\\ k & (\text{for } k\geq /1\sqrt{2})\end{cases}$$

(Rossby) (13)

$$\omega_3=\begin{cases}k & (\text{for } k\leq 1/\sqrt{2})\\ \sqrt{\left(\frac{k}{2}\right)^2+1}-\frac{k}{2} & (\text{for } k\geq 1/\sqrt{2})\end{cases}$$

In this case, for $n=0$, the frequency of the westward propagating gravity wave is not separated from that of the Rossby wave, but they coincide with each other at $k=1/\sqrt{2}$,

$$\omega_2(k=1/\sqrt{2})=\omega_3(k=1/\sqrt{2})=k=1/\sqrt{2}$$

Except this point ω_3 is always smaller in magnitude than ω_2.

Here we should claim that one of the three roots of (12), $\omega=k$, cannot be adopted as an eigenvalue of the original equations (5), from the following reasons. In the process of deriving the equation (6) and consequently the frequency equation (8), we assumed implicitly the following relation holds

$$u=\frac{\omega yv+k\dfrac{dv}{dy}}{i(\omega-k)(\omega+k)}$$

Therefore it is demanded that the denominator $(\omega-k)(\omega+k)$ does not vanish, unless the numerator is identically zero. The solution obtained by equating the numerator to zero does not satisfy the boundary conditions, if we solve the equation to ϕ.

From the above reason the one of the roots for $n=0$, $\omega=k$, should be rejected. Then, from (13) we see that the westward propagating gravity wave does not exist for $k>1/\sqrt{2}$ and the Rossby wave does not exist for $k<1/\sqrt{2}$.

In other words, in the case of the lowest mode, $n=0$, we have only two waves, one of which is considered as the intertio-gravity wave propagating eastward, and the other wave, that propagates westward, can be identified neither with the Rossby wave nor with the inertio-gravity wave. The frequency of this wave ranges from the value which is compared to that of the inertio-gravity wave, to the value which is close to that of the Rossby wave.

As observed from Fig. 3a and 3b this wave connects the two families of waves, and because of the existence of this wave, the vacant space in the frequency diagram is lost.

Special solution not included in the equation (8)

Next we shall consider another solution which is not included in the general form (8). The frequency equation (8) was obtained by reducing the original simultaneous equation (5) to the equation for v only.

There may exist a solution which has no meridional velocity v. Putting $v(x, y)\equiv 0$ in (5) we get

$$\begin{aligned} &i\omega u+ik\phi=0\\ &yu+\frac{d\phi}{dy}=0\\ &i\omega\phi+iku=0 \end{aligned} \tag{14}$$

Since both u and ϕ appear in the first and the third equations and they are algebraic equations, the above equations have solutions only when

$$(\omega-k)(\omega+k)=0$$

Then the solutions are obtained as follows,

$$\begin{aligned} &\phi=u=Ce^{-\frac{1}{2}y^2}, \quad \text{for} \quad \omega=-k\\ &\phi=-u=Ce^{\frac{1}{2}y^2}, \quad \text{for} \quad \omega=k \end{aligned} \tag{15}$$

Clearly the lower one does not satisfy the boundary condition (7), and it is rejected. It is interesting that the upper solution, $\omega=-k$, is obtained if we put $n=-1$ in (8). So we shall label this solution by $n=-1$. In Fig. 3a and 3b the value of ω of this mode is drawn with a thick dashed line. The frequency of this wave reaches zero, when the wave number becomes zero, whereas the frequencies of the other eastward propagating waves have the lower bound (approximately $\sqrt{2n+1}$) in magnitude.

At any rate, it seems very important that for the lowest mode solutions the frequencies of Rossby type wave continues to that of inertio-gravity wave and there is no gap in the spectrum.

4. Eigenfunctions

In the last section we got a set of eigenvalues, ω_{nl}, of the equations (3). They are labelled by double subscripts n and l where n stands for n in equation (8) or meridional mode of solutions, while l distinguishes the three roots of (8) for definite value of n. It decides the type of solutions, that is, whether it is of inertio-gravity waves (eastward and westward propagating) or of Rossby wave.

The eigensolutions of u and ϕ will be obtained from (9) and the following relations,

$$\begin{aligned} u&=\frac{1}{i(\omega^2-k^2)}\left(\omega yv+k\frac{dv}{dy}\right)\\ \phi&=\frac{1}{-i(\omega^2-k^2)}\left(kyv+\omega\frac{dv}{dy}\right) \end{aligned} \tag{16}$$

Making use of the recurrence formulas for Hermite's polynomials

$$\frac{dH_n(\xi)}{d\xi}=2nH_{n-1}(\xi)$$

$$H_{n+1}(\xi)=2\xi H_n(\xi)-2nH_{n-1}(\xi),$$

eigenfunctions belonging to eigenvalue ω_{nl} are written as;

$$\begin{pmatrix} v \\ u \\ \phi \end{pmatrix}_{nl}=\begin{pmatrix} i(\omega_{nl}{}^2-k^2)\phi_n \\ \frac{1}{2}(\omega_{nl}-k)\phi_{n+1}+n(\omega_{nl}+k)\phi_{n-1} \\ \frac{1}{2}(\omega_{nl}-k)\phi_{n+1}-n(\omega_{nl}+k)\phi_{n-1} \end{pmatrix} \quad (17)$$

where

$$\phi_n=e^{-\frac{1}{2}y^2}H_n(y)$$

We note that from the above expression that if n is an odd number then v is an odd function and u and ϕ are even functions with respect to y, and if n in even, parity of each component is reversed. From this point it seems adequate to consider that the solution obtained by putting $v\equiv 0$ (the upper one of (15)) corresponds to $n=-1$.

Special attention must be paid to the solutions for $n=0$. In this case expression (17) is valid, but as ω_{ol} we have only two roots as described earlier. Putting $n=0$ in (17) they are simplified as follows;

$$\begin{pmatrix} v \\ u \\ \phi \end{pmatrix}_{ol}=\begin{pmatrix} 2i(\omega_{ol}+k)\phi_0 \\ \phi_1 \\ \phi_1 \end{pmatrix} \quad (18)$$

In addition to the above mentioned family of solutions we have the solution labelled with $n=-1$.

$$\begin{pmatrix} v \\ u \\ \phi \end{pmatrix}_{-1}=\begin{pmatrix} 0 \\ \phi_0 \\ \phi_0 \end{pmatrix} \quad (19)$$

These eigensolutions including (19) form an orthogonal complete set.

