Problem of Coleman-Mazur on p-adic families of L-functions

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Abstract

For a prime number $p \geq 5$, consider a primitive cusp eigenform $f = f_k$ of weight $k \geq 2$, $f = \sum_{n=1}^{\infty} a_n q^n$, and consider a family of cusp eigenforms $f_{k'}$ of weight $k' \geq 2$, $k' \mapsto f_{k'} = \sum_{n=1}^{\infty} a_n(k') q^n$, containing f for $k' = k$, such that the Fourier coefficients $a_n(k')$ are given by certain p-adic analytic functions $k' \mapsto a_n(k')$ for $(n, p) = 1$, and let $\alpha_p(k')$ be a Satake p-parameter of $f_{k'}$.

In "The Eigencurve" (1998), R.Coleman and B.Mazur stated the following problem:

Given a prime p and a family $\{f_{k'}\}$ of cusp eigenforms of a fixed positive slope $\sigma = \mathrm{ord}_p(\alpha_p(k')) > 0$, to construct a two variable p-adic L-function interpolating on all k' the Amice-Vélu p-adic L-functions $L_p(f_{k'})$ studied in [Am-Ve] , [Vi76] and in [MTT].

A solution (2003, see [PaTV]) is described using the Rankin-Selberg method and the theory of p-adic integration with values in a p-adic Banach algebra $\mathcal{A}$.

*A talk held in Kyoto on September 22, 2005, 10:30–12:00.

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Our p-adic L-functions are Mellin transforms of certain measures with values in $\mathcal{A}$. We construct such measures from products of classical Eisenstein series, which produce distributions with values in certain Banach $\mathcal{A}$-modules $\mathcal{M} = \mathcal{M}(N; \mathcal{A})$ of modular forms with coefficients in the algebra $\mathcal{A}$.

Another approach, based on modular symbols, was developed by Glenn Stevens. Applications of these results to the p-adic Birch and Swinnerton-Dyer conjecture were discussed by P.Colmez (Bourbaki talk, June 2003, [Colm03]).

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0 Statement of the problem of Coleman-Mazur

This talk is about the paper [PaTV] by A.P., *Two variable p-adic L functions attached to eigenfamilies of positive slope*, Invent. Math. v. 154, N3 (2003), pp. 551 - 615.

The Tate field $\mathbb{C}_p$

Fix a prime p, and let $\mathbb{C}_p = \widehat{\overline{\mathbb{Q}}}_p$ be the Tate field (the completion of the field of p-adic numbers)

We fix an embedding $i_p : \overline{\mathbb{Q}} \to \mathbb{C}_p$, and view algebraic numbers as p-adic numbers via i_p.

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A primitive cusp eigenform f

$$f = f_k = \sum_{n\geq 1} a_n q^n \in \mathcal{S}_k(\Gamma_0(N), \psi),$$

(where $q = e(z) = \exp(2\pi i z)$, $\mathrm{Im}(z) > 0$)

a primitive cusp eigenform $f = f_k$ of weight $k \geq 2$ for $\Gamma_0(N)$ with a Dirichlet character $\psi \pmod N$.

The special values of the L-function attached to f at $s = 1, \cdots, k-1$:

$$L_f(s, \chi) = \sum_{n\geq 1} \chi(n) a_n n^{-s},$$

(χ are Dirichlet characters)

where $1 - a_p X + \psi(p) p^{k-1} X^2 = (1 - \alpha X)(1 - \alpha' X)$ is the Hecke polynomial

α and α' are called the Satake parameters of f

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Periods of f

Following a known theorem of Manin [Ma73], there exist two non-zero complex constants $c^+(f), c^-(f) \in \mathbb{C}^\times$ (the *periods* of f) such that for all $s = 1, \cdots, k-1$ and for all Dirichlet characters χ of fixed parity, $(-1)^{k-s}\chi(-1) = \pm 1$, the normalized special values are *algebraic numbers*:

$$L^*_f(s,\chi) = \frac{(2i\pi)^{-s}\Gamma(s)L_f(s,\chi)}{c^\pm(f)} \in \overline{\mathbb{Q}}. \tag{0.1}$$

