Persistent Currents for Interacting Bosons on a Ring with a Gauge Field

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Bose-Einstein condensates with cold atoms

Anderson et al., Science 95
Davis et al. PRL 95

Typical parameters

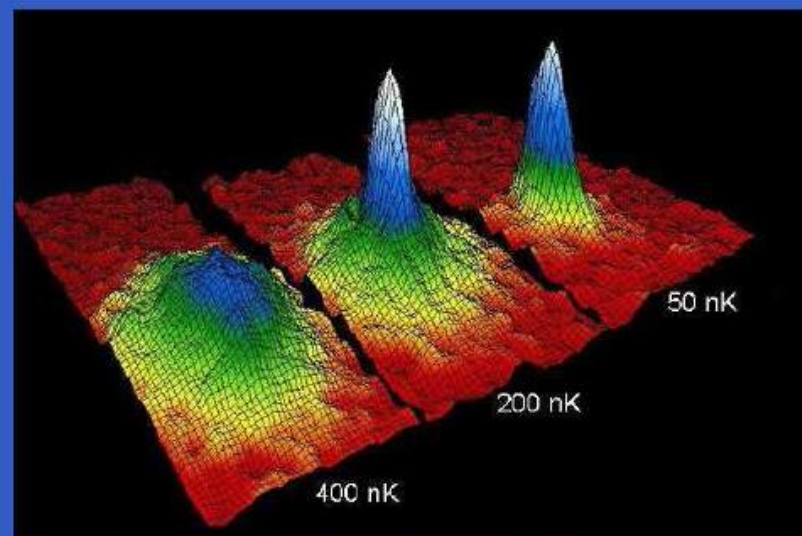
temperature 10 - 100 nK

density 10^{13} - 10^{14} cm^{-3}

number of atoms 10^3 - 10^7

size 10 μm - 1 mm

lifetime 10 s



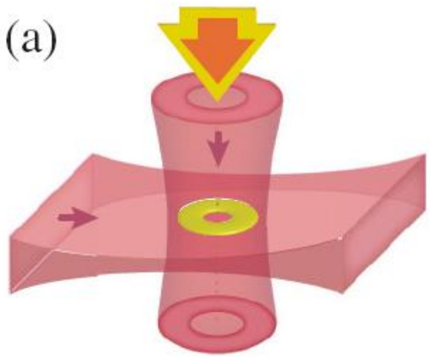
JILA & MIT 1995

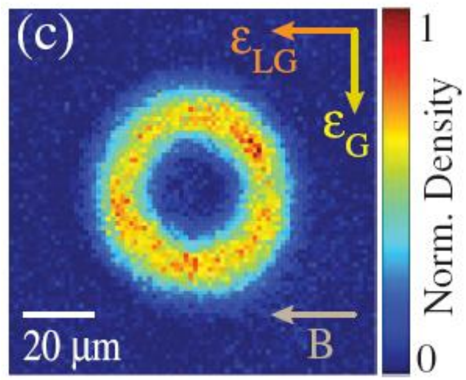
Condensed species

^{87}Rb   Na ^{7}Li   H ^{85}Rb ^{4}He* ^{41}K   Cs   Yb   Cr ^{39}K

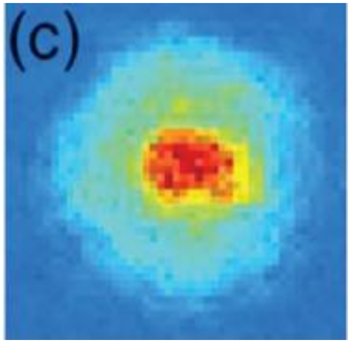
Persistent flow of BEC in torroidal trap with a weak link

Ryu et al. PRL 07
Ramanathan et al. PRL 11
Murray et al., PRA 13

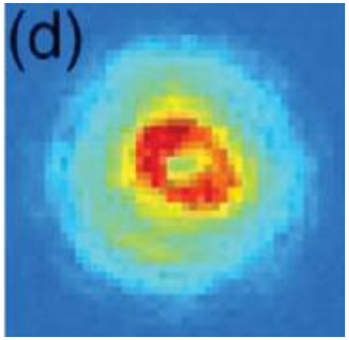


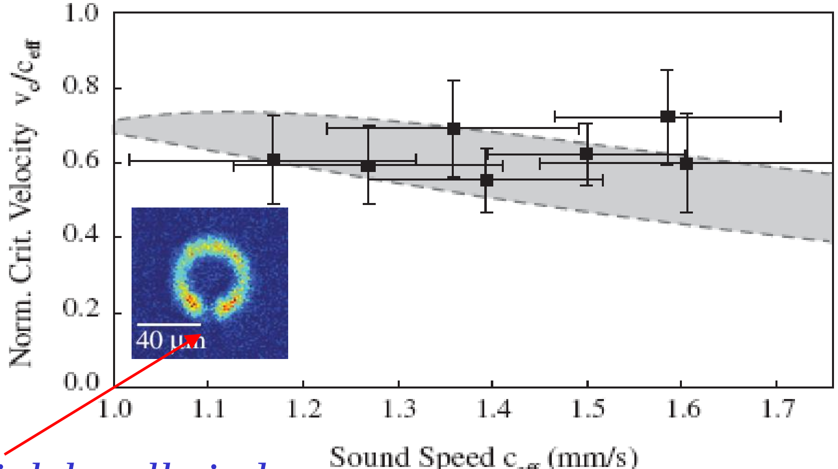


non-rotating state after expansion



rotating state induced using optical fields; after expansion





weak link locally induces flow instability

Outline

1. Single particle on a rotating ring

Defining concepts

2. Interacting bosons on a rotating ring

Discovering the interplay between backscattering and interactions

3. Atomic SQUID (AQUID)

Optimizing performance while playing with interaction and barrier strengths

4. Dynamics of AQUID

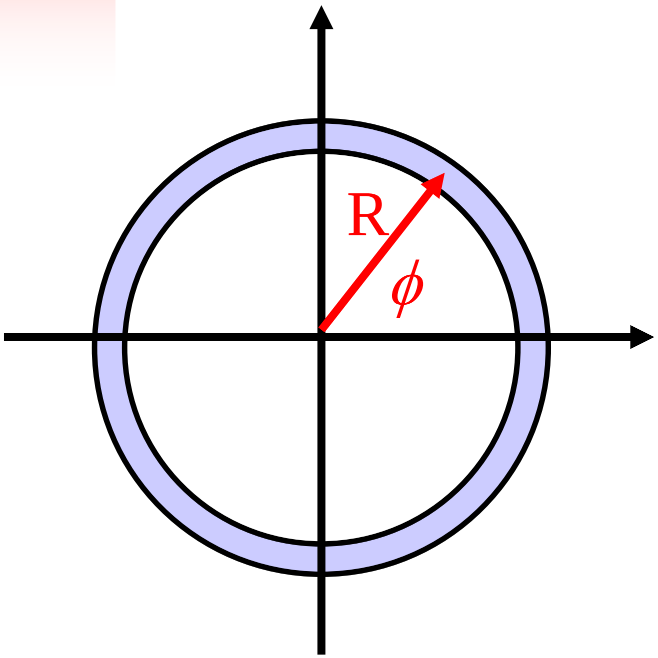
Interacting bosons + barrier yields an open quantum system

Conclusions

I. Single particle on a rotating ring

Defining concepts

Particle on a uniform ring: angular momentum states



Hamiltonian and Schrödinger equation

$$H = -\frac{\hbar^2}{2mR^2}\frac{\partial^2}{\partial\phi^2}$$

$$H\psi(\phi) = E\psi(\phi)$$

$$-\frac{\hbar^2}{2mR^2}\frac{\partial^2}{\partial\phi^2}\psi = E\psi$$

Eigenfunctions and eigenvalues

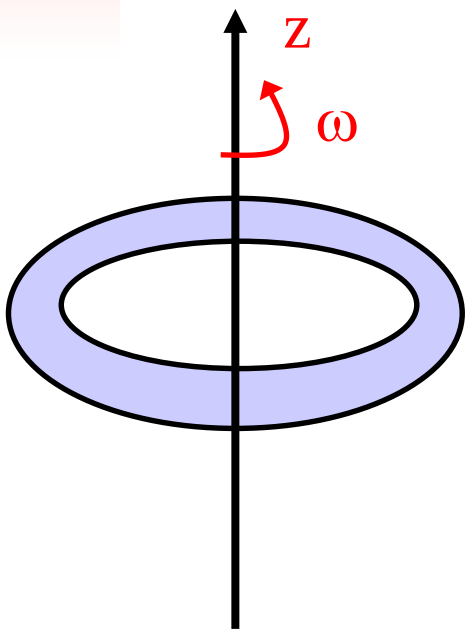
$$\psi(\phi) = \psi_n(\phi) = \frac{1}{\sqrt{2\pi}}e^{in\phi} = \psi(\phi + 2\pi)$$

$$E = E_n = \frac{\hbar^2 n^2}{2mR^2}$$

Wave function is strictly 2π periodic

Spectrum is discrete

Particle on a uniform rotating ring



Hamiltonian in rotating frame

$$H = -\frac{\hbar^2}{2mR^2}\left(\frac{\partial}{\partial\phi} - i\frac{m\omega R^2}{\hbar}\right)^2$$

$$= -\frac{\hbar^2}{2mR^2}\left(\frac{\partial}{\partial\phi} - i\frac{\Phi}{\Phi_0}\right)^2$$

Coriolis flux and flux quantum

$$\Phi = \omega R^2 \qquad \Phi_0 = \hbar/m$$

Eigenfunctions and eigenvalues

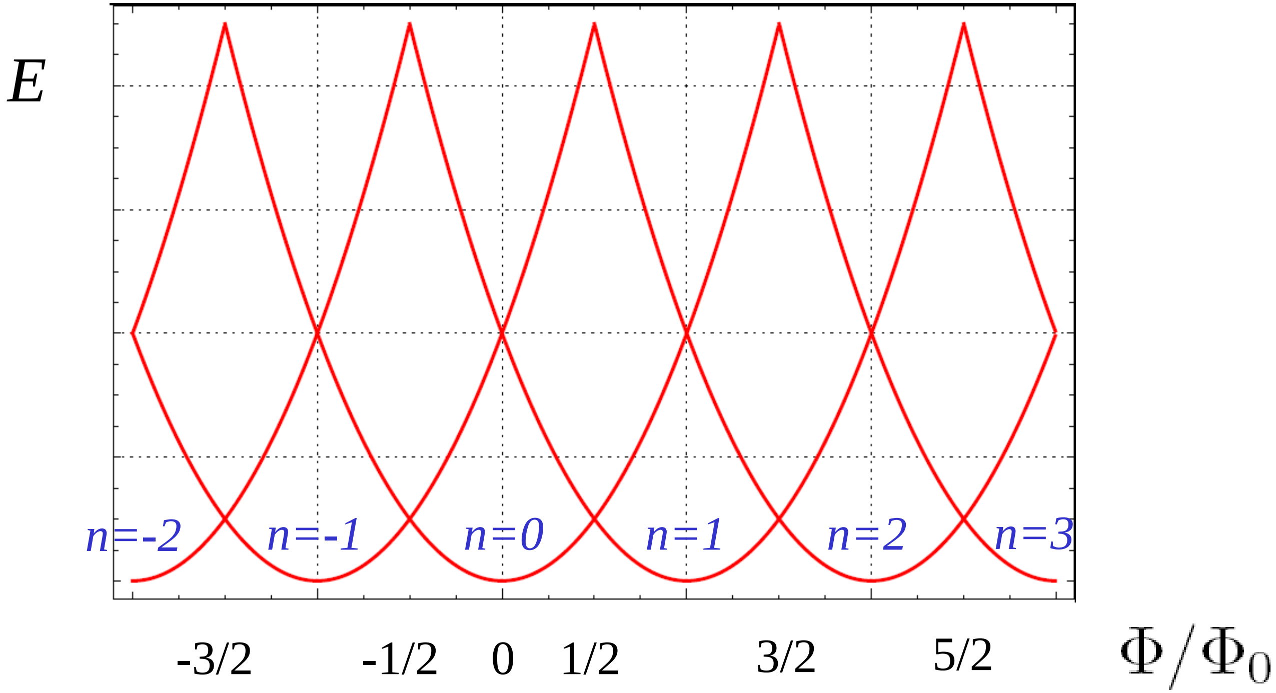
$$\psi(\phi) = \psi_n(\phi) = \frac{1}{\sqrt{2\pi}}e^{in\phi} = \psi(\phi + 2\pi)$$