The orthogonality is derived directly from (5). We can rewrite (5) in the following way

$$\Omega\xi+i\omega\xi=0 \quad (20)$$

$$\Omega=\begin{pmatrix} 0 & -y & ik \\ y & 0 & \frac{d}{dy} \\ ik & \frac{d}{dy} & 0 \end{pmatrix} \qquad \xi=\begin{pmatrix} u \\ v \\ \phi \end{pmatrix}$$

In the above expression the operator Ω is skew-Hermitic, *i.e.*, Hermitian adjoint operator of Ω is the same as Ω except that the sign is reversed. It is known that all eigenvalues of such operators are pure imaginary, and corresponding eigenfunctions are orthogonal to each other unless degeneracy occurs.

The completeness is proved by use of the completeness of the Hermite functions.

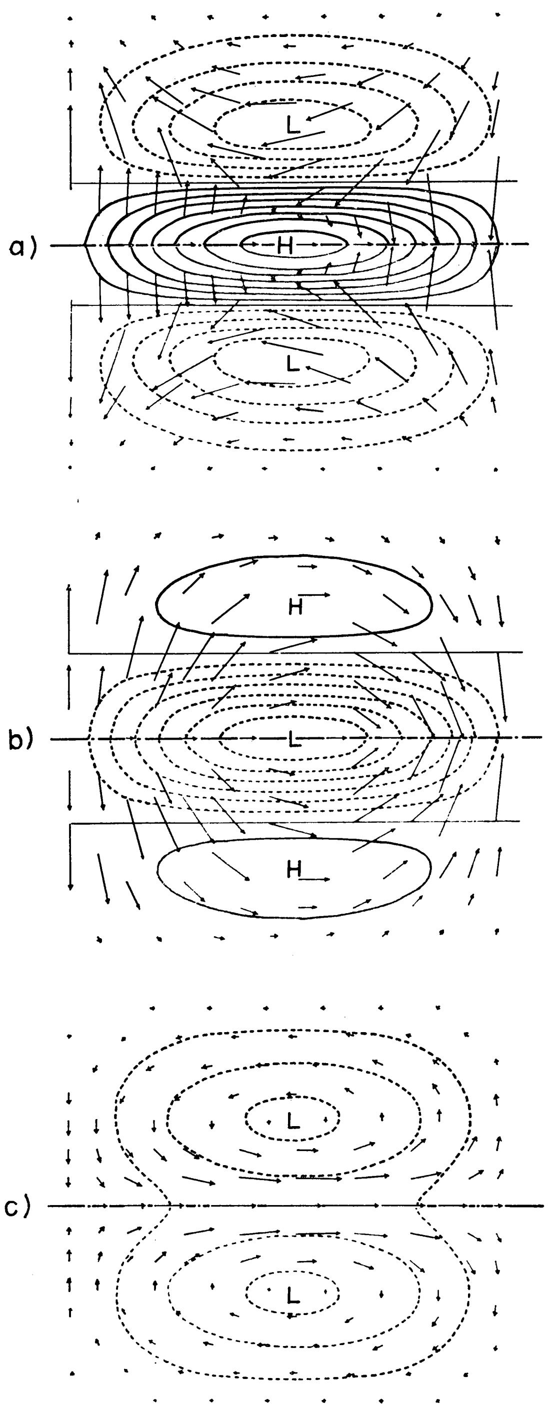
5. Rossby waves and gravity waves in the equatorial area

Some examples of eigenfunctions obtained in the last section are shown in Fig. 4 through 8. Units of wind velocity and pressure (surface elevation) are arbitrary. In all figures half of the one wave length in the x-direction is shown. In Figs. 4 and 5 the eastward propagating inertio-gravity wave, the westward propagating inertio-gravity wave, and the Rossby wave are indicated by a, b and c, respectively.

At first we shall note the pattern for $n=1$ shown in Fig. 4a—4c. We notice clear distinction between the Rossby-type wave and the gravity waves. The former is characterized by the geostrophic relationship between pressure (surface elevation) and velocity fields, while the latters have the features of inertio-gravity waves. It is interesting that in the Rossby wave solution, strong zonal velocity is found along the equator, which is expected from approximate geostrophic balance between the pressure gradient and wind.

The situation is similar for $n=2$, as shown in Fig. 5a—5c. Approximate geostrophic balance holds for the Rossby type solution, though somewhat curious wind field is observed in the vicinity of the equator. Namely counter-clockwise vortex which is located at the equator has no counterpart in the pressure fields.

Those peculiarities found in the Rossby type solution might simply be attributed to vanishing of the Coriolis force. Anyhow it is noteworthy that we can get "quasi-geostrophic"



|———| unit length

Fig. 4. Pressure and velocity distributions of eigensolutions for $n=1$
a: Eastward propagating inertio-gravity wave
b: Westward propagating inertio-gravity wave
c: Rossby wave.