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A family of slope $\sigma > 0$ of cusp eigenforms $f_{k'}$ of weight $k' \geq 2$ containing f

$$k' \mapsto f_{k'} = \sum_{n=1}^{\infty} a_n(k')q^n \in \overline{\mathbb{Q}}[\![q]\!] \subset \mathbb{C}_p[\![q]\!]$$

1) the Fourier coefficients $a_n(k')$ of $f_{k'}$ and the Satake p-parameter $\alpha_p(k')$ are given by certain p-adic analytic functions $k' \mapsto a_n(k')$ for $(n,p) = 1$

2) the slope is constant and positive: $\mathrm{ord}(\alpha_p(k')) = \sigma > 0$

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A model example of a p-adic family (not cusp and $\sigma = 0$): Eisenstein series

$$a_n = \sum_{d|n} d^{k'-1}, f_{k'} = E_{k'}$$

the Fourier coefficients $a_n(k')$
and one of the Satake p-parameters
$\alpha_p(k') = 1$
are p-adic analytic functions,
and $\text{ord}_p(\alpha_p(k')) = \text{ord}_p(1) = 0$

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The existence of families of slope $\sigma > 0$: R.Coleman, [CoPB]

He gave an example with
$p = 7,\ f = \Delta,\ k = 12$
$a_7 = \tau(7) = -7 \cdot 2392, \sigma = 1,$
and a program in PARI for computing such families is contained in [CST98] (see also the Web-page of W.Stein, `http://modular.fas.harvard.edu/`)

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The Problem, ***see [Co-Ma] R. Coleman, B. Mazur, The eigencurve. Galois representations in arithmetic algebraic geometry, (Durham, 1996), London Math. Soc. Lecture Note Ser., 254, at p.6***

Given a p-adic analytic family $k' \mapsto f_{k'} = \sum_{n=1}^{\infty} a_n(k')q^n \in \overline{\mathbb{Q}}[\![q]\!]$ of positive slope $\sigma > 0$, to construct a two-variable p-adic L-function interpolating $L^*_{f_{k'}}(s, \chi)$ on (s, k').

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Known cases:

- One-variable case ($k = k'$ is fixed, $\sigma > 0$), treated in [Am-Ve] by Y. Amice, J. Vélu, in [Vi76] by M.M. Višik, and in [MTT] by B. Mazur; J. Tate; J. Teitelbaum
- $\sigma = 0$ (H.Hida) ("ordinary families") (see in [Hi93])

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• Special values of L-functions attached to families f_k of Yu.I. Manin and M. M.Vishik, [Ma-Vi] : $f_k = \sum_{\mathfrak{a} \subset \mathcal{O}_K} \lambda^{k-1}(\mathfrak{a}) q^{N\mathfrak{a}}$ and of N.M.Katz, [Kat]), which are are certain ordinary families

they correspond to powers of a grössen-character λ of an imaginary quadratic field K at a *splitting prime* p, (resp. to grössencharacters of type A_0 of the idèle class group $\mathbb{A}_K^*/K^*$ (in the sense of Weil [We56],) of a CM-field K.

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Motivation comes from the conjecture of Birch and Swinnerton-Dyer, see in [Colm03] , Colmez, P.: La conjecture de Birch et Swinnerton-Dyer p-adique. Séminaire Bourbaki. [Exposé N°.919] (Juin 2003). For a cusp eigenform $f = f_2$, corresponding to an elliptic curve E by Wiles [Wi], we consider a family containing f.

