Laboratoire de Physique et Modélisation des Milieux Condensés
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The best quantum thermoelectric at finite power output

Robert S. Whitney

+ Phys. Rev. Lett. **112**, 130601 (2014),
+ Phys. Rev. B **91**, 115425 (2015) (all the details),
+ arXiv:1603.09216 (three-terminal)

CPTGA Workshop "Open Quantum Systems" Grenoble — May 2016

OVERVIEW

♣ Question about textbook $thermodynamics$

♣ INTRODUCTION — Thermoelectrics

♣ Landauer scattering theory

$\Longrightarrow$ *Most efficient* thermoelectric at *given* power output

QUESTIONS about basic THERMODYNAMICS

$\eta = \dfrac{\text{power output}}{\text{heat input}}$ less than Carnot: $\eta_{\mathrm{Carnot}} \equiv 1 - T_{\mathrm{cold}}/T_{\mathrm{hot}}$

Carnot $\Leftrightarrow$ $reversibility$ $\Rightarrow$ "zero" power output

♣ What does "zero" mean?

$\Longrightarrow$ system specific or universal?

♣ Stricter upper bound at finite power?

THERMOELECTRICS MACHINES

HEAT FLOW $\Longleftrightarrow$ *CHARGE* FLOW

POWER GENERATION or REFRIGERATION

Seebeck or Peltier

EXAMPLE : NASA POWER SOURCE

Heat source: 5kg plutonium α-decay

$$T_{\mathrm{hot}} \simeq 1000\mathrm{K}$$
$$T_{\mathrm{cold}} \simeq 230\mathrm{K}$$

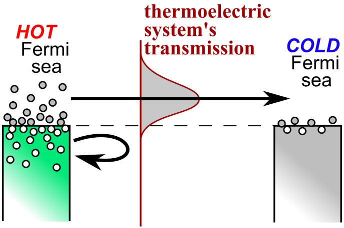
Power output $= 120\mathrm{W}$

Efficiency $\eta = 6\%$

ORIGIN of THERMOELECTRICITY

Thermoelectric effects

= *difference* between dynamics *above* and *below* Fermi surface



Origin of *energy filter*: MESOSCOPIC or CRYSTAL STRUCTURE

Thermoelectric effects

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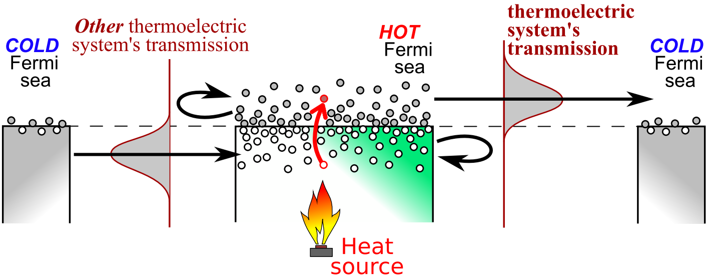
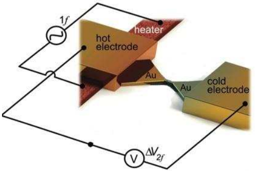


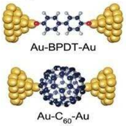
Origin of *energy filter*: MESOSCOPIC or CRYSTAL STRUCTURE

QUANTUM THERMOELECTRICS

TWO RESERVOIRS

Quantum thermoelectrics
in traditional thermocouple

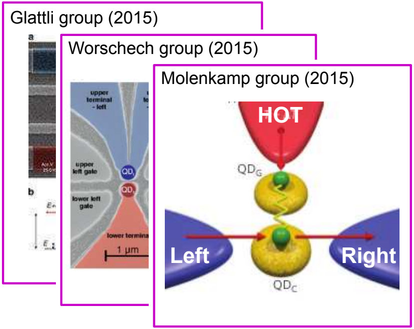




Reddy group
(2015)

THREE RESERVOIRS 1 for heat & 2 for current

"Quantum Thermocouple"



2 reservoir theory (& older expts) = Molenkamp group, Mahan-Sofo, Linke group

3 reservoir theory = Entin-Wohlmann et al, Sánchez & Büttiker

PLAYGROUND

DEFINITIONS

♢ Heat?

♢ Work?

♢ Entropy?