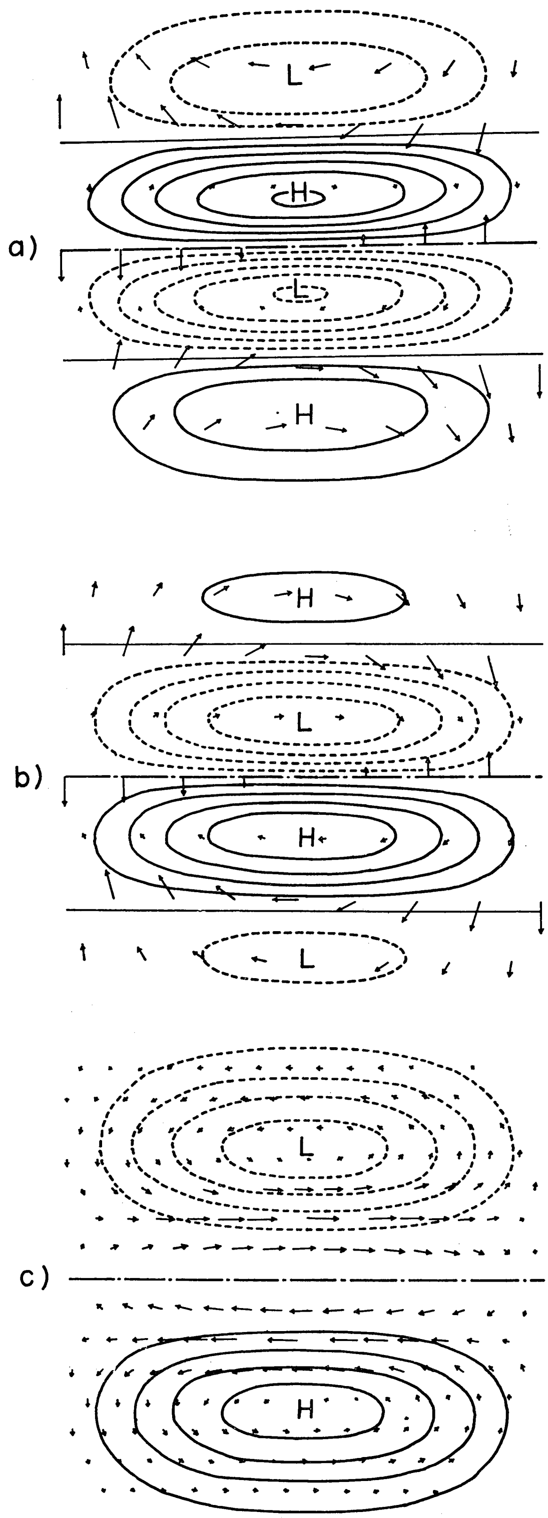


Fig. 5. Same as Fig. 4 but for $n=2$.

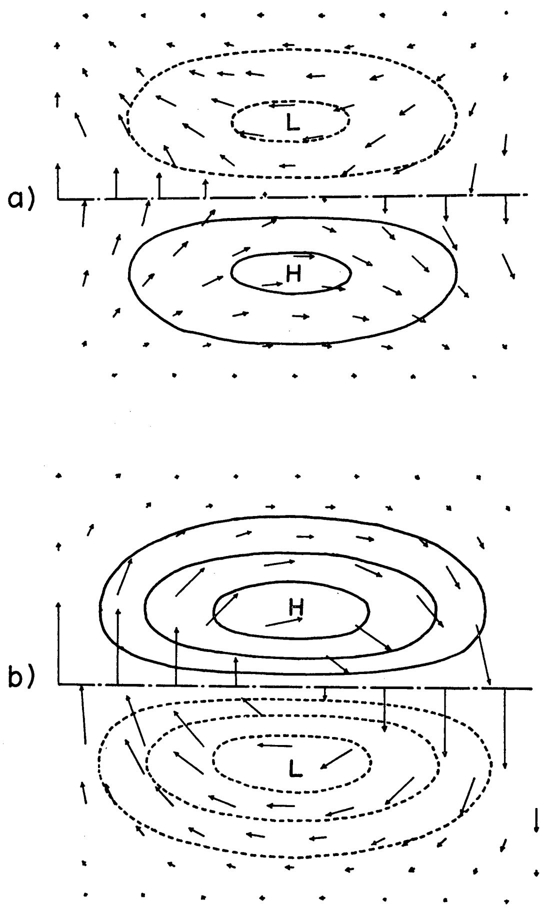


Fig. 6. Pressure and velocity distributions of eigensolutions for $n=0$ and $k=0.5$
a: Eastward moving inertio-gravity wave
b: Westward moving inertio-gravity wave.

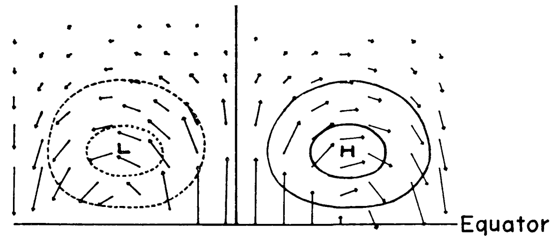


Fig. 7. Rossby type wave ($n=0$ $k=1.0$).

motions in the domain including the equator.

The most interesting are the behaviours of Rossby and gravity waves of the lowest modes. Fig. 6a and 6b show the pressure and wind patterns for $n=0$ and $k=0.5$. For this case, as mentioned earlier, only two solutions, *i.e.* eastward (6a) and westward (6b) moving waves exist. The westward moving wave was taken as a gravity wave in this case from the frequency diagram shown in Fig. 3. In this diagram westward moving wave is classified as the Rossby wave for $k>1/\sqrt{2}$ and as the gravity wave for $k<1/\sqrt{2}$, from the reasons described previously. The pressure and wind distributions for $n=0$, $k=1.0$ are shown in Fig. 7. This solution belongs to the Rossby type according to the frequency diagram. As observed from these figures there is no marked difference between the "gravity" wave of $k=0.5$ and the Rossby wave of $k=1.0$. It is expected from the expression for eigensolution (18), where k and ω are the parameters which control the type of patterns, and if the differences are little amount the resultant patterns will show little difference, too. In our case the frequency ω changes continuously with wave number, k, and consequently eigenfunction changes its form gradually with change of wave number.

If we examine these figures, we note that the westward moving waves have somewhat mixed characteristics of the Rossby and gravity waves. The relationship between the pressure and the velocity fields is approximately geostrophic in the higher latitudes, while near the equator ageostrophic wind components predominate. The configuration of wind and pressure fields near the equator resemble those of the gravity wave for $n=2$. With increase of wave number k, the former character tends to predominate, therefore the overall feature of the wind and pressure fields becomes those of the quasi-geostrophic wave.

The situation is similar for the wave labeled with $n=-1$, the solution obtained by putting $v(x, y)\equiv 0$. Namely, it is impossible to classify this wave as either of the two wave types. The solution of this type behaves like as a pure gravity wave in the x-direction, while in the y-direction the geostrophic relation holds between zonal velocity and meridional pressure gradient. This feature is observed in Fig. 8, which shows pattern for $n=-1$ and $k=0.5$. At the both ends of the cell where the longitudinal pressure gradient is large, the feature of pure gravity wave is marked, while in the middle part where

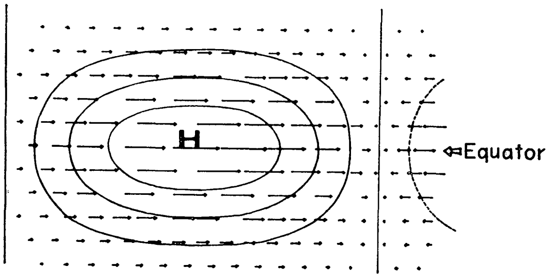


Fig. 8. Pressure and velocity distributions of eigensolution for $n=-1$ and $k=0.5$. This wave behaves like as the Kelvin wave.

longitudinal pressure gradient is small and zonal velocity is large, the other aspect, the geostrophic balance between the pressure and the wind fields is pronounced. It is plausible that the former character predominates in short waves and the latter becomes more pronounced in long waves.