$$E = E_n = \frac{\hbar^2}{2mR^2}(n - \Phi/\Phi_0)^2$$

Wave function is strictly 2π periodic

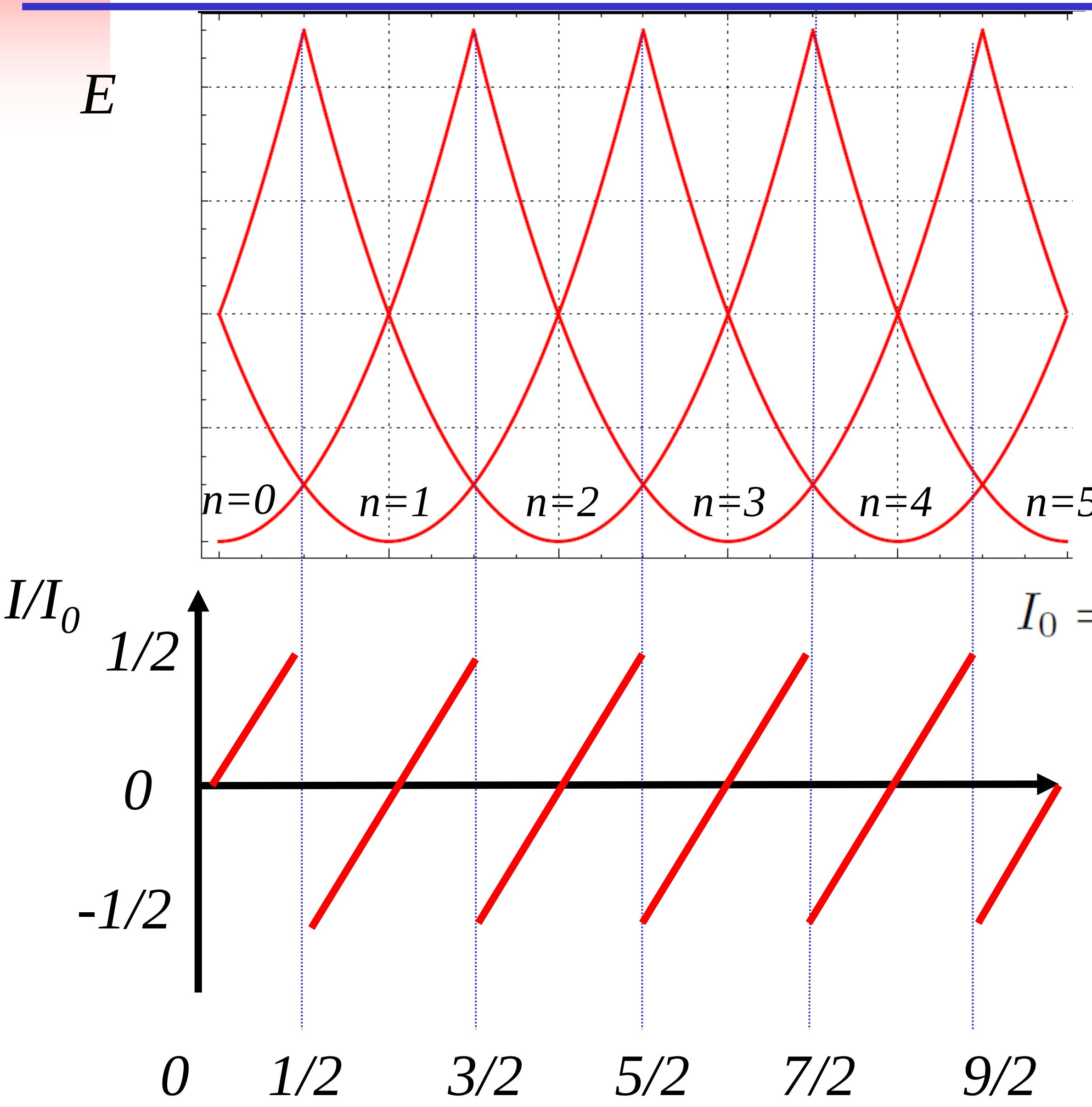
Energy depends on Φ: ring sustains persistent current $I \sim \partial E/\partial\Phi$

Spectrum for a particle on a uniform rotating ring: periodicity with Coriolis flux



Periodicity is Coriolis flux quantum Φ_0

Spectrum and persistent currents (rotating frame)



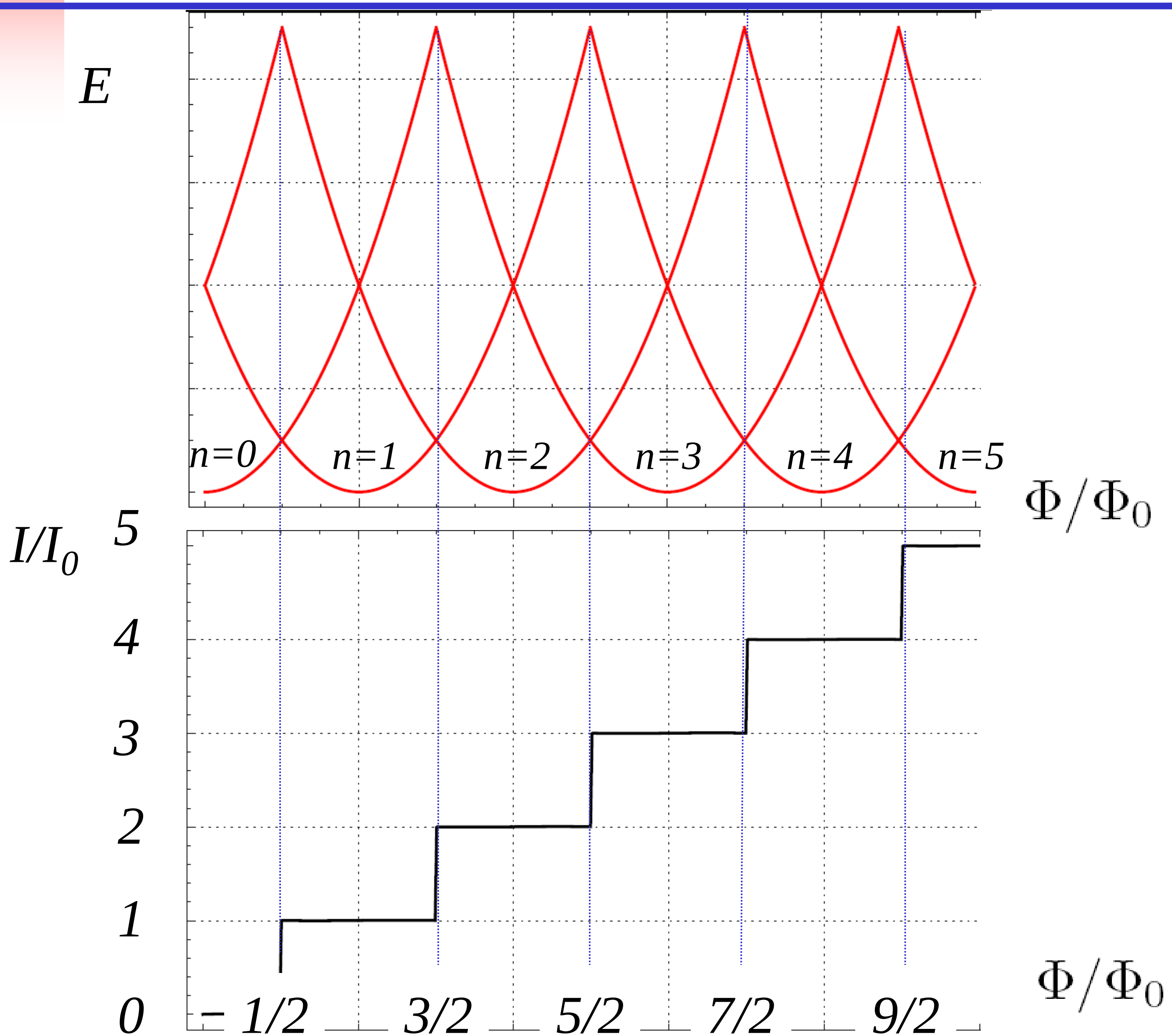
Φ/Φ_0

$I_0 = \hbar/(2\pi m R^2)$

lowest energy states: well-defined angular momentum

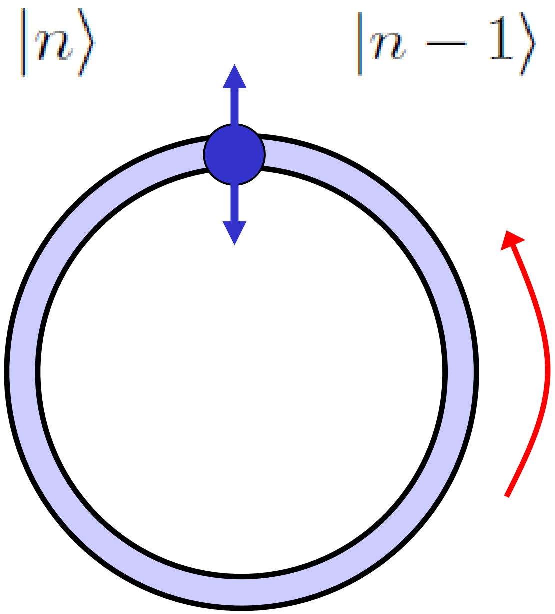
persistent current is a saw-tooth as a function of Coriolis flux

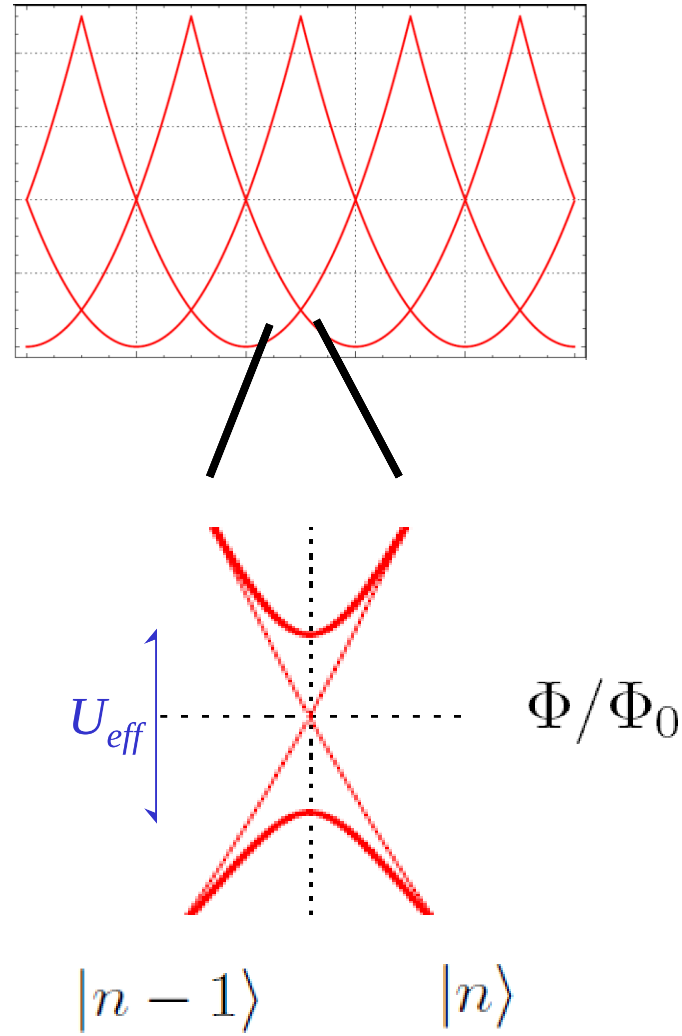
Spectrum and persistent currents (laboratory frame)