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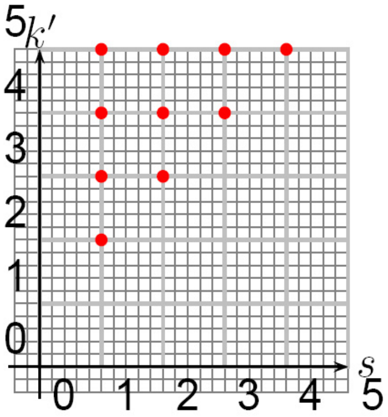
One can try to approach $k = 2, s = 1$ from the other direction, taking $k' \to 2$ instead of $s \to 1$, this leads to a formula linking the derivative over s at $s = 1$ of the p-adic L-function with the derivative over k' at $k' = 2$ of the p-adic analytic function $\alpha_p(k')$, see in [CST98]:

$$\boxed{L'_{p,f}(1) = \mathcal{L}_p(f) L_{p,f}(1)}$$

with $\mathcal{L}_p(f) = -2 \frac{d\alpha_p(k')}{dk'}\Big|_{k'=2}$



The validity of this formula needs the existence of our two variable L-function!

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Our method

is a combination of the Rankin-Selberg method, the theory of p-adic integration with values in p-adic Banach algebras $\mathcal{A}$ and the spectral theory of Atkin's U-operator: $U = U_p : \mathcal{A}[\![q]\!] \to \mathcal{A}[\![q]\!]$ defined by:

$$U\left(\sum_{n\geq 1} a_n q^n\right) = \sum_{n\geq 1} a_{pn} q^n \in \mathcal{A}[\![q]\!].$$

Here $\mathcal{A} = \mathcal{A}(\mathcal{B})$ is a certain p-adic Banach algebra of functions on an open analytic subspace $\mathcal{B} \subset X$ of the weight space $X = \mathrm{Hom}_{cont}(Y, \mathbb{C}_p^*)$. This is an *analytic space over* $\mathbb{C}_p$, which consists of all continuous characters of a profinite group $Y \cong (\mathbb{Z}/N\mathbb{Z})^* \times \mathbb{Z}_p^*$.

The classical analogue of the weight space is the whole complex plane

$$\mathbb{C} = \mathrm{Hom}_{cont}(\mathbb{R}_+^*, \mathbb{C}^*), s \mapsto (y \mapsto y^s).$$

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The weights k' correspond to certain points in $\mathcal{B} \subset X$. Any series $f = \sum_{n\geq 1} a_n q^n \in \mathcal{A}[\![q]\!]$ produces a family of q-expansions $\{f_{k'} = ev_{k'}(f) = \sum_{n\geq 1} ev_{k'}(a_n) q^n \in \mathbb{C}_p[\![q]\!]\}$, which can be classical modular forms in $\overline{\mathbb{Q}}[\![q]\!]$.

• We construct first an analytic function $\mathcal{L}_\mu : X \to \mathcal{A} = \mathcal{A}(\mathcal{B})$ as the Mellin transform

$\mathcal{L}_\mu(x) = \int_Y x(y)\, d\mu(y)$ (where $x \in X = \mathrm{Hom}_{cont}(Y, \mathbb{C}_p^*), x = x(y)$),

μ is a certain measure with values in $\mathcal{A}$, on the profinite group Y.

• For each $s \in \mathcal{B} \subset X$, there is the evaluation homomorphism $ev_s : \mathcal{A}(\mathcal{B}) \to \mathbb{C}_p$; we obtain $\mathcal{L}_\mu(x, s)$ by evaluation of an $\mathcal{A}$-valued integral:

$$\mathcal{L}_\mu(x, s) = \mathcal{L}_\mu(x)(s) = ev_s\left(\int_Y x d\mu\right) \quad (x \in X, \quad \mathcal{L}_\mu(x) \in \mathcal{A}).$$

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This gives a p-adic analytic L-function in two variables $(x, s) \in X \times \mathcal{B} \subset X \times X$:

$$(x, s) \longmapsto \mathcal{L}_\mu(x, s).$$

• We check an equality relating the algebraic numbers $L^*_{f_{k'}}(s, \chi)$ $(s = 1, \cdots, k' - 1)$ with the values $\mathcal{L}_\mu(x, k')$ at certain points $x \in X$ (more precisely, at $x = \chi \cdot y_p^{k'}$).