In conclusion we might say that there is no marked difference between the Rossby and the gravity waves for the lowest modes, the wave confined near the equator. Since we have no physical reason to distinguish "quasi-geostrophic wave" and "gravity wave" we cannot apply the concept of filtering to the motions in the equatorial area.

6. Trapping of waves in the equatorial area

As mentioned previously, one of the characteristic phenomena concerning the equatorial disturbances is trapping of the waves of low frequencies. This problem was first discussed by Yoshida (1959), based upon the same set of equations as treated in this paper. But his analysis was not complete, because he derived an equation of surface elevation which was difficult to be dealt with and he suggested the existence of trapped waves from the asymptotic behaviors of the solution. Stern (1963) treated inertia oscillation in the low latitudes and Bretherton (1964) discussed inertio-gravity oscillations in a two-layered ocean. Either of them confined the problem to the motions in the meridional plane, *i.e.*, they assumed the motion is uniform in the longitudinal direction. Therefore it might be said that they did not discuss about "waves", but motions of fluid ring, so to say.

Here we shall discuss about the trapping of waves in the equatorial area in more detail. As understood from the expression for eigensolutions (17) the waves of both gravity and Rossby type of lower modes are confined in the region near the equator. From the equation (6) we can recognize that the solution v has an appreciable amplitude only in the domain,

$$y=\frac{y^*}{\sqrt{\frac{c}{\beta}}}<\sqrt{2n+1} \tag{21}$$

Within this domain the solution becomes wavy, and in the outer part of this domain it approaches to 0. y^* denotes the approximate north-south extent of the wave. The equation (6) is equivalent to a equation which describes the wave motions in an inhomogeneous medium and condition (21) determines the domain where refractive index is positive. Bretherton (1964) gave an explanation to the trapping phenomenon of pure intertia oscillation from the view point of reflection of waves at the top and the bottom of the fluid.

In our case, the existence of trapped modes of waves may be understood as a result of refraction of primary waves. Though the medium is not inhomogeneous, the variation of the Coriolis effects give arise the variation of propagation velocity of inertio-gravity waves. The phase velocity of inertio-gravity wave is given as;

$$c=\pm\sqrt{c_g{}^2+f^2/\boldsymbol{k}^2} \tag{22}$$

where $\boldsymbol{k}$ is the vector wave number in horizontal plane and the velocity of pure gravity wave is denoted by c_g. The relation (22) is valid in this discussion, because now we are concerning propagation of wavelets from a wave front, so we may take the Coriolis parameter is constant in such a small range. Since (22) states that propagation velocity is larger for higher latitudes, the wave generated near the equator will be refracted and reflected toward the equator.

In this meaning the equator plays the role of a duct in the propagations of inertio-gravity waves. The solutions of lower mode we have obtained are the guided waves through this duct.

From the same view point we shall consider the trapping of the Rossby waves. The propagation velocity of the Rossby waves by β-plane approximation is written as

$$c_R = -\frac{\beta}{k^2 + \dfrac{f^2}{gH}}$$

where H is the mean depth of the fluid. Here we note that the larger the f is the smaller (in magnitude) c_R becomes, *i.e.*, the velocity of the Rossby waves is smaller in the higher latitudes. Then we cannot expect reflection of the Rossby waves at higher latitude, though in the solutions obtain in the Section 4 we see that the Rossby waves of lower modes are confined in the equatorial region, too. The simple explanation of trapping phenomena in terms of refraction of waves is not correct for this case.

Next we shall discuss the nature of the wave labelled by $n=-1$. This wave is very similar to the Kelvin wave. The Kelvin wave is a wave which propagates along the coastal line with the velocity of long gravity wave. The deviation of the surface elevation is just in geostrophic balance with the motions associated with the waves. Particle velocities are parallel to the coastal line and have no transverse component. In these points the two waves are the same. In the case of Kelvin wave, however, coastal boundary is essential, because the amplitude of the wave increases exponentially towards the coast. In the case of the equatorial Kelvin wave there is no coastal boundary but owing to changing of sign of the Coriolis parameter, the wave can be confined only in the vicinity of the equator.

Finally the author should stress that there is a possibility that the inertio-gravity waves of long periods could propagate through the equatorial duct, so to say. It seems promising to explain the long period variation of sea level observed in the equatorial ocean, in terms of these guided waves.

Concerning the problems, in what way an incident wave is refracted and trapped, or by what cause the trapped modes of waves could be excited, it is necessary to carry out more detailed analyses and will be discussed elsewhere.

7. Validity of the approximations in connection with the tidal theory

In this article in Sections 2 through 4 we have developed a theory on free waves in the equatorial area. Since we have been dealing with the motions of fluid of constant depth under hydrostatic balance, and which is subject to the earth's rotation, the problem should be included in the theories on tides. In fact, it will be demonstrated that the equation we have treated is an approximate form of the Laplace's tidal equation which is valid near the equator, and that the solutions we have obtained in Section 4 are the free wave solutions of the approximated tidal equation. Therefore the waves which we called inertio-gravity waves and the Rossby waves are just equivalent to approximate forms of free oscillations of the first and the second kind, respectively.

Here we shall examine to what extent our solutions are applicable as approximate solutions of the rigorous tidal equation.

In the mathematical analyses performed in Section 2, we treated the equation to v, the meridional component of velocity. For the sake of comparison with the tidal theory, it is convenient to deal with the equation to ϕ, geopotential height of the top surface, which is equivalent to pressure. Eliminating u and v between the equations (5) we get the equation to ϕ as follows,

$$\frac{d^2\phi}{dy^2} + \frac{2y}{\omega^2 - y^2}\frac{d\phi}{dy} + \left[-\frac{k}{\omega}\frac{\omega^2 + y^2}{\omega^2 - y^2} - k^2 + (\omega^2 - y^2) \right]\phi = 0 \tag{23}$$

where ω and k are the frequency and the wave number in the longitudinal direction, respectively. All quantities are non-dimensionalized by use of scaling given as (2).