Effect of an impurity

Impurity induces mixing of angular momentum eigenstates



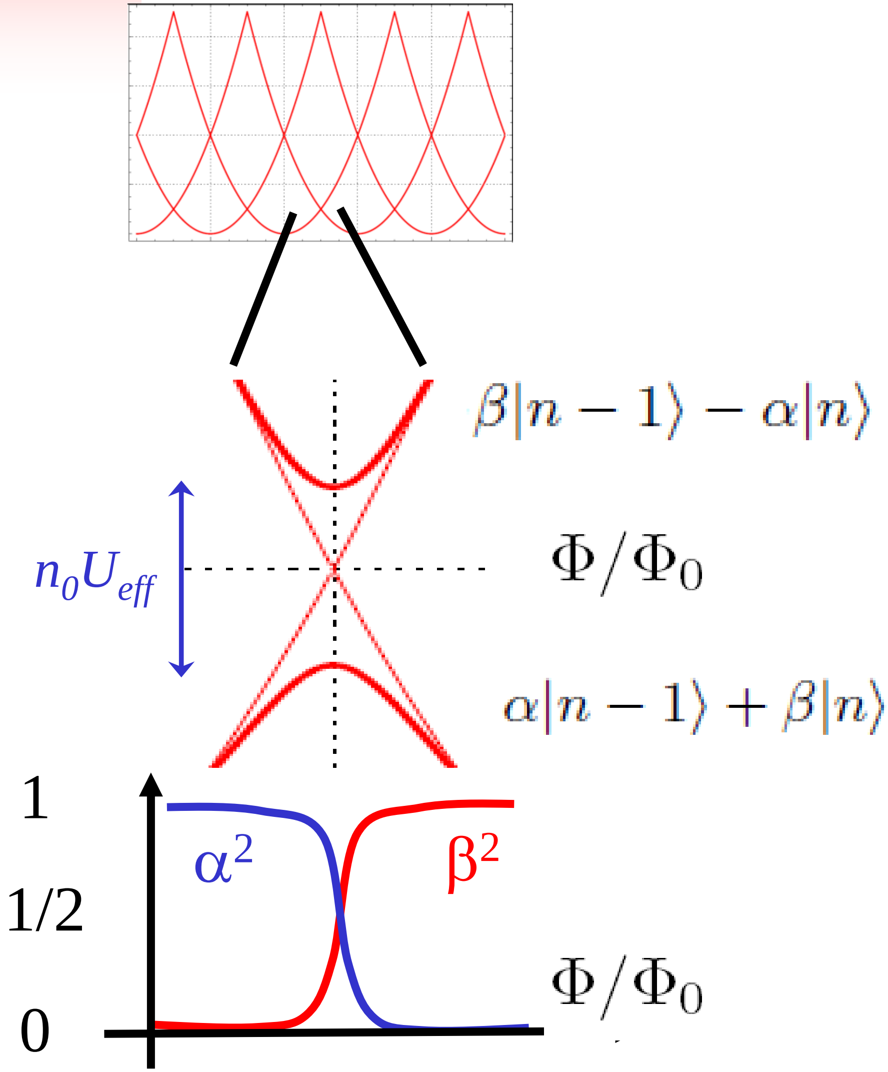


Effective Hamiltonian near degeneracy point

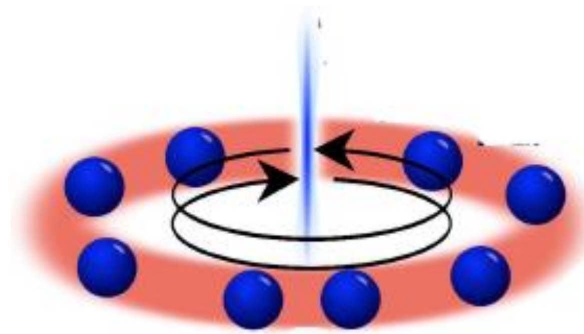
$$H_{\text{eff}} = \begin{pmatrix} E_{n-1} & U_{\text{eff}} \\ U_{\text{eff}} & E_n \end{pmatrix}$$

cf. band structure in solids

Mixing angular momentum states: superpositions



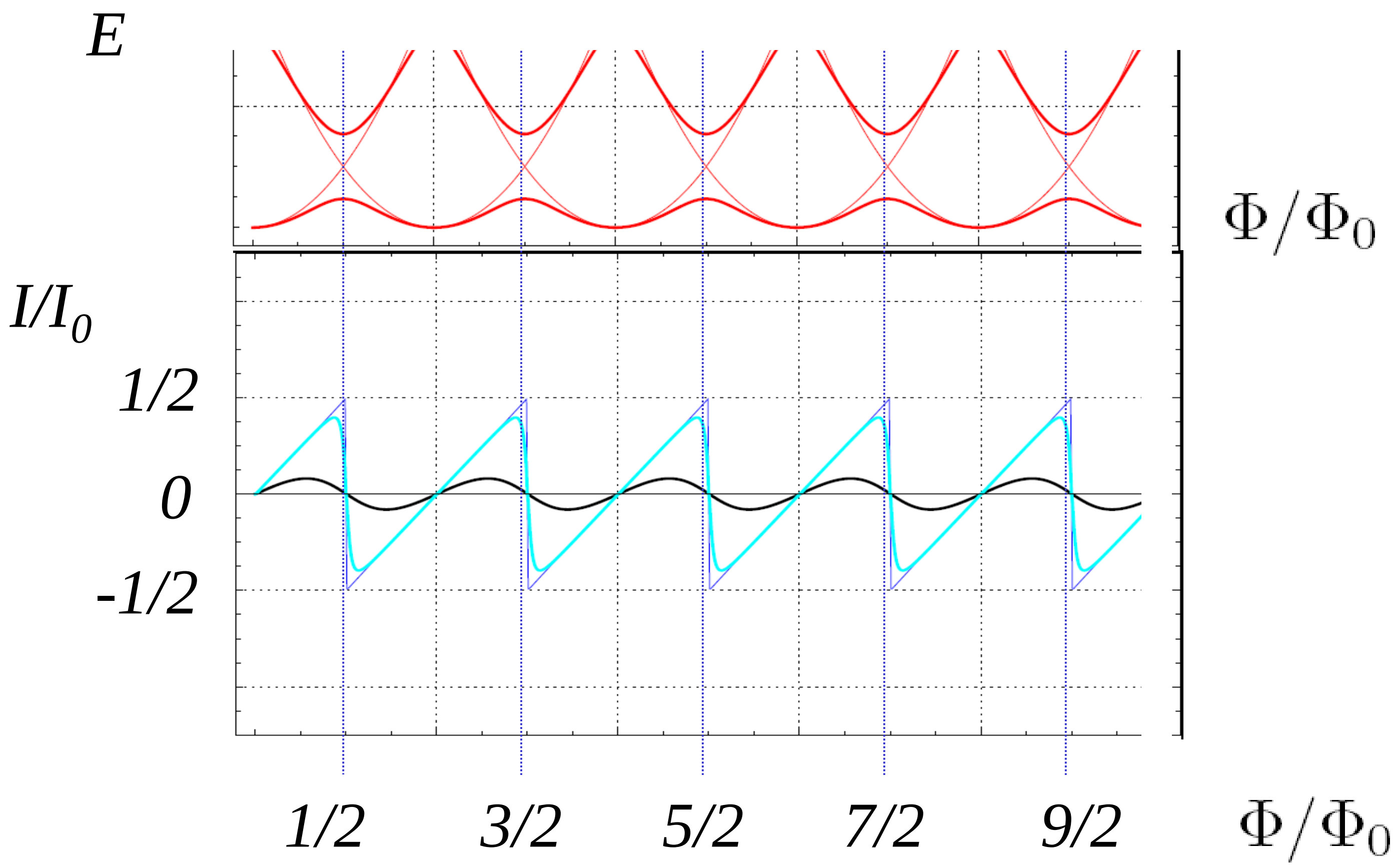
Coherent impurity scattering: formation of superpositions of angular momentum states.



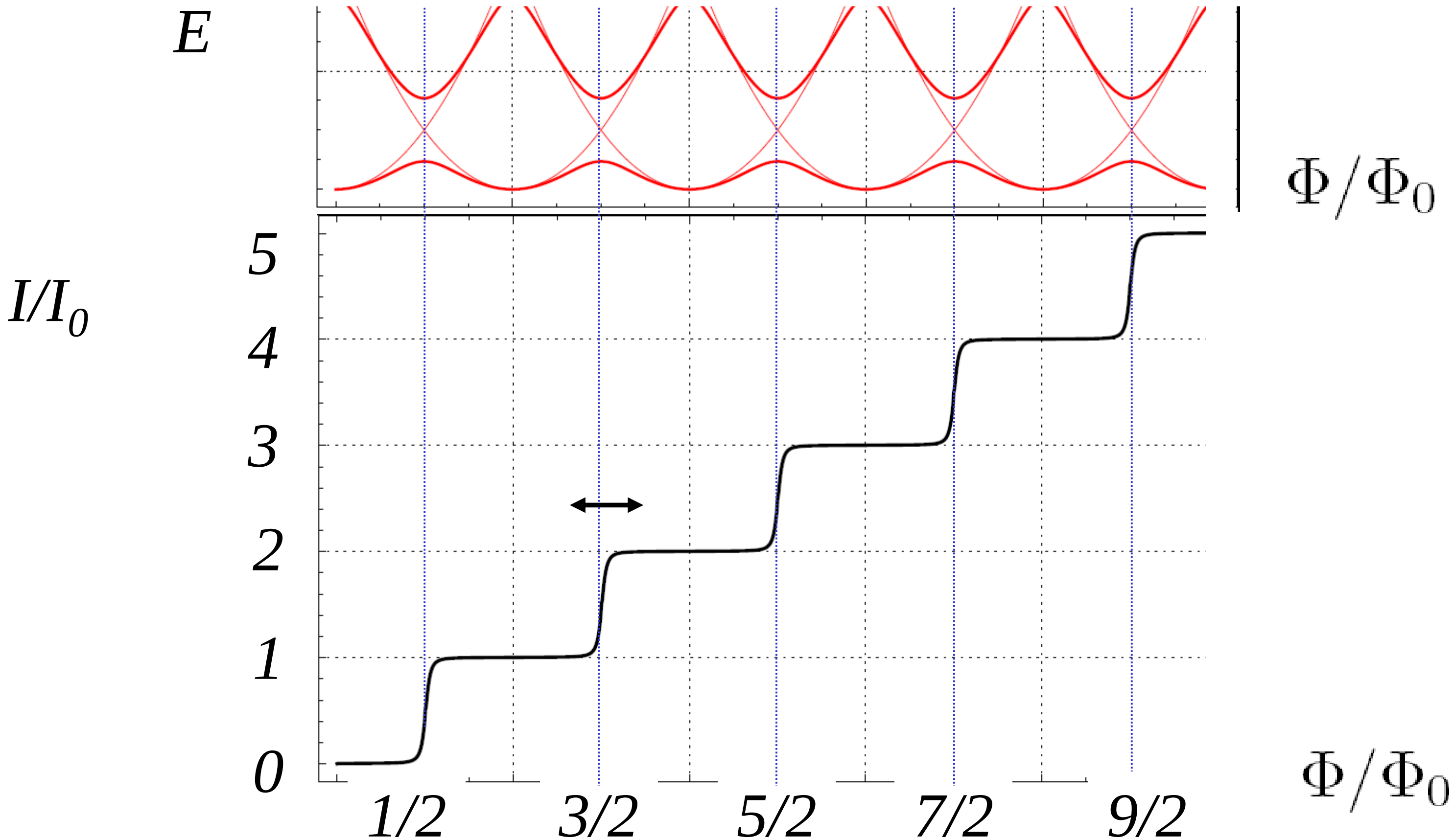
Weight factors in superpositions are tunable with Coriolis flux