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1 p-adic integration and the p-adic weight space

Consider the group

$Y = \varprojlim_v (\mathbb{Z}/Np^v\mathbb{Z})^\times \cong (\mathbb{Z}/N\mathbb{Z})^\times \times \mathbb{Z}_p^\times$ (a profinite group with a projection $y_p : Y \to \mathbb{Z}_p^\times$)

and the group of p-adic characters

$X = X_N = \mathrm{Hom}_{\mathrm{cont}}(Y, \mathbb{C}_p^\times) \ni \chi, y_p^n$, (the p-adic weight space, which is a $\mathbb{C}_p$-analytic group)

where

$\chi \bmod Np^v\mathbb{Z} : (\mathbb{Z}/Np^v\mathbb{Z})^\times \to \mathbb{C}_p^\times$ (a Dirichlet character)

$y_p : Y \to \mathbb{Z}_p^\times$ (the canonical projection, a p-adic character of Y)

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The analytic structure on $X = X_N = \mathrm{Hom}_{\mathrm{cont}}(Y, \mathbb{C}_p^\times)$ over $\mathbb{C}_p$ is given by the decomposition:

$$X \xrightarrow{\sim} \mathrm{Hom}((\mathbb{Z}/Np\mathbb{Z})^\times, \mathbb{C}_p^\times) \times \mathrm{Hom}_{\mathrm{cont}}(\Gamma, \mathbb{C}_p^\times)$$

where $Y \cong (\mathbb{Z}/Np\mathbb{Z})^\times \times \Gamma$, $\Gamma = (1 + p\mathbb{Z}_p)^\times$, is a procyclic group of generator $\gamma = 1 + p$, and we see that X is a finite cover of the p-adic unit disc:

$$X \twoheadrightarrow \mathrm{Hom}_{\mathrm{cont}}(\Gamma, \mathbb{C}_p^\times) \xrightarrow{\sim} \mathfrak{U} = \{t \in \mathbb{C}_p \mid |t-1|_p < 1\} \cong \{\chi_t : \gamma \mapsto t \mid t \in \mathfrak{U}\}.$$

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Notation

$(k,\psi) = y_p^k \psi \in X$	is a point on the weight space X (we write simply k)
$\mathcal{A}$	(a p-adic Banach algebra)
V	(an $\mathcal{A}$-module)
$\mathcal{C}(Y,\mathcal{A})$	(the $\mathcal{A}$-Banach algebra
$\cup$	of *continuous functions* on Y)
$\mathcal{C}^{loc-const}(Y,\mathcal{A})$	(the $\mathcal{A}$-algebra
	of *locally constant functions* on Y)

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Definition 1.1 *a) A distribution $\mathcal{D}$ on Y with values in V is an $\mathcal{A}$-linear form*

$$\mathcal{D} : \mathcal{C}^{loc-const}(Y,\mathcal{A}) \to V, \quad \varphi \mapsto \mathcal{D}(\varphi) = \int_Y \varphi d\mathcal{D}.$$

b) A measure μ on Y with values in V is a continuous $\mathcal{A}$-linear form

$$\mu : \mathcal{C}(Y,\mathcal{A}) \to V, \quad \varphi \mapsto \int_Y \varphi d\mu.$$

The integral $\int_Y \varphi d\mu$ can be defined for any continuous function φ, and any bounded distribution μ, using the Riemann sums.

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Admissible measures of Amice-Vélu

A more delicate notion of an h-admissible measure was introduced in [Am-Ve] by Y. Amice, J. Vélu (see also [MTT], [Vi76]):

DEFINITION 1.2

a) For $h \in \mathbb{N}, h \geq 1$ let $\mathcal{P}^h(Y, \mathcal{A})$ denote the $\mathcal{A}$-module of locally polynomial functions of degree $< h$ of the variable $y_p : Y \to \mathbb{Z}_p^\times \hookrightarrow \mathcal{A}^\times$; in particular,

$$\mathcal{P}^1(Y, \mathcal{A}) = \mathcal{C}^{loc-const}(Y, \mathcal{A})$$

(the $\mathcal{A}$-submodule of locally constant functions). Let also denote $\mathcal{C}^{loc-an}(Y, \mathcal{A})$ the $\mathcal{A}$-module of locally analytic functions, so that

$$\mathcal{P}^1(Y, \mathcal{A}) \subset \mathcal{P}^h(Y, \mathcal{A}) \subset \mathcal{C}^{loc-an}(Y, \mathcal{A}) \subset \mathcal{C}(Y, \mathcal{A}).$$

b) Let V be a normed $\mathcal{A}$-module with the norm $|\cdot|_{p,V}$. For a given positive integer h an h-admissible measure on Y with values in