Now according to the tidal theory (for instance, see Siebert, 1961), the Laplace's tidal equation which determines the meridional distributions of horizontal velocity divergence and pressure is written as;

$$\frac{d}{d\mu}\left(\frac{1-\mu^2}{f^2-\mu^2}\frac{d\Theta}{d\mu}\right)$$

$$-\frac{1}{f^2-\mu^2}\left[\frac{s}{f}\frac{f^2+\mu^2}{f^2-\mu^2}+\frac{s^2}{1-\mu^2}\right]\Theta+\frac{4a^2\Omega^2}{gh}\Theta=0 \tag{24}$$

where f is the frequency of oscillations divided by twice of the earth's angular frequency 2Ω, s is the wave number in the longitudinal direction, a the radius of the earth, g the accerelation of the gravity and μ is $\cos\theta$ where θ is the co-latitude. Here noted that h is the so-called equivalent depth, and in the tidal theory it is an eigenvalue to be determined, because f is a specified quantity as one of the periods of the tidal oscillations. In our problem, however, we are treating free oscillations and therefore the procedure is reversed, *i.e.*, h is the depth of the fluid that is prescribed and the problem is to find out eigenvalues of f as function of s, by solving (24) with the suitable boundary conditions at $\mu=\pm1$.

Since we are consideridg the problems of the equatorial region or $\mu=0$, the approximate form to (24) is obtained by neglecting μ against 1 as follows;

$$\frac{d^2\Theta}{d\mu^2}+\frac{2\mu}{f^2-\mu^2}\frac{d\Theta}{d\mu}(1-f^2)+\left[-\frac{s}{f}\frac{f^2+\mu^2}{f^2-\mu^2}-s^2+(f^2-\mu^2)\frac{4a^2\Omega^2}{gh}\right]\Theta=0 \tag{25}$$

One would note that equation (25) is very similar to (23), if one reminds that the following correspondences hold; $f\to\omega$, $s\to k$, $\mu\to y$. Discrepancies are found in the two points: The second term is multiplied by factor $(1-f^2)$ in (25), and the last term in the brackets is multiplied by non-dimensional parameter $4a^2\Omega^2/gh$. Since f is the ratio of the frequency of the oscillation considered to that of the earth's rotation, we may neglect it against unity so far as we concern with the low frequency oscillations, say oscillations of which period are order of days or so. As to the next point, if we use the same scaling for corresponding quantities in the two equations the two terms become equal. Then we may conclude that the equation we have treated becomes identical with the tidal equations if we make approximations valid near the equator.

Next the boundary conditions should be discussed. Since the rigorous tidal equation is formulated on the spherical cordinates the boundary conditions to (24) is;

$$\Theta=0 \quad \text{at} \quad \mu=\pm1 \quad (\text{for } s\neq0) \tag{26}$$

This condition implies that Θ should vanish at the both poles which are the singular points of the coordinate system. In our treatment the boundary conditions imposed were

$$\Theta\to0 \quad \text{when} \quad \mu\to\pm\infty \tag{27}$$

In order to compare the above two conditions we shall consider the behaviors of the solutions in the region distant from the equator or $\mu\gg0$. Considering the situations

$$f^2\ll1 \quad \frac{4a^2\Omega^2}{gh}\gg1 \tag{28}$$

the solution of (24) in the region $\mu\gg0$ behaves as a convex function and should grow exponentially with μ, unless f is an eigenvalue. Then the eigenvalue which is determined so as to satisfy the boundary condition (26) should not differ so much from the value which is determined to satisfy (27). Otherwise the exponentially growing component will become remarkable with increase of μ.

Summarizing the above discussions the solutions obtaind in this article would be justified, *a posteriori*, as approximate solutions of the tidal equation, if the conditions (28) are fulfilled.

The validity of these assumptions for

		Oscillations of the 1st kind	Oscillations of the 2nd kind
Atmosphere	External	NO	NO
	Internal	Valid for the lower modes	Valid
Ocean	External	Valid for the lower modes	Valid
	Internal	Valid for the lower modes	Valid

various kinds of motions are examined and listed below.

8. Forced stationary motion

So far we have discussed free waves in the equatorial region. In this section we shall examine what motion is caused when some external forces are working, for one simple example. The problem is; what motions and surface elevations will be caused when the mass sources and sinks are put alternatingly along the equator. By solving this problem we will get some informations on particular properties of atmospheric and oceanic motions in the low latitude area. The methods of mathematical analysis are explained below, in a generalized form.

Here we shall start again from the same model as treated in Section 2. Now we are considering stationary state resulted from some external causes, the equations of motion and continuity are written as follows;

$$-fv+g\frac{\partial h}{\partial x}=-\alpha u+F_x$$

$$fu+g\frac{\partial h}{\partial y}=-\alpha v+F_y \qquad (29)$$

$$H\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=-\alpha h+Q$$

where u, v, h, f, g and H are the same as in Section 2. In this case in right hand side of the equations two sets of terms are added. F_x, F_y, and Q are the x and y components of forces and mass source (or sink) respectively. They are to be given as external forces and tend to cause motions and undulations of the surface elevations. If such external causes are given and the fluid is set in motion, there will appear some restitutive effects, say frictional forces, diffusions. They will counteract the motion and at last the balance will be reached. In order to simulate this situation, we add the terms ($-\alpha u$, $-\alpha v$, $-\alpha h$) in the right side of (29) as the simplest form that express these effects.

The equations (30) may describe the internal mode of the atmospheric motions, as mentioned in the last part of Section 2. In this case, h expresses the thickness between two isobaric surfaces, which is equivalent to the temperature, and Q should be interpreted as the heat sources and sinks.

Equations (29) are transformed in dimensionless forms in the same manner as adopted in Section 2.

Here we shall assume that all external forces have sinusoidal variation in the x-direction. Then the solutions will have the same variation, and factor e^{ikx} will be separable. Then equations (29) are reduced to;

$$\alpha u-yv+ik\phi=F_x$$

$$\alpha v+yu+\frac{d\phi}{dy}=F_y \qquad (30)$$

$$\alpha\phi+iku+\frac{dv}{dy}=Q$$

here various quantities are nondimensionalized and common factor e^{ikx} is omitted. The same symbol as in (29) are used, for no confusion will occur.