System is a tunable two-level system (qubit)

Persistent current as a function of Coriolis flux: rotating frame



Persistent current as a function of Coriolis flux: laboratory frame

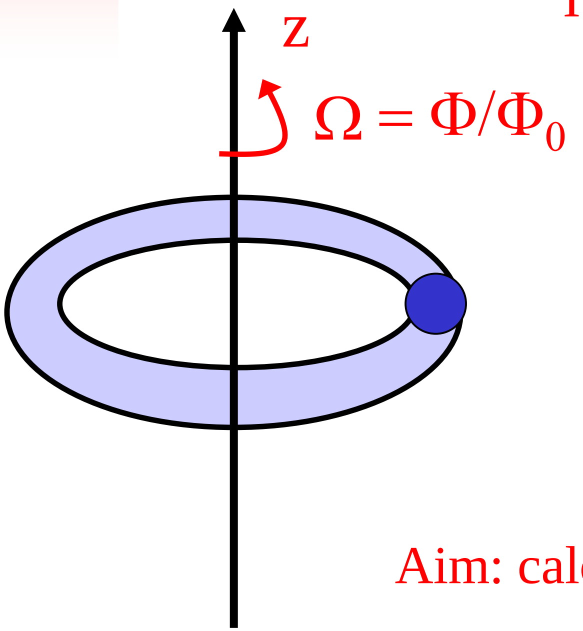


II. Interacting bosons on a rotating ring

Discovering the interplay between backscattering and interactions

Rotating interacting 1D bosons on a ring with a barrier

Cominotti et al. PRL 14



Hamiltonian

$$\mathcal{H} = \sum_{j=1}^{N} \frac{\hbar^2}{2M}\left(-i\frac{\partial}{\partial x_j} - \frac{2\pi}{L}\Omega\right)^2 + U_0\delta(x_j) + \frac{g}{2}\sum_{j,l=1}^{N}\delta(x_l - x_j).$$

kinetic energy in rotating frame — *barrier* — *repulsive interaction*

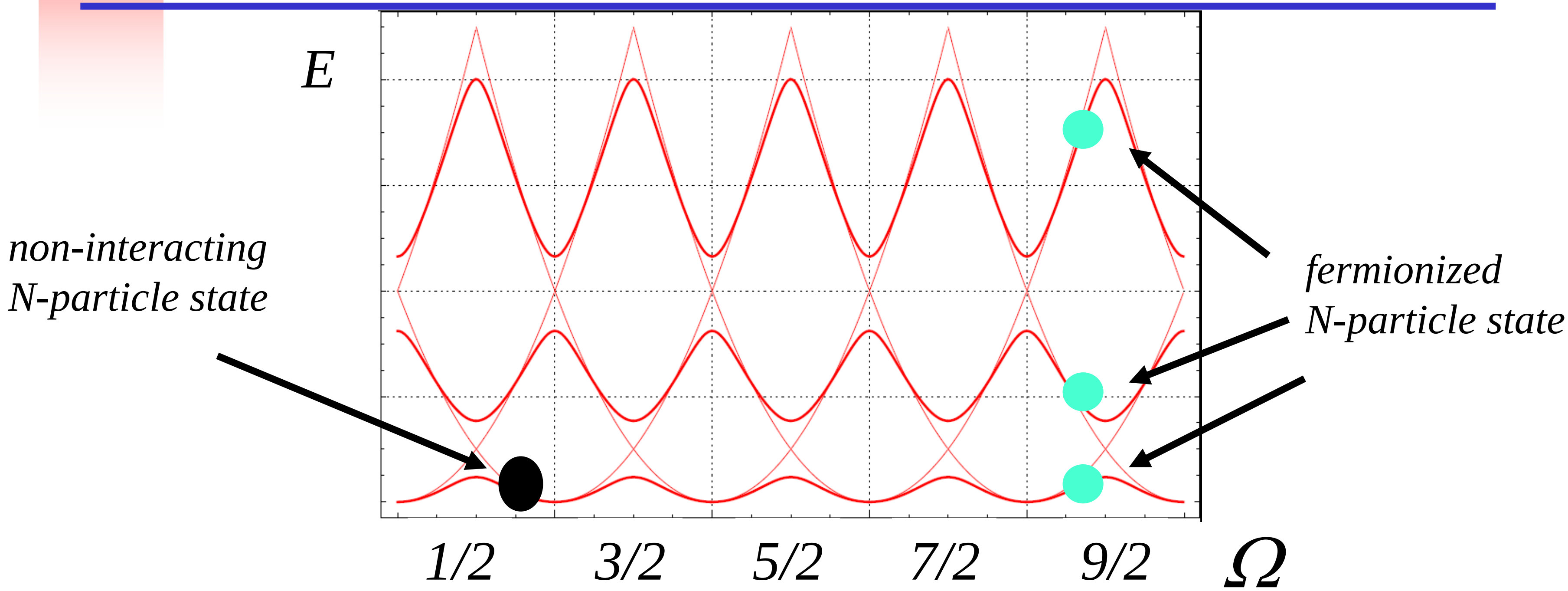
Aim: calculate persistent current $I(\Omega) = -\frac{1}{2\pi\hbar}\frac{\partial E(\Omega)}{\partial \Omega}.$

Various techniques:

- non-interacting bosons & Tonks-Girardeau bosons
- Gross-Pitaevskii equation (weak interactions)
- Luttinger liquid (strong interactions)
- DMRG (intermediate interactions)

Analytical solutions

Cominotti et al. PRL 14



Noninteracting bosons

$$\Psi_{\rm NI}(x_1,\ldots,x_N) = \prod_{i=1}^{N} \psi_0(x_i)$$

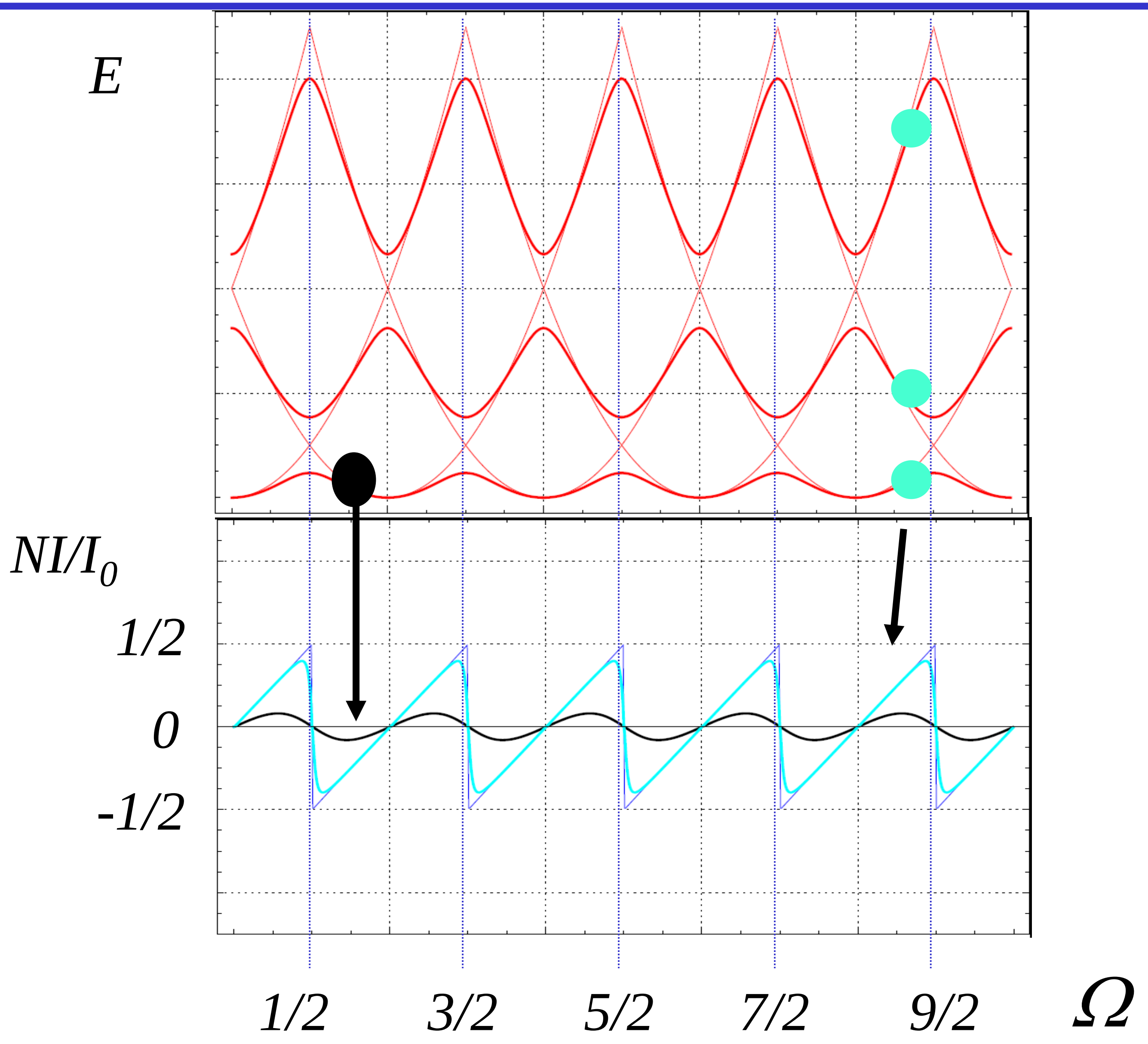
single-particle wave function on a ring with a barrier

Tonks-Girardeau bosons (infinite hard-core repulsion)

$$\Psi_{\rm TG}(x_1,\ldots,x_N) = \prod_{1\le j<\ell\le N} {\rm sgn}(x_j - x_\ell) \times \det[\psi_k(x_i)]$$

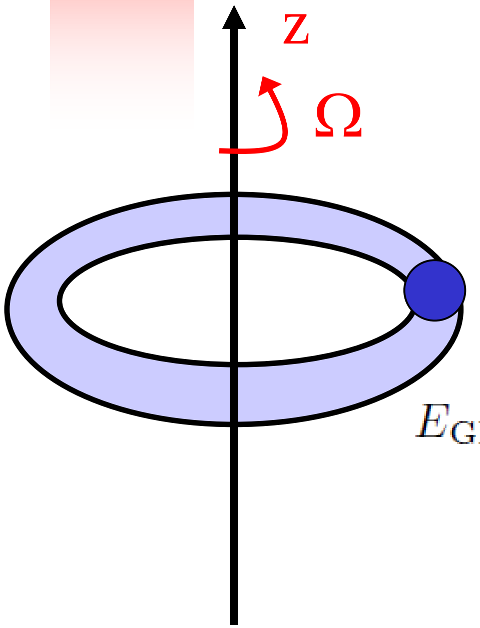
Persistent current amplitude *larger* for interacting bosons