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V is an $\mathcal{A}$-module homomorphism

$$\tilde{\Phi} : \mathcal{P}^h(Y, \mathcal{A}) \to V$$

such that for fixed $a \in Y$ and for $v \to \infty$ the following growth condition is satisfied:

$$\left| \int_{a+(Np^v)} (y_p - a_p)^{h'} d\tilde{\Phi} \right|_{p,V} = o(p^{-v(h'-h)}) \qquad (1.1)$$

for all $h' = 0, 1, \ldots, h-1, a_p := y_p(a)$

The condition (1.1) allows to integrate only the locally-analytic functions: there exists a unique extension of $\tilde{\Phi}$ to $\mathcal{C}^{loc-an}(Y, \mathcal{A}) \to V$ (via the embedding $\mathcal{P}^h(Y, \mathcal{A}) \subset \mathcal{C}^{loc-an}(Y, \mathcal{A})$). The integral is defined using generalized Riemann sums: take the beginning of the Taylor expansion of a locally-analytic function $\phi \in \mathcal{C}^{loc-an}(Y, \mathcal{A})$ (of order $h-1$) instead of just values of a function ϕ.

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The p-adic Mellin transform and two variable p-adic analytic functions

Any h-admissible measure $\tilde{\mu}$ on Y with values in a p-adic Banach algebra $\mathcal{A}$ can be caracterized by the logarithmic growth $o(\log^h(\cdot))$ of its *Mellin transform* $\mathcal{L}_{\tilde{\mu}}(x)$ (see [Am-Ve], [Vi76], [HaH]):

$$\mathcal{L}_{\tilde{\mu}} : X \to \mathcal{A}, \text{ defined by } \mathcal{L}_{\tilde{\mu}}(x) = \int_Y x(y) d\tilde{\mu}(y),$$

where $x \in X,\ \mathcal{L}_{\tilde{\mu}}(x) \in \mathcal{A},\ X \subset \mathcal{C}^{loc-an}(Y, \mathcal{A})^\times$

Key property of h-admissible measures $\tilde{\mu}$: its Mellin transform $\mathcal{L}_{\tilde{\mu}}$ is analytic with values in $\mathcal{A}$.

Then we obtain the function $\mathcal{L}_\mu(x, s)$ by evaluation at (s, ψ): this is a p-adic analytic function in two variables $(x, s) \in X \times \mathcal{B} \subset X \times X$:

$$\mathcal{L}_{\tilde{\mu}}(x, s) = ev_s(\mathcal{L}_{\tilde{\mu}}) \quad (x \in X, \quad \mathcal{L}_{\tilde{\mu}}(x) \in \mathcal{A}).$$

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EXAMPLE 1.3 ([AM-VE], [MTT], [VI76]) *For a primitive cusp eigenform* $f = f_k = \sum_{n\geq 1} a_n q^n \in \mathcal{S}_k(\Gamma_0(N), \psi)$ *of weight* $k \geq 2$ *for* $\Gamma_0(N)$ *with a Dirichlet character* ψ *and positive slope* $\sigma = \mathrm{ord}_p(\alpha)$ *define the integer* $h = [\sigma] + 1$ *(where* $\sigma < k - 1$*, and* $1 - a_p X + \psi(p)p^{k-1}X^2 = (1 - \alpha X)(1 - \alpha' X)$ *as above).*