Next we shall consider the boundary conditions. It may be plausible to assume that external forces or inhomogeneous terms in (30) are not zero only in the finite distance from $y=0$. Then the solution should have no-zero value in the finite domain, *i.e.*, the boundary conditions to solve (30) are;

$$u,\ v,\ \phi\to 0 \quad \text{when} \quad y\to\pm\infty$$

The free wave solutions obtained in Section 4 satisfy the same boundary conditions and it is proved that the set of all eigenfunctions form a complete set, that is, any arbitrary set of three functions (which satisfies the condition that integration of square of absolute value over the whole domain remains finite) can be expressed by the linear combination of eigenfunctions. Therefore it is possible to express both solution and forcing functions of (30) byserieses of free wave solutions. We shall write the equations (30) symbolically in the following way;

$$(\Omega+\alpha I)\chi=\dot{\sigma} \qquad (31)$$

where χ and σ are solution and the inhomogeneous terms of (30), respectively. $(\Omega+\alpha I)$ is the operater which corresponds to the left hand side of (30). I is the unit operator and

αI expresses the first terms in each equation of (30) and Ω expresses the rest terms. We shall denote free wave solutions obtained in the previous sections by ξ_m, of which frequency is ω_m. Then the following relation holds

$$\Omega\xi_m = -i\omega_m\xi_m \tag{32}$$

Here it is noted that $\xi's$ are labelled by one subscript m, because it is possible to rearrange the set of eigensolutions obtained in Section 4 in a single array.

Expanding χ and σ in terms of ξ,

$$\chi = \sum a_m\xi_m \quad \sigma = \sum b_m\xi_m$$

and inserting this expression into (31), we have

$$\sum (\Omega + \alpha I) a_m\xi_m = \sum b_m\xi_m$$

Making use of (32) the above equation turns to

$$\sum a_m(-i\omega_m + \alpha)\xi_m = \sum b_m\xi_m \tag{33}$$

Since $\xi's$ are orthogonal with each other we have following relation between the two expansion coefficients;

$$a_m = \frac{1}{\alpha - i\omega_m} b_m \tag{34}$$

This relation represents the "response" of the system (30), *i.e.*, it shows that when b_m or external causes are given, how this model reacts to them. It is understood from (34) that the low frequency modes will have large amplitudes while the high frequency modes will be suppressed if the input is of the same amount. It is simply due to the fact that we are treating stationary motions or the input of zero frequency. Therefore the lower the frequency of a free wave is, the more resonant it is to the excitation. Since b_m is got by the following relation,

$$b_m = \left[\int \bar{\xi}_m(y)\sigma(y)dy\right] \Big/ \left[\int |\xi_m(y)|^2 dy\right]$$

we can get $\{a_m\}$ and consequently the solution of (30) for any arbitrarily given external forces, (F_x, F_y, Q).

Here we shall treat one particular solution of (30).

Namely we shall seek for the solution of (30) which is caused by the inhomogeneous terms as following;

$$\sigma = \begin{pmatrix} 0 \\ 0 \\ Q(y) \end{pmatrix} \qquad Q(y) = \psi_0 \equiv e^{-\frac{1}{2}y^2}$$

In this case the series of solution in terms of ξ terminates up to the forth term.

In Fig. 9 the distributions of the surface elevation and the circulation are shown, together with the distribution of mass source and sink given as forcing term. Numerical values adopted are $k=0.5$ and $\alpha=0.2$.

An outstanding feature of the circulation pattern is the strong zonal flow confined in the vicinity of the equator. Associated with this flow ridge and trough are located along the equator which divide the pressure cells into two petals.

Mathematically speaking, we can explain this result by the fact that Rossby wave solution predominates while gravity waves are suppressed. It is quite natural because the frequency of Rossby wave is much smaller than that of the gravity wave and consequently more resonant to the excitation.

In the physical point of view, the situation may be explained in the following way. For the sake of understanding, we shall imagine the series of events which would take place if the mass sources and sinks are given at a certain moment.

The surface elevation will be subject to the distribution of mass sources and sinks, *i.e.*, where the mass is added the surface tends to be raised and where the mass is extracted the surface is depressed. The fluid motions induced by the surface inclination will be deflected by the Coriolis force, in higher latitudes. Then flow field is settled as to be geostrophic flow corresponding to the pressure field. In this way anticyclonic or cyclonic flow fields are established where high or low pressure cells are located. In the vicinity of the equator, we see that a strong zonal flow exists. This flow is caused by the impression of the mass sources and sinks, *i.e.*, the flow is directing from the mass source to the mass sink. It is interesting that this equatorial zonal flow is intensified by the