Macroscopic persistent current as a function of Coriolis flux:



Mean-field theory of condensate on a rotating ring

Cominotti et al. PRL 14



Macroscopic wave function

$$\Phi(x) = |\Phi(x)| e^{i\phi(x)}$$

superfluid density ($|\Phi(x)|$)

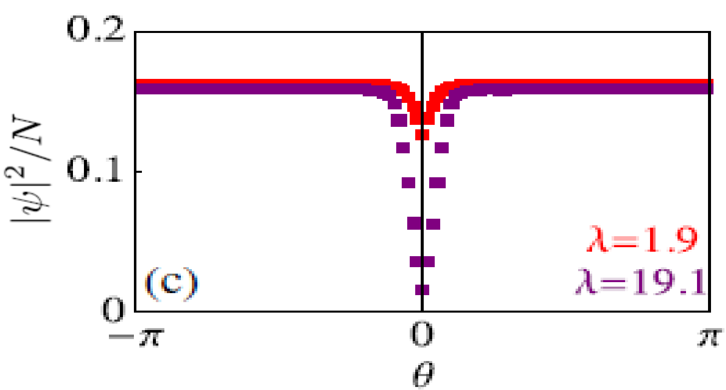
superfluid phase ($\phi(x)$)

Gross-Pitaevskii energy

$$E_{\mathrm{GP}} = \int dx \, \{ \Phi^* (\hbar^2/2M)[-i\partial_x - (2\pi/L)\Omega)^2 \Phi] + U_0 \delta(x) |\Phi|^2 + g|\Phi^4| \}$$

kinetic energy in rotating frame — *barrier* — *repulsive interaction*

Configuration that minimizes energy @ Ω =0.4



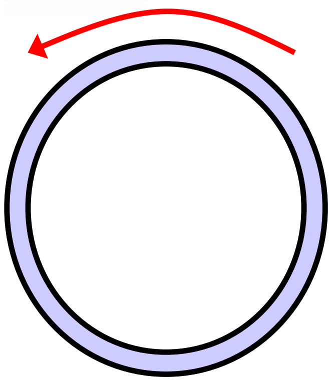
Characteristic scale: healing length

$$\xi = \hbar / \sqrt{2Mgn_0}$$

Persistent current amplitude
increases
with interaction strength

Luttinger liquid approach

Cominotti et al. PRL 14



Hamiltonian of the liquid

$$\mathcal{H}_0 = \frac{\hbar v_s}{2\pi}\int_0^L dx\left[K\left(\partial_x\phi(x) - \frac{2\pi}{L}\Omega\right)^2 + \frac{1}{K}(\partial_x\theta(x))^2\right]$$

fluctuating phase — *fluctuating density* — *kinetic energy* — *interaction term*

$$v_s K = \hbar\pi n_s/m$$

Mode expansion for fluctuating fields

$$\theta(x) = \theta_0 + \frac{1}{2}\sum_{q\neq 0}\left|\frac{2\pi K}{qL}\right|^{1/2}[e^{iqx}b_q + e^{-iqx}b_q^\dagger],$$

$$\phi(x) = \phi_0 + \frac{2\pi x}{L}(J-\Omega) + \frac{1}{2}\sum_{q\neq 0}\left|\frac{2\pi}{qLK}\right|^{1/2}\mathrm{sgn}(q)[e^{iqx}b_q + e^{-iqx}b_q^\dagger],$$

zero modes — *fluctuating parts* $\delta\phi$, $\delta\theta$ — *topological part*

Commutation relations

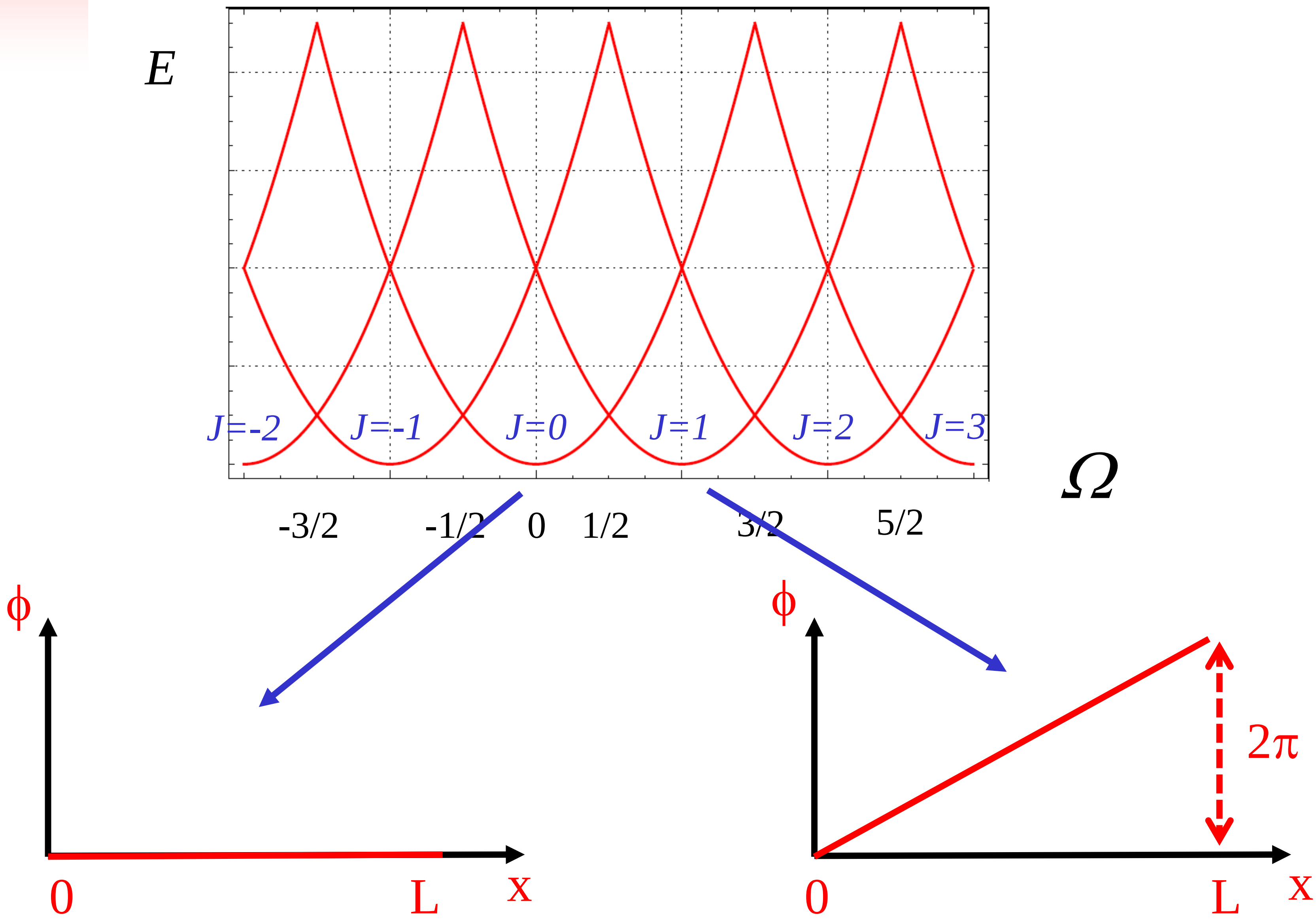
$$[b_q, b_{q'}^\dagger] = \delta_{q,q'}$$

bosonic low-energy excitations (phonons)

$$[J, e^{-2i\theta_0}] = e^{-2i\theta_0} \longrightarrow e^{-2i\theta_0}|J\rangle = |J+1\rangle$$

raising angular momentum (phase-slip)

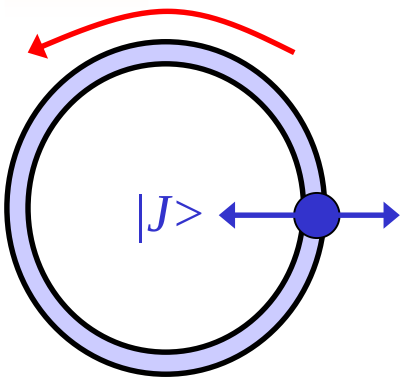
Changing angular momentum: phase-slips



Luttinger liquid approach

Cominotti et al. PRL 14

Hamiltonian of the liquid



$$\mathcal{H}_0 = \frac{\hbar v_s}{2\pi}\int_0^L dx\left[K\left(\partial_x\phi(x) - \frac{2\pi}{L}\Omega\right)^2 + \frac{1}{K}(\partial_x\theta(x))^2\right]$$

fluctuating phase — *fluctuating density* — *kinetic energy* — *interaction term*

$$v_s K = \hbar\pi n_s/m$$

Hamiltonian of the barrier

$$\mathcal{H}_b = \int dx \Psi^\dagger(x) U_0 \delta(x) \Psi(x) = U_0 \rho(0)$$

$$\sim 2U_0 n_0 \cos(2\theta(0)) = n_0 U_0 \sum_J |J-1\rangle\langle J| e^{2i\delta\theta(0)} + |J\rangle\langle J+1| e^{-2i\delta\theta(0)}$$

Barrier induces transitions between angular momentum states (phase-slips)

Role of interactions: barrier renormalization

$$\langle e^{\pm 2i\delta\theta(0)}\rangle = e^{-2\langle\delta\theta^2(0)\rangle} = \left(\frac{\alpha}{L}\right)^K$$

$$U_0 \longrightarrow U_{eff} = U_0(\alpha/L)^K$$

Persistent current amplitude *decreases* with interaction strength

Optimal persistent current

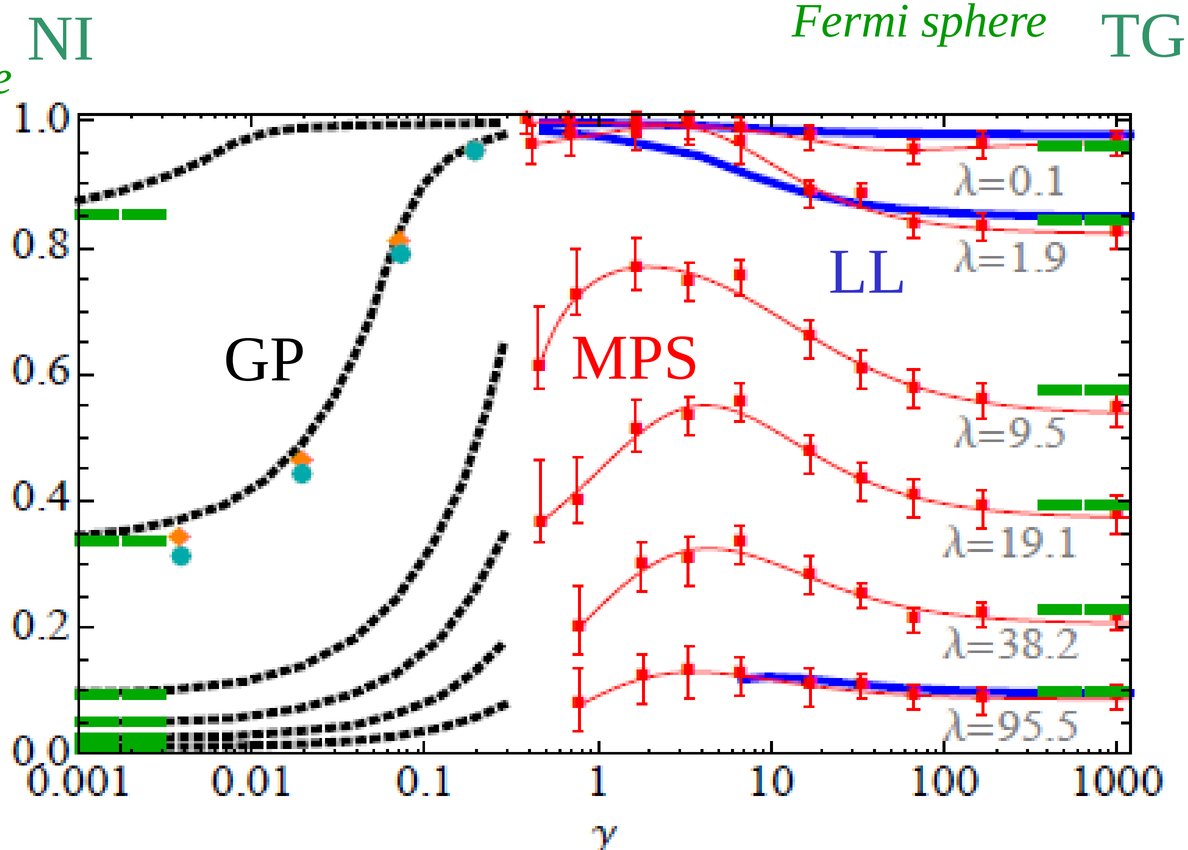
Cominotti et al. PRL 14

Current amplitude as a function of interaction strength for various barrier strengths

