Giac/Xcas, a swiss knife for mathematics

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Trophées du Libre 2007

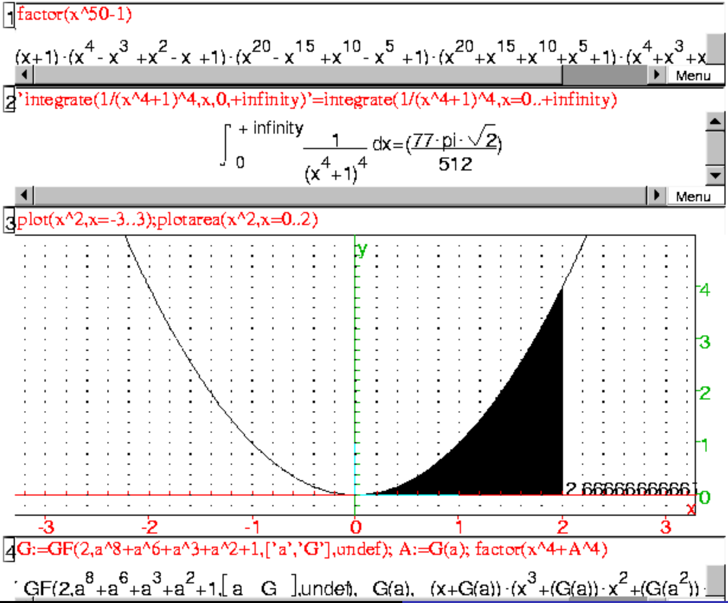
Plan

1. Xcas: interface for CAS, dynamic geometry and spreadsheet, audience: scientific students to research
2. Giac: a C++ library, the computation kernel, for C++ programmers and other interfacable languages.

CAS

From highschool to university. . .

- integer arithmetic : primes, GCD, extended GCD, cryptography...
- polynomials: GCD, factorization, fractions, finite fields...
- linear algebra: vectors, matrices, reduction, factorizations
- calculus: derivatives, integration, limits, series, ...
- numeric and symbolic solvers (equations, systems)
- 2-d and 3-d graphs: functions, parametric curves, level curves, ...
- ...



1 factor(x^50-1)

$(x+1)\cdot(x^4-x^3+x^2-x+1)\cdot(x^{20}-x^{15}+x^{10}-x^5+1)\cdot(x^{20}+x^{15}+x^{10}+x^5+1)\cdot(x^4+x^3+x$

2 'integrate(1/(x^4+1)^4,x,0,+infinity)'=integrate(1/(x^4+1)^4,x=0..+infinity)

$$\int_0^{+\ infinity} \frac{1}{(x^4+1)^4} dx=(\frac{77\cdot pi\cdot \sqrt{2}}{512})$$

3 plot(x^2,x=-3..3);plotarea(x^2,x=0..2)

4 G:=GF(2,a^8+a^6+a^3+a^2+1,['a','G'],undef); A:=G(a); factor(x^4+A^4)

$GF(2,a^8+a^6+a^3+a^2+1,[a\ \ G\ ],undef),\ G(a),\ (x+G(a))\cdot(x^3+(G(a))\cdot x^2+(G(a^2))\cdot$

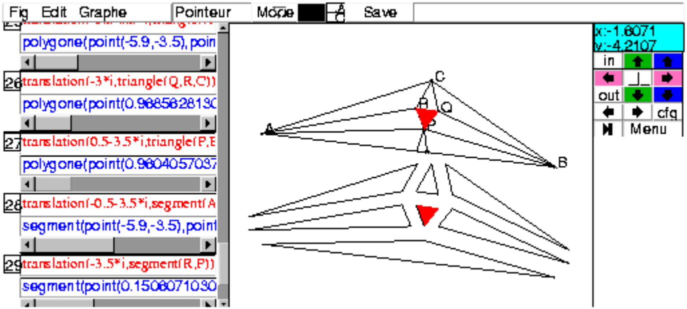
Documentation (R. De Graeve)

French (70 000 lines) and partially in English (13 000 lines)

- tutorial
- commands by themes in the menus or by alphabetic order in index
- command completion
- short help with examples to paste
- more complete help inside the browser
- examples sessions
- manuels, exercices
- Internet ressources

Geometry.