Then there exists an h*-admissible* $\mathbb{C}_p$*-valued measure* $\tilde{\mu} = \tilde{\mu}_{\alpha,f}(y)$ *on* Y *such that for all couples* (j, χ) *with* $0 \leq j \leq k - 2$*, and for any nontrivial primitive Dirichlet character* $\chi \bmod p^v$ *satisfying* $\chi\xi(-1) = (-1)^{k-1-j}$*, there is the following equality (in* $\mathbb{C}_p$*):*

$$\int_Y \chi(y)\, y_p^j\, d\tilde{\mu} = i_p\left(\frac{p^{vj}G(\chi)}{\alpha^v} L_f^*(1 + j, \bar{\chi})\right) \quad \left(= \mathcal{L}_{\tilde{\mu}}(\chi\, y_p^j)\right), \quad (1.2)$$

where $G(\chi)$ *is the Gauss sum of the character* $\chi \bmod p^v$*, and* $L_f^*(1 + j, \bar{\chi})$ *is given by a choice of periods (0.1). In other words, the complex L-values (1.2) attached to* f *coincide with the values* $\mathcal{L}_{\tilde{\mu}}(\chi\, y_p^j)$ *of the p-adic Mellin transform of* $\tilde{\mu}$*.*

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2 Coleman's families

The proof of the existence of families of slope $\sigma > 0$ by R.Coleman, [CoPB], uses the following ideas:

Notation

$[K : \mathbb{Q}_p] < \infty$	– a finite extension of $\mathbb{Q}_p$ contaning all the Fourier coefficients $i_p(a_n)$ of f
$\mathcal{A} = \mathcal{A}_K(\mathcal{B})$	– *the K-Banach algebra of rigid-analytic functions*

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$ev_{k'} : \mathcal{A} \to K$	– the evaluation map defined for all $(k', \psi) \in \mathcal{B}$ (a neigbourhood around $(k, \psi) \in X$).
$\mathcal{M}^\dagger(N; \mathcal{A}) = \bigcup_{v \geq 1} \mathcal{M}^\dagger(Np^v, \psi; \mathcal{A}) \subset \mathcal{A}[\![q]\!]$	– a Banach $\mathcal{A}$-module of overconvergent families of modular forms: this module is generated by some $g = \sum_{n=1}^\infty b_n q^n \in \mathcal{A}[\![q]\!]$ such that $ev_{k'}(g) \in K[\![q]\!]$ are classical cusp eigenforms for all k' with (k', ψ) *in a neighbourhood* $\mathcal{B}$ of $(k, \psi) \in X$.

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Coleman proved:

- The operator U acts as a completely continuous operator on each $\mathcal{A}$-submodule $\mathcal{M}^\dagger(Np^v;\mathcal{A}) \subset \mathcal{A}[\![q]\!]$ (i.e. U is a limit of finite-dimensional operators)

$\Longrightarrow$ there exists the Fredholm determinant $P_U(T) = \det(Id - T\cdot U) \in \mathcal{A}[\![T]\!]$

- there is a version of the Riesz theory: for any inverse root $\alpha \in \mathcal{A}^*$ of $P_U(T)$ there exists an eigenfunction g, $Ug = \alpha g$

such that $ev_{k'}(g) \in K[\![q]\!]$ are classical cusp eigenforms for all k' such that (k', ψ) is in a neigbourhood $\mathcal{B}$ around $(k,\psi) \in X$ (see in [CoPB])

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DEFINITION 2.1

a) *A function $g \in \mathcal{M}^\dagger(Np^v;\mathcal{A}) \subset \mathcal{A}[\![q]\!]$ is called Coleman's family if $Ug = \alpha g$, and the functions $ev_{k'}(g) \in K[\![q]\!]$ are cusp eigenforms for all k' such that (k',ψ) is in a neigbourhood $\mathcal{B}$ around (k,ψ) in the p-adic weight space X, and $\mathrm{ord}_p(\alpha(k')) = \sigma > 0$ is constant and positive, where $\alpha(k') = ev_{k'}(\alpha) \in K \cap i_p(\overline{\mathbb{Q}})$*

b) *Let $f_{k'} \in \overline{\mathbb{Q}}[\![q]\!]$ denote the primitive cusp eigenform attached to $ev_{k'}(g) \in K[\![q]\!]$. Then the family $\{f_{k'}\}$ of classical primitive cusp forms is also called Coleman's family.*

REMARK 2.2 *Hida's families correspond to $\sigma = 0$, they were constructed in [Hi86] (see also [Hi93]).*

There exist analogues of Hida's families in the Siegel modular case (see [Bue], [Hi04]).