TG: Bosons occupy Fermi sphere

NI: all bosons in the lowest band

GP: interactions induce healing



MPS: competiton between healing and quantum fluctuations

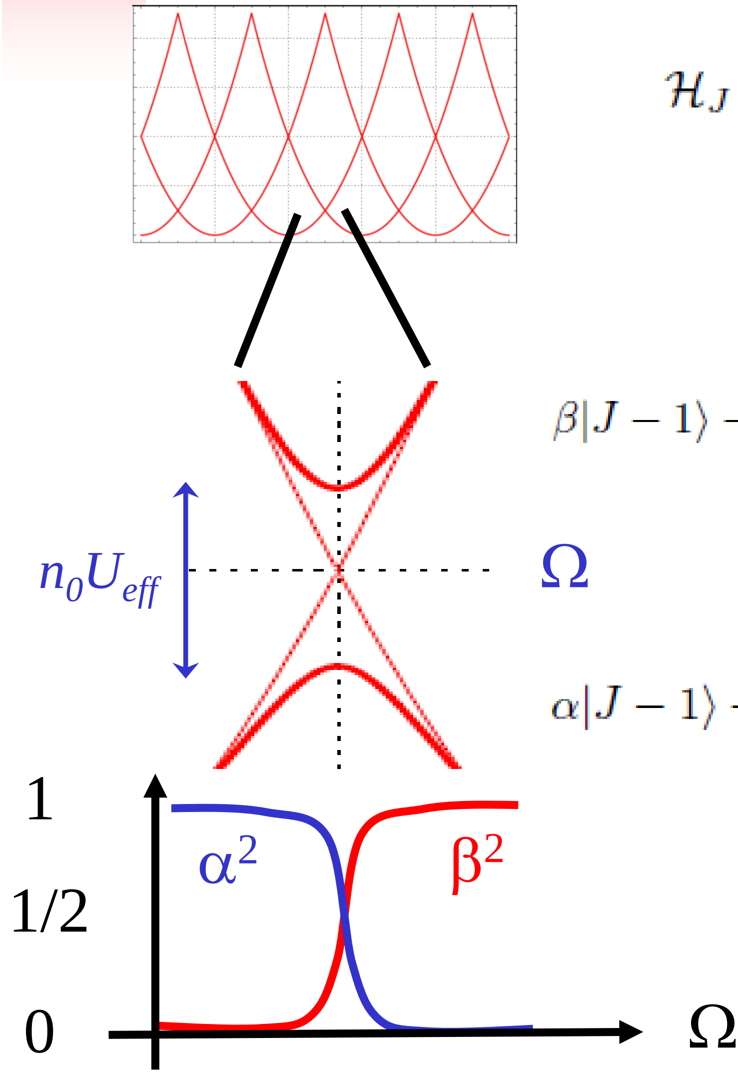
LL: interactions induce quantum phase fluctuations

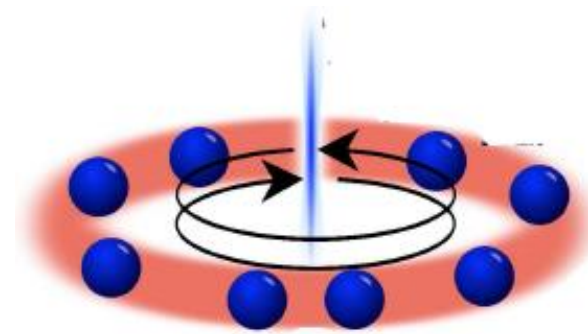
III. Atomic SQUID (AQUID)

Optimizing performance while playing with interaction and barrier strengths

Macroscopic persistent-current qubit

$$\mathcal{H}_J = E_0(J-\Omega)^2 + n_0 U_{\rm eff} \sum_J |J+1\rangle\langle J| + \text{H.c.}$$





Weight factors in macroscopic superpositions are tunable with Coriolis flux

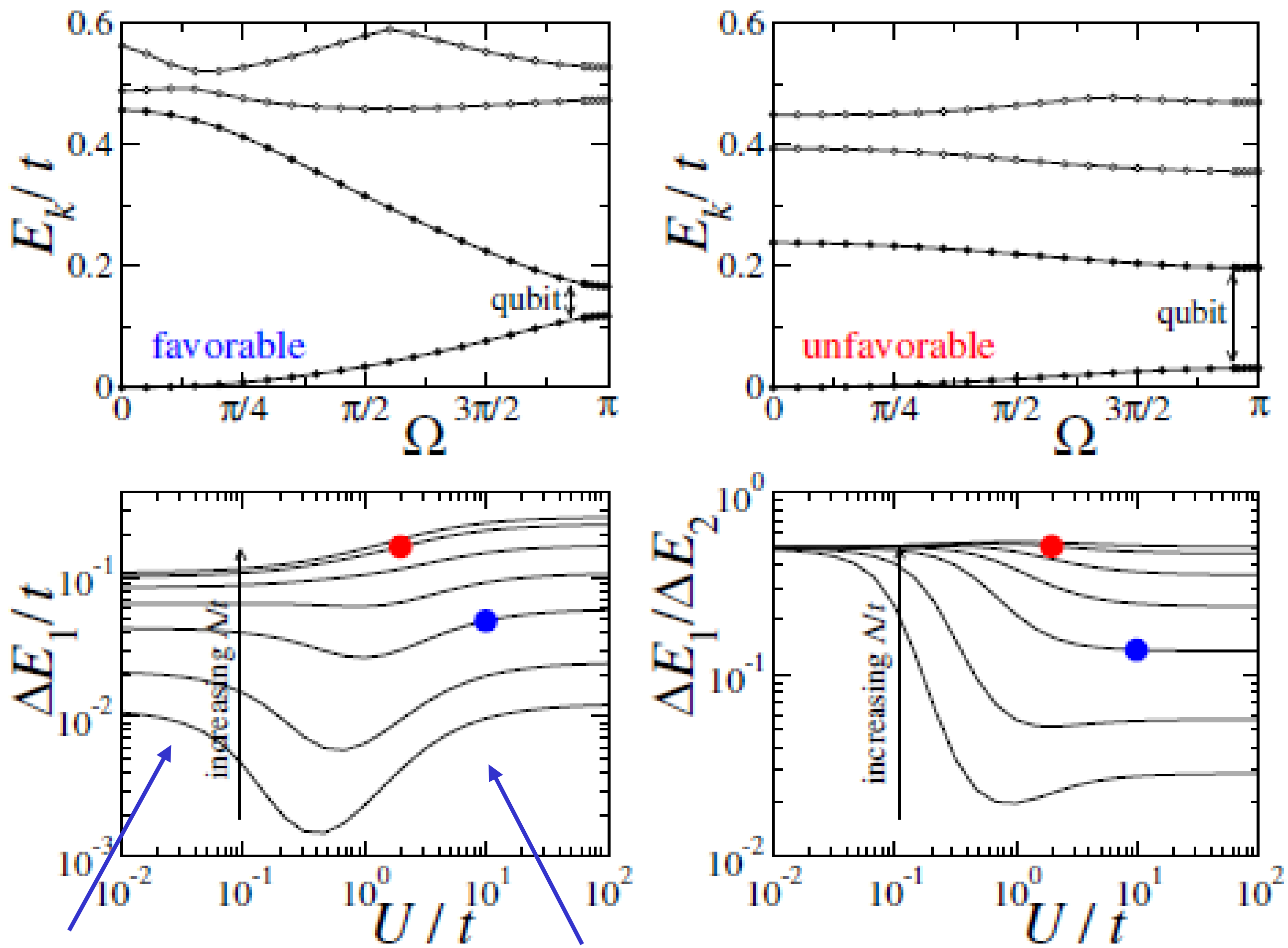
System is a tunable two-level system (qubit)

Qubit energy states: role of interaction and barrier strengths

Aghamalyan, Cominotti et al. NJP 15

Spectrum
as a function of Ω
@U/Λ = 20 & 0.4

Level splitting
& anharmonicity
as a function of
interaction and
barrier strengths
@ Ω = 0.5