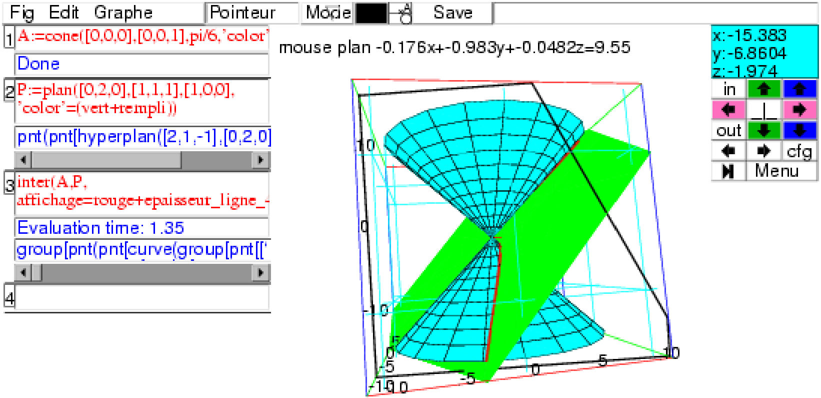
- Make constructions with the mouse or/and by commands
- Interactive figures (pointer mode and parameters)
- In the plane or in the space
- 3-d visualization options inherited from OpenGL
- Analytic proofs of theorems using the CAS



```
polygone(point(-5.9,-3.5),poin
26 translation(-3*i,triangle(Q,R,C))
polygone(point(0.98858281130
27 translation(0.5-3.5*i,triangle(P,E
polygone(point(0.98040570937
28 translation(-0.5-3.5*i,segment(A
segment(point(-5.9,-3.5),point
29 translation(-3.5*i,segment(R,P))
segment(point(0.15080710930
```

x:-1.6071
y:-4.2107

3-d example.



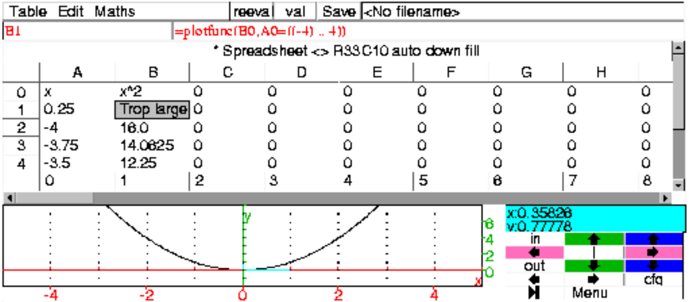
```
1 A:=cone([0,0,0],[0,0,1],pi/6,'color'
Done
2 P:=plan([0,2,0],[1,1,1],[1,0,0],
'color'=(vert+rempli))
pnt(pnt[hyperplan([2,1,-1],[0,2,0]
3 inter(A,P,
affichage=rouge+epaisseur_ligne_
Evaluation time: 1.35
group[pnt(pnt[curve(group[pnt[['
4
```

mouse plan -0.176x+-0.983y+-0.0482z=9.55

x:-15.383
y:-6.8604
z:-1.974

Spreadsheet.

- cells may have a symbolic value
- celles may have a graphic value.
- import/export with other modules
- but few formatting options



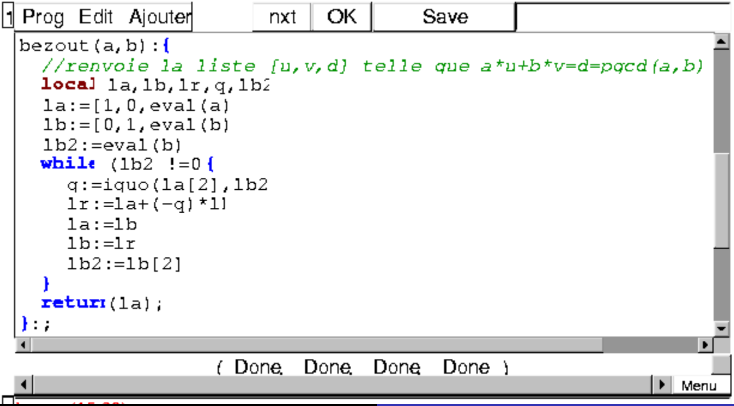
B1 =plotfunc(B0,A0=((-4) .. 4))

* Spreadsheet <> R33C10 auto down fill

	A	B	C	D	E	F	G	H	
0	x	x^2	0	0	0	0	0	0	0
1	0.25	Trop large	0	0	0	0	0	0	0
2	-4	16.0	0	0	0	0	0	0	0
3	-3.75	14.0625	0	0	0	0	0	0	0
4	-3.5	12.25	0	0	0	0	0	0	0
	0	1	2	3	4	5	6	7	8

Programmation

- interpreted language, not typed
- syntax choice: Xcas, maple, mupad, TI89.
- interactive debugger