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In the ordinary case such p-adic families of Siegel modular forms were studied by K.Buecker (Dissertation of Cambridge University, UK, 1994, under the direction of R. Taylor, see in [Bue]), and by J.Tilouine and E.Urban [Ti-U]. A more general approach is developed in new Hida's book [Hi04].

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3 Main results

MAIN THEOREM 3.1 *Consider a nonzero analytic function* $\alpha = \alpha(s) \in \mathcal{A}^\times$ *defined in a neigbourhood* $\mathcal{B}$ *of* $(k, \psi) \in X$*, and consider Coleman's family*

$$f = \left\{ f_{k'} = \sum_{n=1}^{\infty} a_n(k')q^n \right\} \in \mathcal{A}[\![q]\!]$$

(with coefficients in the algebra $\mathcal{A} = \mathcal{A}(\mathcal{B})$*)* $\alpha \in \mathcal{A}^\times$ *is the corresponding eigenvalue of* U*. Suppose that the slope* $\mathrm{ord}_p(\alpha) = \sigma > 0$ *is fixed for all* $\alpha = \alpha(k')$ *with* (k', ψ) *in* $\mathcal{B}$*, and define the integer* $h = [\sigma] + 1$*.*

Then there exists an h*-admissible measure* $\tilde{\mu} = \mu_{\alpha,f}$ *with values in* $\mathcal{A}$ *on the group* Y*, determind by the following property: for all couples* (j, χ) *with* $0 \le j \le k' - 2$*,* $k' > 2\sigma + 2$*, any primitive Dirichlet character* $\chi \bmod p^v$ *satisfying* $\chi\xi(-1) = (-1)^{k'-1-j}$*, there*

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is the following equality

$$ev_{k'}\left(\int_Y \chi(y)\, y_p^j\, d\tilde{\mu}\right) = i_p\left(R_{k'}\cdot\frac{p^{vj}G(\chi)}{\alpha_p(k')^v}L^*_{f_{k'}}(1+j,\bar{\chi}),\right) \quad (3.1)$$

where $G(\chi)$ *is the Gauss sums of* $\chi \bmod p^v$, *and* $R_{k'} \in \mathbb{Q}^\times$ *is an elementary factor coming from an explicit choice of periods* $c^\pm(f_{k'})$.

The choice of periods:

$$c^\pm(f_{k'}) = \frac{(-2i\pi)^{k'-1}\langle f_{k'}, f_{k'}\rangle_{Np}}{\Gamma(k'-1)L_{f_{k'}}(k'-1,\bar{\xi})}, \text{ where } \xi(-1) = \pm(-1)^j. \quad (3.2)$$

Recall that by [Ra52], [Za77] and [Sh77], the numbers

$\dfrac{L_f(1+j,\bar{\chi})L_f(k'-1,\bar{\xi})}{\pi^{k'+r}\langle f_{k'}, f_{k'}\rangle_{Np}}$ are algebraic for all $j \in \mathbb{Z}$ with $0 \le j \le k'-2$, $\chi\xi(-1) = (-1)^{k'-1-j}$ (here $\langle f_{k'}, f_{k'}\rangle_{Np}$ denotes the Petersson scalar product.