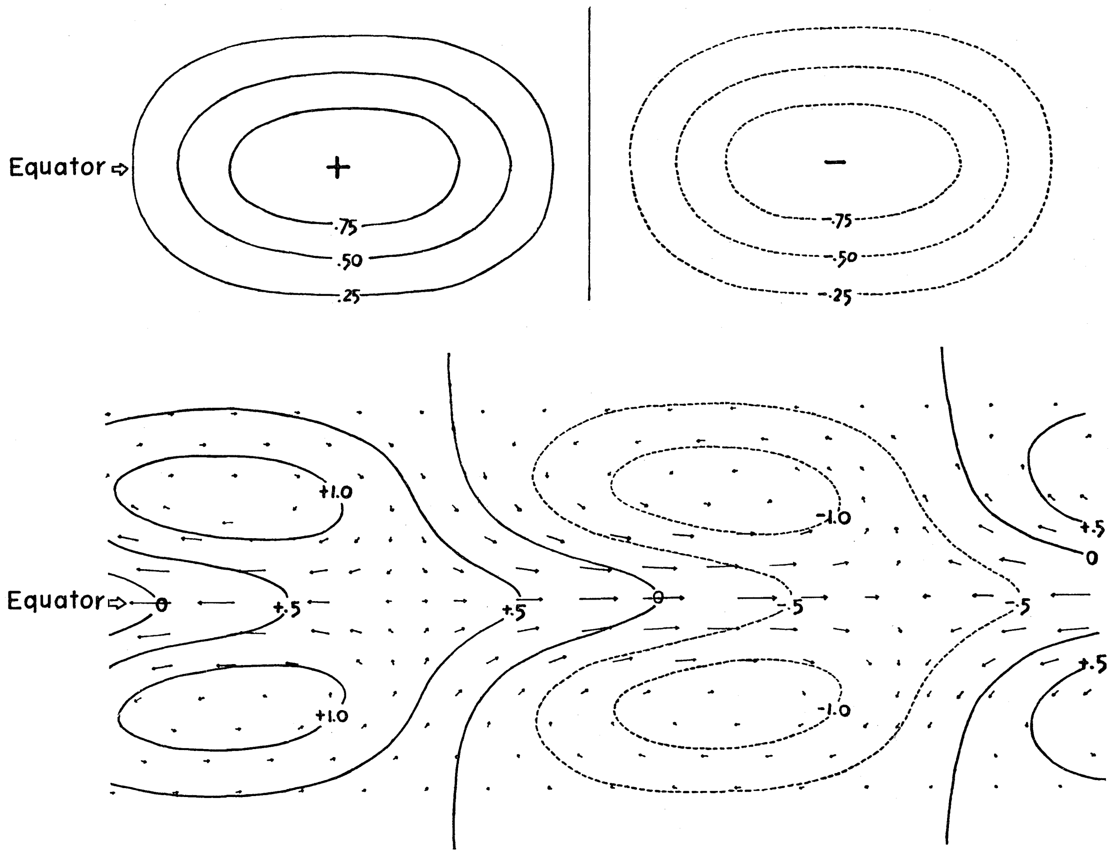


Fig. 9. Stationary circulation pattern (lower) caused by the mass source and sink (upper).

turning of flows from the higher latitude. Namely the circulations in the high latitude regions bring converging or diverging motions towards or from the equator at the end of each cell, because the sense of rotation is opposite in each hemisphere for the same pressure pattern. If we note, for instance, the western edge of the low pressure cell in Fig. 9 the flow is converging towards the equator, and this flow turns to the east.

On the other hand, horizontal velocity convergence brings surface elevation and makes a ridge along the equator. In this way geostrophic balance between the pressure and the flow fields is attained in the vicinity of the equator, too.

In other words, when the fluid is supplied at some place and extracted at the other place the compensating current prefers to flow through the equator.

If we speak in terms of "adjustment problem", the above process may be summerized as follows: In the higher latitudes the flow field is set up so as to balance geostrophically with the pressure field which was generated by mass sources and sinks. On the contrary, in the equatorial region, pressure distribution tends to follow wind field, ignoreing the impressed mass sources. It is noteworthy that, in this example, the surface elevation pattern near the equator is not the reflection of the external forces, though the external forces have the maximum magnitude at the equator. It seems to be very important that, in the equatorial area, pressure or temperature fields could be opposite in sense to the external heatings.

9. Formal development of the theory for general stratified fluid

So far our discussions were confined to the so-called divergent barotropic model as described in the second section. It is thought that this model simulates the atmosphere or the ocean in fundamental hydrodynamic properties. We cannot apply the results obtained in this study directly to the actual atmospheric or oceanic phenomena. But fortunately the mathematical discussions made in the previous sections are applicable to some particular cases of stratified fluids if we formulate the problem in the following manner.

Let us consider a stably stratified fluid on the rotating earth. With some approximations conventionally adopted, we can derive the system of equations governing small perturbations of the fluid motion and the density as follows;

$$\frac{\partial u}{\partial t}-fv=-\frac{1}{\bar{\rho}_0}\frac{\partial p}{\partial x} \tag{35}$$

$$\frac{\partial u}{\partial t}+fu=-\frac{1}{\bar{\rho}_0}\frac{\partial p}{\partial y} \tag{36}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{37}$$

$$\frac{\partial \rho}{\partial t}+w\frac{\partial \bar{\rho}}{\partial z}=0 \tag{38}$$

$$\frac{\partial p}{\partial z}=-\rho g \tag{39}$$

Here (u, v, w) are the velocities in the x, y, z directions respectively. p the pressure, ρ the density. All these symbols stand for small perturbation quantities, while $\bar{\rho}(z)$ is the basic density and $\bar{\rho}_0$ is the constant mean density, f being the Coriolis parameter and g the acceleration of gravity. By eliminating p and w between (37), (38), and (39) the equation for p is obtained as follows,

$$\frac{\partial}{\partial t}\left[\frac{\partial}{\partial z}\left(\frac{1}{g\bar{\rho}_z}\frac{\partial p}{\partial z}\right)\right]+\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{40}$$

where $\bar{\rho}_z$ is the abreviation of $\partial\bar{\rho}/\partial z$, the basic stability factor. Combining this equation with (35) and (36) we get three equations for three variables u, v, and p.

We note that in this set of equations, differentiation with respect to z appears only within the brackets in (40), and if we assume that this term is proportional to $(p/\bar{\rho}_0)$, the system (35), (36) and (40) are reduced to the differential equations with respect to the horizontal coordinates only. Further they are equivalent to (1) which we have treated. Namely, if we put

$$\frac{\partial}{\partial z}\left(\frac{1}{g\bar{\rho}_z}\frac{\partial p}{\partial z}\right)=\frac{1}{gH_*}\frac{p}{\bar{\rho}_0} \tag{41}$$

(40) is written as

$$\frac{\partial}{\partial t}\left(\frac{p}{\bar{\rho}_0}\right)+gH_*\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=0 \tag{42}$$

Clearly this equation corresponds to the mass conservation equation in the one-layer model treated in the previous sections, and (35), (36) and (42) form an equivalent set of equations to (1), or the set of equations for two-dimensional quasi-horizontal motions of the homogeneous fluid with (apparent) depth H_*.

Here H_* is to be determined in the following way. In solving (41), the boundary conditions are

$$w=0 \quad \text{at} \quad z=0$$

$$w=\frac{d\zeta}{dt}\approx\frac{d\zeta}{dt} \quad \text{at} \quad z=H$$

where ζ is the surface elevation.

These conditions are transformed into,

$$\frac{\partial p}{\partial z}=0 \quad \text{at} \quad z=0$$

$$H_*p(z=H)=\int_0^H p(z)\,dz \tag{43}$$

Then the equation (41) with the above conditions poses an eigenvalue problem, and H_* will be determined as eigenvalues. The corresponding eigenfunctions will form a set of orthogonal functions. If they are complete we can express any function of z in terms of them.

The mathematical discussions mentioned above are the same as usually found in the theory of wave motions of long wave length (Eckart, 1960).

Summing up, the equations (35) through (39) can be solved by the method of separation of variables. Then equations concerning the horizontal coordinates turn to be the same as those for homogeneous density with a parameter H_*, apparent depth of that fluid. Therefore the analyses made in the previous sections are valid, if we note one particular mode in the vertical structure.

10. Summary and conclusions

Wave motions in the equatorial area are discussed. Based upon the so-called divergent barotropic model, linearized equations for small perturbations are solved.

The following results are obtained from the mathematical analyses.