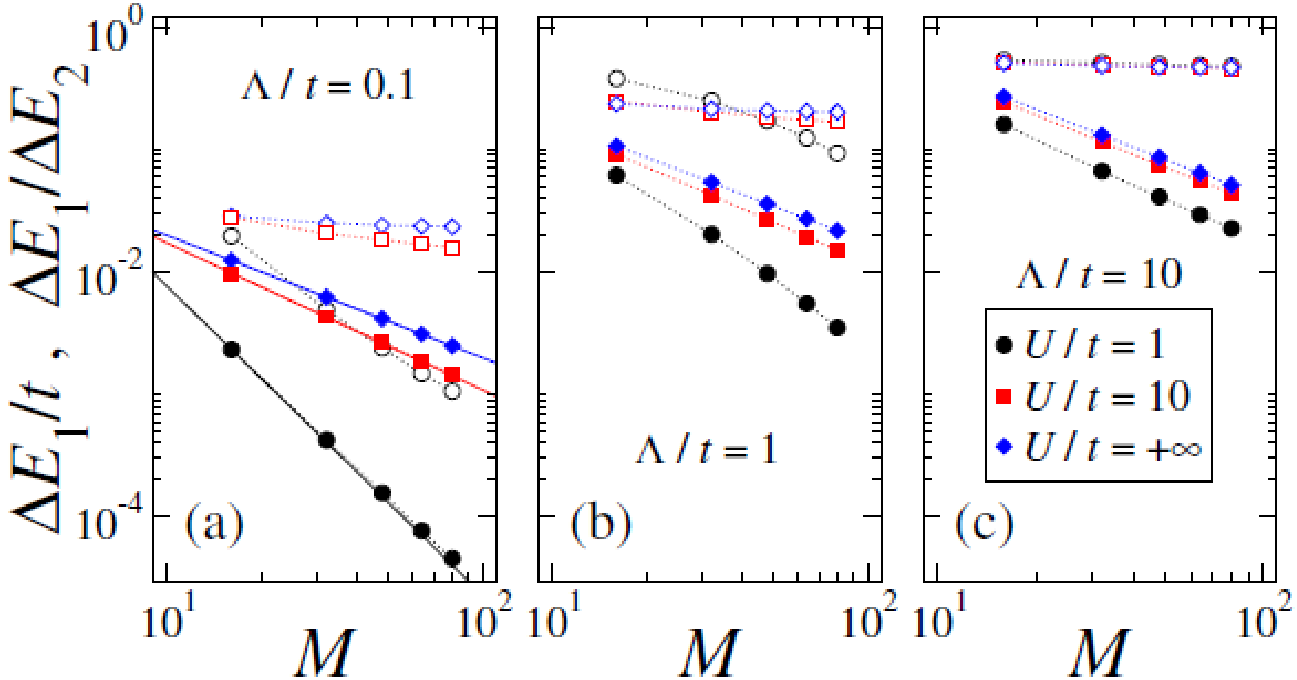
healing

quantum fluctuations

Qubit energy states: mesoscopic finite-size scaling

Aghamalyan, Cominotti et al. NJP 15

Level splitting and anharmonicity as a function of system size



Interactions « protect » qubit splitting & anharmonicity against scaling

Ground-state momentum distribution

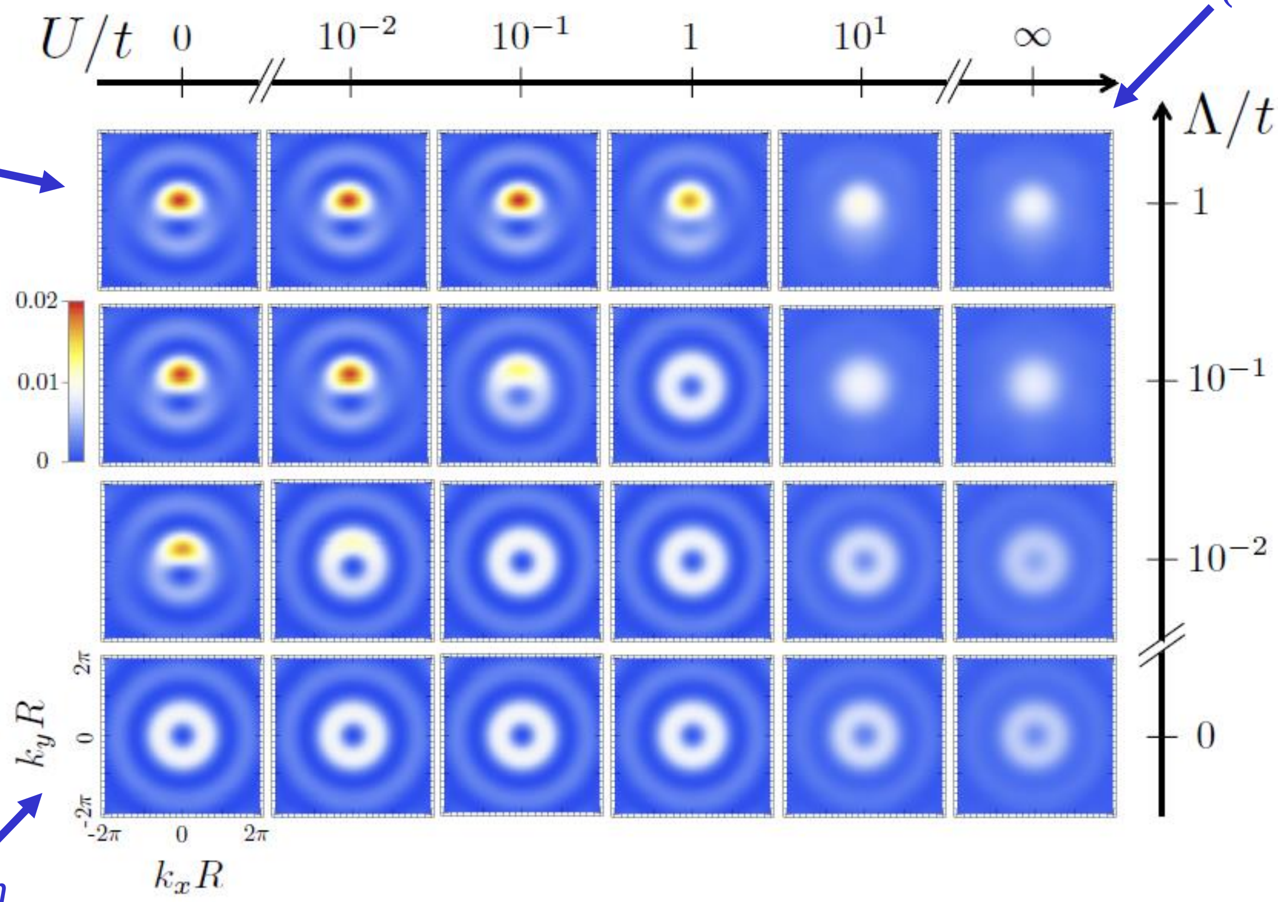
Aghamalyan, Cominotti et al. NJP 15

Definition $$n(\mathbf{k}) = \int d\mathbf{x} \int d\mathbf{x}' \langle \hat{\psi}^\dagger(\mathbf{x}) \hat{\psi}(\mathbf{x}') \rangle e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}$$

Response at Ω = ½ +δ (just away from perfect balance)

Superposition state involving many angular momentum states (Fermi gas)

Superposition state involving two angular momentum states



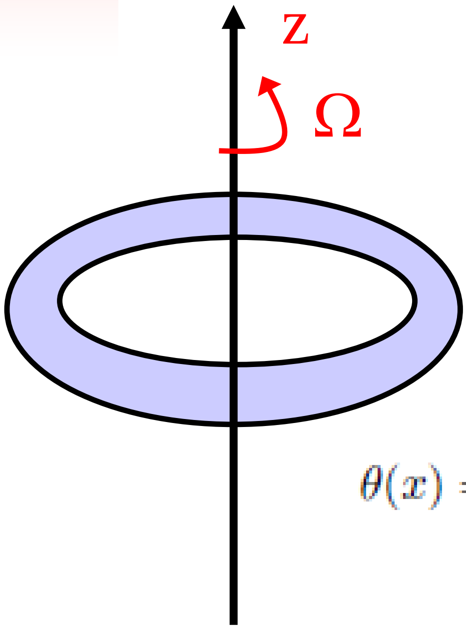
State with one angular momentum quantum

IV. Dynamics of AQUID

Interacting bosons + barrier yields
an open quantum system

Phonon modes on a uniform ring

Polo et al. in progress



Hamiltonian of harmonic fluid

$$\mathcal{H}_0 = \frac{\hbar v_s}{2\pi}\int_0^L dx\left[K\left(\partial_x\phi(x) - \frac{2\pi}{L}\Omega\right)^2 + \frac{1}{K}(\partial_x\theta(x))^2\right]$$

Mode expansion: propagating phonon modes

$$\theta(x) = \theta_0 + \frac{1}{2}\sum_{q\neq 0}\left|\frac{2\pi K}{qL}\right|^{1/2}[e^{iqx}b_q + e^{-iqx}b_q^\dagger],$$

$$\phi(x) = \phi_0 + \frac{2\pi x}{L}(J-\Omega) + \frac{1}{2}\sum_{q\neq 0}\left|\frac{2\pi}{qLK}\right|^{1/2}\mathrm{sgn}(q)[e^{iqx}b_q + e^{-iqx}b_q^\dagger],$$

Harmonic fluid equivalent to harmonic oscillator bath

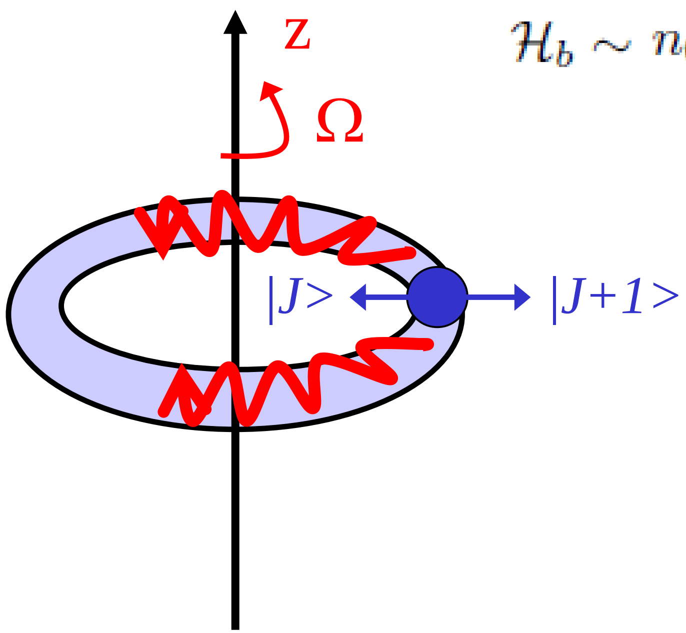
$$\mathcal{H}_0 = E_0(J-\Omega)^2 + \sum_{q\neq 0}\hbar\omega_q b_q^\dagger b_q = E_0(J-\Omega)^2 + \frac{1}{2}\sum_{q\neq 0}(P_q^2 + \omega_q^2 Q_q^2)$$

linear spectrum $\omega_q \sim v_s\,|q|$

Effect of the barrier

Polo et al. in progress

Angular momentum tunneling (phase-slips)
excites phonon modes



$$\mathcal{H}_b \sim n_0 U_0 \sum_J |J-1\rangle\langle J| e^{2i\delta\theta(0)} + |J\rangle\langle J+1| e^{-2i\delta\theta(0)}$$

coupling to density fluctuations

$$\delta\theta(0) = \sum_{q\neq 0} \lambda_q Q_q$$

particle-bath coupling

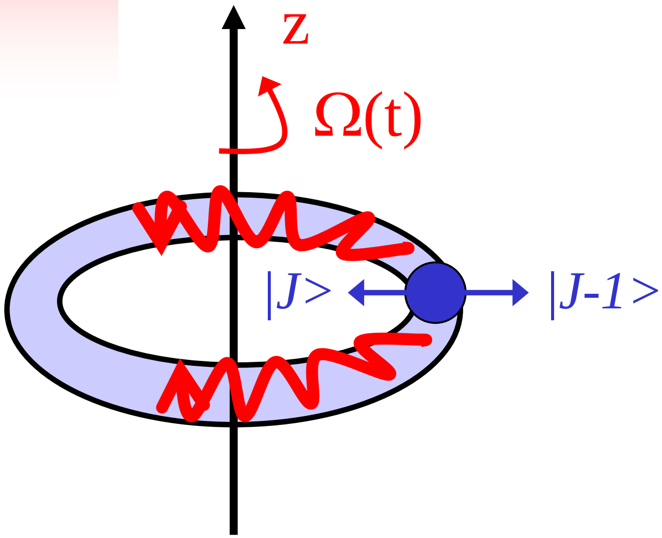
Caldeira-Leggett Hamiltonian

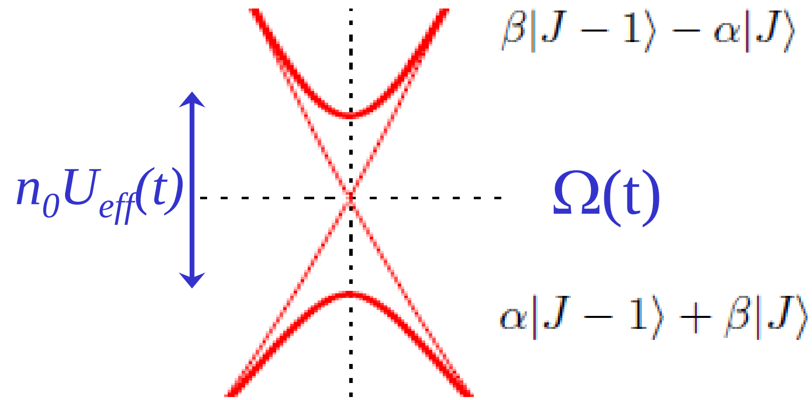
$$\mathcal{H} = E_0(J-\Omega)^2 - 2U_0 n_0 \cos(2\theta_0) + \frac{1}{2}\sum_{q\neq 0}\{[P_q - \mu_q(J-\Omega)]^2 + \omega_q^2 Q_q^2\}$$