QUANTUM MACHINES

♡ Heat engines

♡ Refrigerators

♡ Cooling by heating

♡ Maxwell demons

Testing quantum theories versus thermodynamics

♣ Markovian (Lindblad) master equation Kosloff's review

♣ Scattering theory R.W. 2013

♣ Keldysh non-equil. Green functions Esposito 2015

MAXIMUM EFFICIENCY

AT GIVEN POWER OUTPUT

Inspired by engineering question:

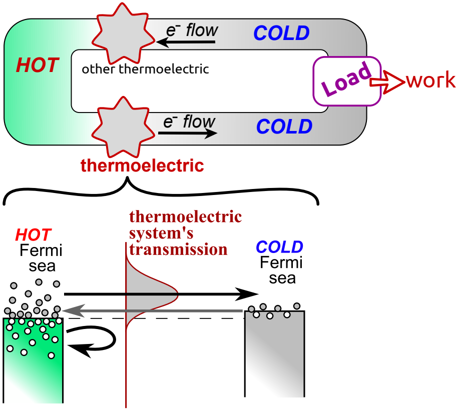
$T_{\text{hot}} \simeq 1000\text{K} \quad T_{\text{cold}} \simeq 230\text{K}$

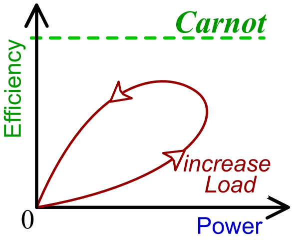
Power output $= 120\text{W}$ $\Longleftarrow$ **NECESSARY**

Efficiency $\eta = 6\%$ $\Longleftarrow$ **“nice” if bigger**

THERMOCOUPLE CIRCUIT

Mahan,Sofo (1996). Humphrey,Linke (2005)
Esposito, Lindenberg, Van den Broeck (2009)





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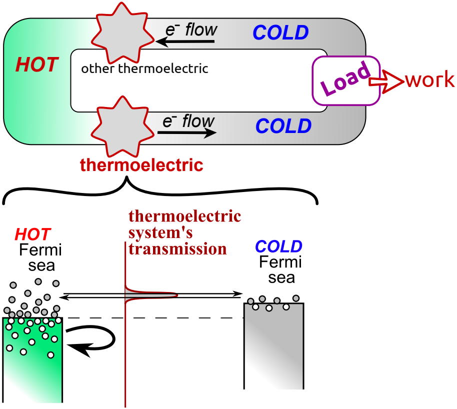
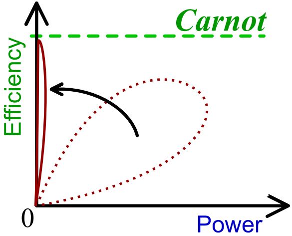
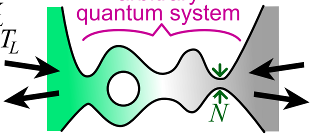




SCATTERING THEORY $\Rightarrow$ *thermodynamics*

Landauer scattering theory



Heat current: $$J_L = \int_{-\infty}^{\infty} \frac{\mathrm{d}E}{h}\,(E - eV_L)\;\mathcal{T}(E)\,\Big(f_{\mathrm{L}}(E) - f_{\mathrm{R}}(E)\Big)$$

Transmission at energy E

Fermi-function

e-e interactions: only mean-field self-consistent Hartree-like

Christen-Büttiker (1996)

No Coulomb blockade, no Kondo Physics ...

Scattering theory $\Longrightarrow$ Thermodynamics

Bruneau Jakšić Pillet, Commun. Math. Phys. **319**, 501 (2013)

R.W., PRB **87**, 115404 (2013)

$$\left.\begin{array}{c}\text{Energy Conservation}\\[1em] \text{HEAT} \Leftrightarrow \text{WORK}\end{array}\right\} \textit{1st law thermodynamics}$$

Proved scattering theory $\Rightarrow$ *2nd law thermodynamics*

... using Clausius definition of entropy, $S = \dfrac{\text{heat}}{T}$

OPTIMIZING EFFICIENCY for GIVEN POWER OUTPUT

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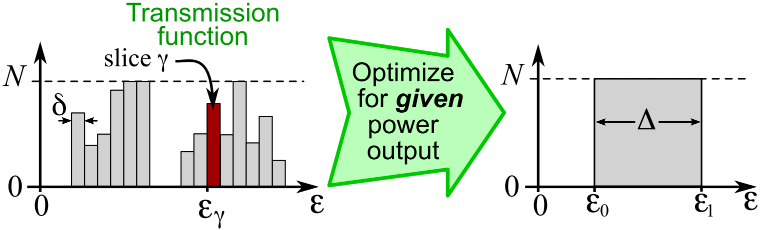
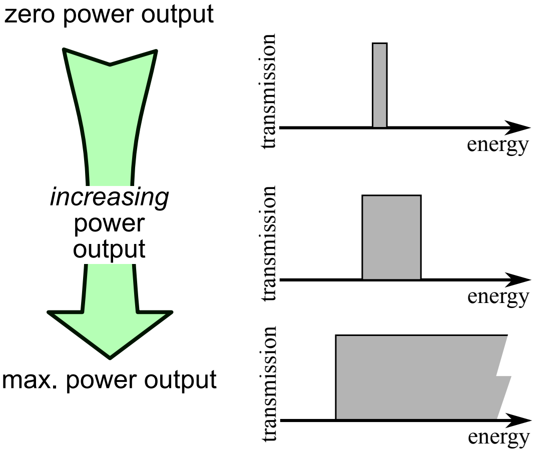