(1) Even in the equatorial area there are two types in the waves motions. The one is the inertio-gravity waves and the other is the Rossby waves. They are distinguished from each other, by the difference of their frequencies. Namely frequencies of the inertio-gravity waves are much larger than that of the Rossby waves.

(2) For the particular mode, however, the distinction between the two waves is not clear. The wave of the lowest mode, which is the smallest in the north-south extent and propagates westward, has somewhat mixed characters of the Rossby wave and of the inertio-gravity wave. The frequency-wave number relationship for this wave is similar to that of the Rossby wave when the longitudinal wave length is smaller than the meridional extent. For long wave part, however, the frequency becomes very large and approaches to that of the gravity wave of the same wave length. The relationship between the pressure and the wind fields also shows the mixed characters of the two types, the Rossby wave type and the inertio-gravity wave type. Either of the two types becomes predominant depending upon the wave length. In these aspects this wave is situated in the intermediate position of the two utterly different wave regimes, and connects them continuously.

It is an interesting problem that whether the wave of such type exists or not in the actual atmospheric conditions, and if it exists what role it plays in the atmospheric motions in the equatorial area.

(3) For lower modes the both waves of the inertio-gravity type and of the Rossby type are confined near the equator. The meridional extent of the wave is of the order of $(c/\beta)^{1/2}$, where c is the velocity of long gravity waves and β is the Rossby parameter. For the inertio-gravity wave this phenomenon is interpreted as the results of refraction at the both sides of the equator. In this meaning the equator can be a wave guide for the propagation of long period gravity waves. It seems that this effect may play some roles in the maintenance of the atmospheric or oceanic disturbances in the equatorial areas.

(4) One particular example of the stationary circulation in the equatorial area is obtained. Considering the same model (one layer of homogeneous fluid) we have calculated the circulations and the pressure distributions when mass sources and mass sinks are imposed alternatingly along the equator. The characteristic features of the circulations and the patterns of surface elevations are:

(i) In the higher latitudes the surface tends to be raised where mass is added and depressed where mass is extracted.

(ii) In the vicinity of the equator, however, the deviations of the surface elevation is less than that in the higher latitude in magnitudes. As a consequence high and low pressure cells are splitted into two parts separated by the equator.

(iii) Strong zonal flow is formed along the equator. The flow directs from the mass source to the mass sink. This equatorial zonal flow is intensified by the turning of the flow associated with the high latitude circulations.

(iv) In the higher latitude region, the velocity fields are in geostrophic balance with the pressure fields.

(v) If we apply the above results to the two level models of the atmosphere, we may deduce the following things.

If the air in the equatorial region is subject to the differential heating in the longitudinal direction, the resultant pressure and wind fields will show the following features: The impressed heating and cooling will produce the low pressure and the high pressure (on the lower level), respectively. The wind blows geostrophically in the high latitude region. The induced vertical motions counteract to the imposed heat sources and sinks, and their effects are stronger near the equator than in the higher latitudes. Consequently the warm or cold air produced by the heating or cooling are splitted into the two parts, by the relatively cold or warm air belt located at the equator.

The present work concerns only with the mathematical analyses of the simplified hydrodynamical equations, and we must be careful in applying the results obtained in this study to the actual atmospheric or the oceanic disturbances.

It is most interesting if we find out some phenomena inferred in this study, for instance, the trapped waves, in the actual atmosphere.

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References

Bretherton, F.P., 1964: Low frequency oscillations trapped near the equator. *Tellus*, **16**, 181–185.

Charney, J.G., 1955: The use of the primitive equations of motion in numerical prediction. *Tellus*, **7**, 22–26.

Eckart, C., 1960: *Hydrodynamics of Oceans and Atmocpheres*, Pergamon Press, New York, 1960, pp. 290.

Hinkelmann, K., 1959: Ein numerisches Experiment mit den primitiven Gleichungen. *The Atmosphere and the Sea in Motion; Scientific Contributions to the Rossby Memorial Volume*, Rockfeller Institute Press, New York, 486–500.

Ichiye, T., On long waves in a stratified, equatorial ocean caused by a travelling disturbance. *Deep Sea Res.*, **6**, 16–37.

Matsuno, T., 1966: Numerical Integrations of primitive equations by use of a simulated backward difference method. *J. meteor. Soc. Japan*, Ser. II, **44**, 76–84.

Phillips, N.A., 1960: On the problem of initial data for the primitive equations, *Tellus* **12**, 121–126.

Siebert, M., 1961: Atmospheric Tides. *Advances in Geophysics* Vol. 7, Academic Press, New York, 1961, 105–187.

Stern, M.E., 1963: Trapping of low frequency oscillations in an equatorial boundary layer. *Tullus*, **15**, 246–250.

Yoshida, K. 1959: Preprints, International Oceanographic Congress, 789–791, American Association for the Advancement of Science, Washington D.C., 1959.

Note added in proof: After the completion of the manuscripts, the author noticed the work by Rosenthal (1965)*. The equatorial waves treated by him are essentially the same as the waves of $n=0$ in the present article, though in the Rosenthal's work uniform basic currents are considered. Rosenthal paid attention mainly to the structures of Rossby type waves of relatively short wave length, whereas the present author treated general properties of the waves in the equatorial region.

* S. L. Rosenthal, "Some preliminary theoretical considerations of tropospheric wave motions in equatorial latitudes", Mon. Wea. Rev. **93**, 605–612, (1965).

赤道近くでの準地衡風的運動

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コリオリの力が働かなくなる赤道近辺での大規模運動の特性を理論的に検討してみた。自由表面をもった単層の流体——いわゆる発散順圧モデル——について線型化された運動方程式を扱い東西方向に動く自由波動の解を求めると，一定のスケールに対して3つの解が得られた。これらは振動数，解の形（圧力及び運動の場）から夫々東向きおよび西向きの慣性重力波およびロスビー波であることがわかる。但し南北スケール最小のものに関してはその区別は明瞭でなく一方の型から他方の型に連続的にかわる。ロスビー波に相当する解は風と圧力の関係が高緯度でほぼ地衡風的であること，および赤道近くで特異なふるまいをするのが特徴である。

次に同じモデル熱冷源に相当するものとして東西に周期的な mass source, sink を与え，定常解と求めた。熱源に相当する所は低圧になるが赤道で分断され，赤道のごく近くはやや逆センスになり，これに伴って高緯度と逆向きの強い流れが生ずることが分った。