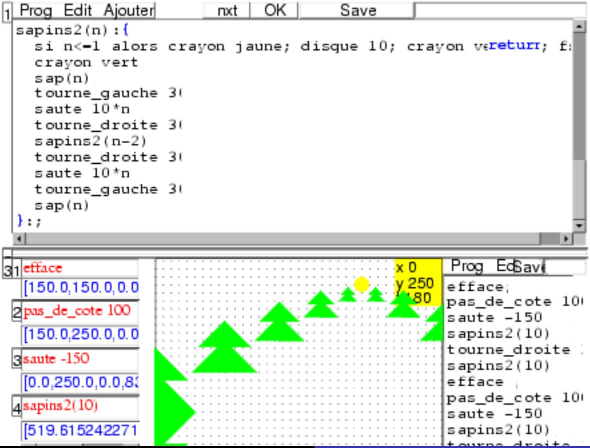


```
bezout(a,b):{
  //renvoie la liste [u,v,d] telle que a*u+b*v=d=pgcd(a,b)
  local la,lb,lr,q,lb2
  la:=[1,0,eval(a)
  lb:=[0,1,eval(b)
  lb2:=eval(b)
  while (lb2 !=0 {
    q:=iquo(la[2],lb2
    lr:=la+(-q)*l
    la:=lb
    lb:=lr
    lb2:=lb[2]
  }
  return(la);
}:;
```

(Done, Done, Done, Done)

Logo

- programming language for primary school
- to test with childrens



```
sapins2(n):{
  si n<=1 alors crayon jaune; disque 10; crayon vereturr; f
  crayon vert
  sap(n)
  tourne_gauche 3(
  saute 10*n
  tourne_droite 3(
  sapins2(n-2)
  tourne_droite 3(
  saute 10*n
  tourne_gauche 3(
  sap(n)
};
```

```
1 effave
  [150.0,150.0,0.0
2 pas_de_cote 100
  [150.0,250.0,0.0
3 saute -150
  [0.0,250.0,0.0,83
4 sapins2(10)
  [519.615242271
```

```
efface;
pas_de_cote 10
saute -150
sapins2(10)
tourne_droite
sapins2(10)
efface
pas_de_cote 10
saute -150
sapins2(10)
tourne_droite
```

Session

- different kind of levels in a session (commandline, comment, spreadsheet, geometry, program, logo)
- organize your work (move, delete/copy/paste, group)
- save, import/export, print

Native part of the code

- About 110 000 + 30 000 lines of code (giac+xcas)
- Parser using bison/flex
- Generic type `gen`, anonymous union of several basic types (hardware floats/int or different kinds of pointers), cf. `giac.info`
- Containers from the standard C++ library (vector, map, ...), for example for polynomials (1-d dense, n-d sparse...)
- Math. algorithms (Cantor-Zassenhauss, Hermite, Yun, mrv, Risch, Gosper, modular, heuristic gcd...)
- Context pointer for multi-threading (not completed)
- Internationalization: gettext + native

Librairies

- GMP: long integers
- MPFR: multiprecision floats
- GSL: numeric algorithms in double precision
- PARI-GP: arithmetic
- NTL: 1-d polynomial factorization
- CoCoA: Groebner basis
- FLTK, FLVW, OpenGL: graphic interface

Performances/tests

- Natives: polynomial arithmetic product (like Trip), GCD (Fermat), ...
- From the libraries: GMP, MPFR, PARI (ifactor(2ˆ 128+1)), NTL (factorization), CoCoA (Groebner)
- Benchmarks Lewis-Wester, Fateman, Zimmermann collection of polynomial to factor, ...
- Small basis of regression tests, including geometry theorems to be proved using the CAS kernel.

Interfaces.

- PC (Linux, Windows) and Mac compatibility
- xcas (graphic), icas (terminal)
- xcas online
- texmacs (via icas)
- emacs (icas+mupad mode)
- PDA: familiar linux, qdcas/StatsNow wince
- LaTeX and mathml export
- Computing with Giac inside a LaTeX document

Programming with Giac

Independant computation kernel as a C++ library:

- C++ programs may use the libgiac
- C++ modules may be loaded at runtime inside a Giac/Xcas session
- PHP/Flash module (Facilimaths)
- probably other languages via SWIG

Abstract

- Maths for education and research
- Compatibility, performances
- Many way to interface

- Roadmap
  - interfaces (more librairies, e.g. Linbox, Atlas, GetFEM++, integration inside SAGE, linux distributions, browser...)
  - more maths algorithms and regression tests, multi-threading