Variables: height of each slice & bias, V

Constraint : power $= P$

OPTIMAL TOP-HAT WIDTH

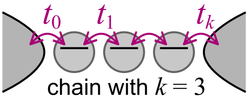
Transcendental equation for top-hat position and width

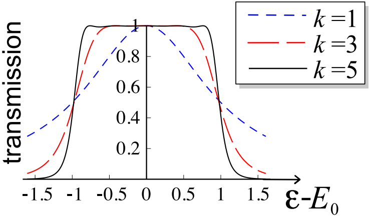


POTENTIAL REALIZATION

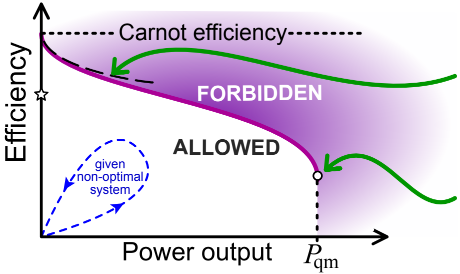
Make chain of sites (tight-binding model) $\Longrightarrow$ states form a *band*

Chains of k quantum dots or molecules





Max. EFFICIENCY for GIVEN POWER



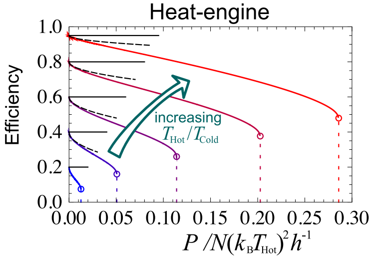
$$\text{Carnot} \times \left(1 - \alpha_1 \sqrt{\frac{P}{P_{\mathrm{qb}}}} + \cdots \right)$$

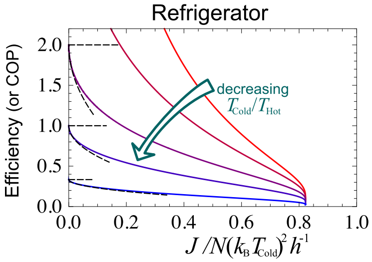
QUANTUM bound (qb)

$$P_{\mathrm{qb}} \propto N \, (T_{\mathrm{hot}} - T_{\mathrm{cold}})^2$$

with “quantum” $N \sim \frac{\text{cross-section}}{\text{wavelength}}$

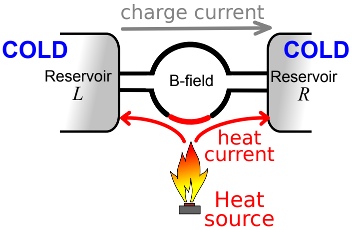
Max. EFFICIENCY for GIVEN POWER





Three-terminal thermoelectric : "quantum thermocouple"

R.W. arxiv:1603.09216



Fully-coherence inside quantum system

Four independent processes :

Hot $\rightarrow$ L
Hot $\rightarrow$ R
L $\rightarrow$ R
R $\rightarrow$ L

RESULT : max. eff. at given power

exactly the same as before

CONCLUSIONS

Max. efficiency at *zero*-power (Carnot) is **classical**

Max. efficiency at *finite*-power is **quantum**

PRL **112**, 130601 (2014), PRB **91**, 115425 (2015)

Preprint : arXiv:1603.09216

Is quantum bound relevant for REAL applications?

Cross-section for $100W$ of power ?

with wavelength $\lambda_{\rm F}\sim 10^{-8}{\rm m}$

$\diamondsuit$ Minimal cross-section $\sim 4{\rm mm}^2$

$\diamondsuit$ 90% of Carnot requires $> 0.4{\rm cm}^2$

=== EXTRAS ===

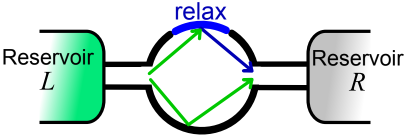
RELAXATION modelled as Buttiker "voltage probe" (1988)

Saito, Benenti, Casati & Prosen, PRB (2011). D. Sánchez & Serra, PRB (2011)

Entin-Wohlman & Aharony, PRB (2012)

VERY DIFFICULT

$\Longrightarrow$ treat only B-field $= 0$



PROOF of same bound

over-estimation in 2 limits

- low power
- high power