quantum particle — *oscillator bath*

Tunable persistent current qubit with dissipation: spin-boson model

Polo et al. in progress





Spin-boson model

$(1/2)(1+\sigma_z)$ $(1/2)\sigma_x$

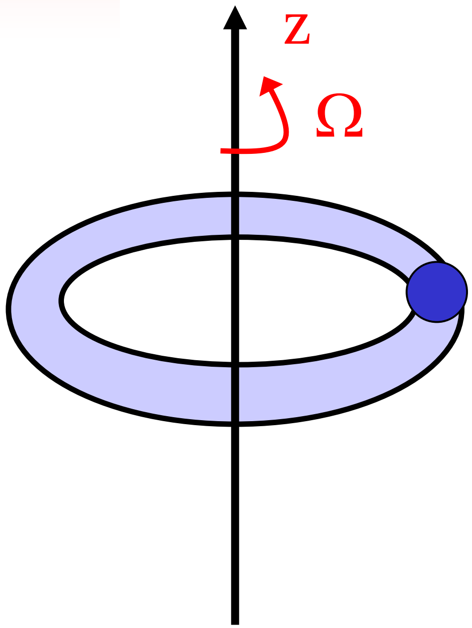
$$\mathcal{H} = E_0(J-\Omega(t))^2 - 2U_0(t)n_0\cos(2\theta_0) + \frac{1}{2}\sum_{q\neq 0}\{[P_q - \mu_q(J-\Omega(t))]^2 + \omega_q^2 Q_q^2\}$$

$$-\frac{1}{2}B_z\,\sigma_z - \frac{1}{2}B_x\,\sigma_x$$

bath coupling linearly to σ_z

Tunable nonlinear oscillator: quantum Langevin problem

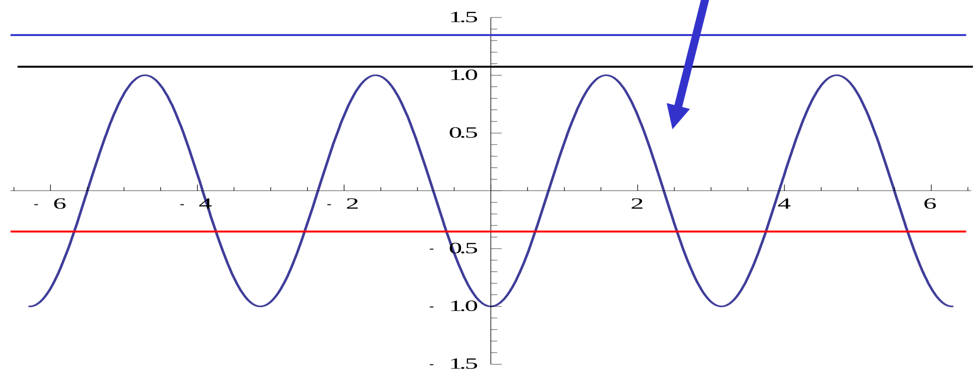
Polo et al. in progress

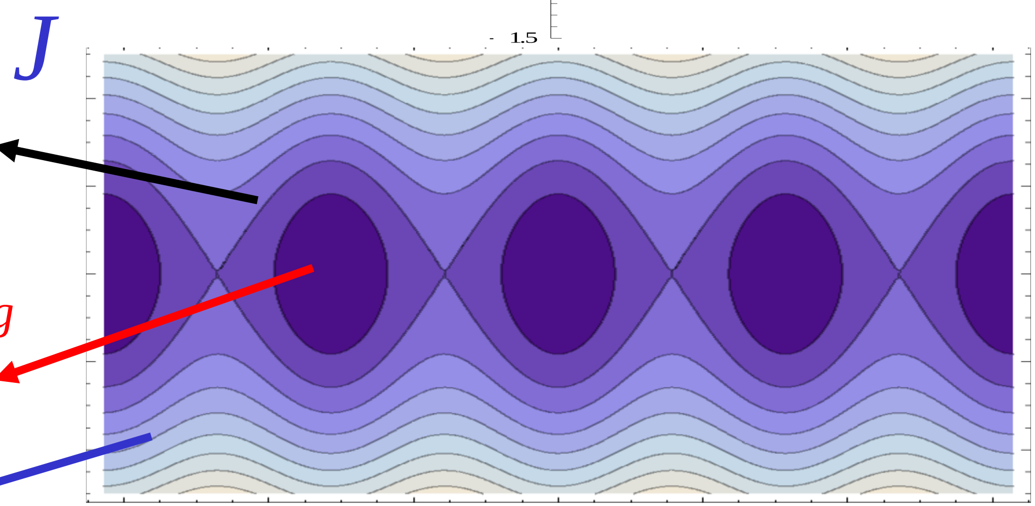


Hamiltonian (no dissipation, no drive)

$$\mathcal{H} = E_0(J-\Omega)^2 - 2U_0 n_0 \cos(2\theta_0)$$

Classical phase portrait





separatrix

oscillating state

Equation of motion

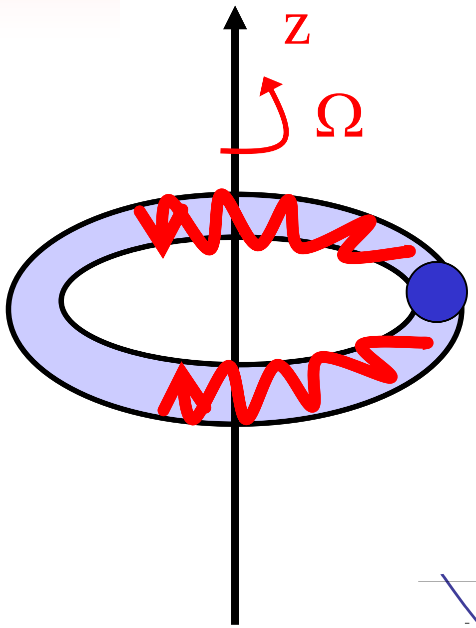
$$\ddot{\theta}_0 = -a \sin 2\theta_0$$

running state (dual to self-trapping)

θ_0

Tunable nonlinear oscillator: quantum Langevin problem

Polo et al. in progress

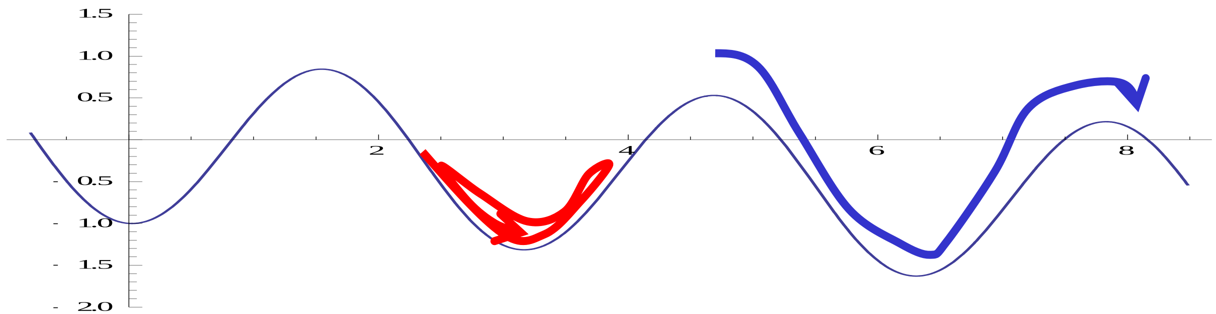


Quantum Langevin equation for driven system

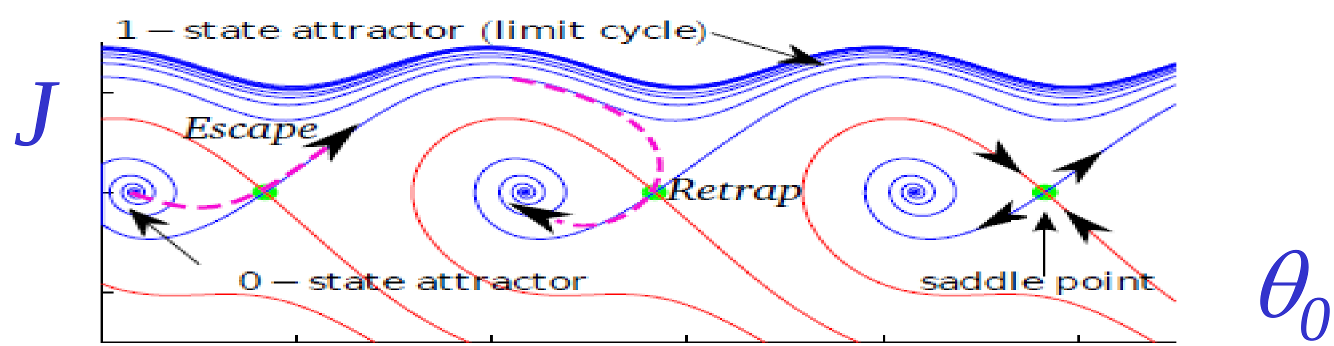
FDT

$$\ddot{\theta}_0 = -a(t)\sin 2\theta_0 - b\dot{\theta} + c\dot{\Omega}(t) + \xi(t)$$

damping — *external drive* — *Langevin force*



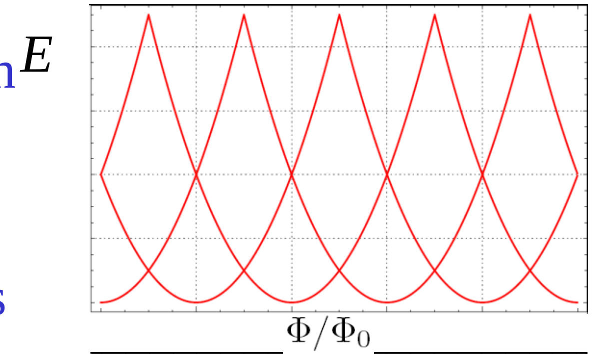
Ergül et al., PRB13



Conclusions

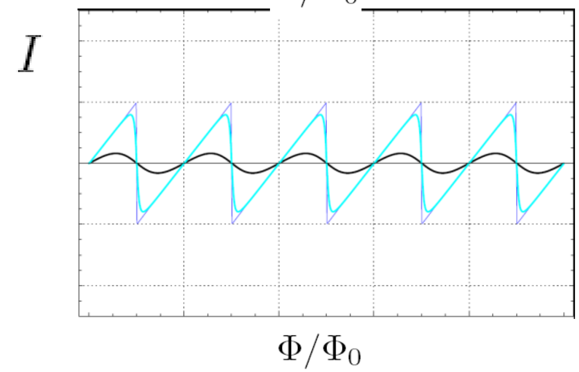
Spectrum of condensate on rotating loop periodic with Coriolis flux

Ring sustains persistent currents



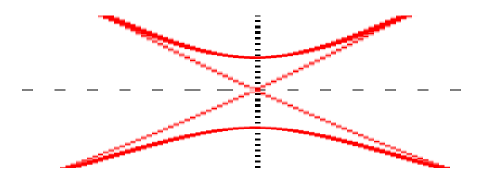
Response depends on barrier and interaction strengths

From sawtooth to sinusoidal behaviour



Coherent phase-slips induce superposition of angular monentum states

Ring can be used as a qubit



Dynamics governed by Caldeira-Legget model

From spin-boson physics to Langevin equation