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A key ingredient in our construction is the use of a linear form

$$\ell_{\alpha(k)} : \mathcal{M}_k(Np, \psi, \overline{\mathbb{Q}}) \to \overline{\mathbb{Q}},$$

such that
$\alpha(k) \in \mathbb{Q}^\times$, $\ell_{\alpha(k)}(U_p h) = \alpha\ell_{\alpha(k)}(h)$ for all $h \in \mathcal{M}_k(Np, \psi, \overline{\mathbb{Q}})$, and $1 - a_p X + \psi(p)p^{k-1}X^2 = (1-\alpha(k)X)(1-\alpha(k)'X)$ for a primitive cusp eigenform $f = \sum_{n=1}^{\infty} a_n q^n \in \mathcal{S}_k(\Gamma_0(N), \psi, \overline{\mathbb{Q}})$ of weight $k \ge 2$ for $\Gamma_0(N)$ with a Dirichlet character $\psi \pmod N$. One can define such linear form by $\ell_\alpha : h \longmapsto \dfrac{\langle f^0, h\rangle}{\langle f^0, f_0\rangle}$, where
f_0 is an eigenfunction of U_p: $f_0|U_p = \alpha(k')f_0$, and
f^0 is the corresponding eigenfunction of U_p^*: $f^0|U_p^* = \overline{\alpha(k)}f^0$,

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$$f_0 = \sum_{n\geq 1} a_n q^n - \alpha \sum_{n\geq 1} a_n q^{pn} = \sum_{n\geq 1} a(f_0, n) q^n \in \mathcal{S}_k(\Gamma_0(Np), \psi, \overline{\mathbb{Q}}), \text{ and}$$

$$f^0 = f_0^\rho\Big|_k \begin{pmatrix} 0 & -1 \\ Np & 0 \end{pmatrix}, \quad f_0^\rho = \sum_{n\geq 1} \overline{a}(f_0, n) q^n \in \mathcal{S}_k(\Gamma_0(Np), \overline{\psi}, \overline{\mathbb{Q}})$$

is an eigenfunction of the adjoint operator U_p^*, and the ratio $\langle f^0, f_0\rangle / \langle f, f\rangle \in \overline{\mathbb{Q}}^\times$ is explicitely computed in [Go-Ro].

An answer to the question of Coleman–Mazur is given by the function (3.3) of the following theorem:

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THEOREM 3.2 *Under the assumptions and notations of Theorem 3.2 there exists a unique p-adic analytic function on $X \times \mathcal{B}$ (of two variables x, s),*

$$\mathcal{L}_{\alpha,f}(\cdot_1, \cdot_2, \xi, f) : X \times \mathcal{B} \to \mathbb{C}_p \qquad (3.3)$$

such that

i) for any fixed $(s, \psi) \in \mathcal{B}$, the function $\mathcal{L}_{\alpha,f}(x, s; \xi, f)$ of the variable x is $\mathbb{C}_p$-analytic and has the logarithmic growth $o(\log^h(x))$,

ii) for each couple (χ, j) with $0 \leq j \leq k' - 2$, $k' > 2\sigma + 2$ and any primitive Dirichlet character $\chi \bmod p^v \in X^{\mathrm{tors}}$ with values in $K^\times$ satisfying $v \geq 2$, $\chi\xi(-1) = (-1)^{k'-1-j}$, the special value $\mathcal{L}(\chi y_p^j, k'; \xi, f_{k'})$ is given by the image under i_p of the algebraic number $R_{k'} \cdot \dfrac{p^{vj} G(\chi)}{\alpha_p(k')^v} L^*_{f_{k'}}(1 + j, \bar{\chi})$, where $G(\chi)$ is the Gauss sums of $\chi \bmod p^v$, and $R_{k'} \in \mathbb{Q}^\times$ is an elementary factor given by the explicit choice of periods $c^\pm(f_{k'})$, as in (3.2).

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4 Construction of the admissible measure $\tilde{\mu}$

Recall that by Definition 1.2, an *h-admissible measure* on a profinite group Y with values in an $\mathcal{A}$-module V is given as an $\mathcal{A}$-module homomorphism

$$\tilde{\mu} : \mathcal{P}^h(Y, \mathcal{A}) \to V,$$

satisfying a certain growth condition (1.1).

This means that $\tilde{\mu}$ is given by a sequence $\{\mu_j\}$ of certain distributions on Y, in such a way that for $j = 0, 1, \cdots, h-1$ and for all compact open subsets $U \subset Y$ one has

$$\int_U y_p^j d\tilde{\mu} = \mu_j(U). \tag{4.1}$$

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The growth condition (1.1) has the form: for $t = 0, 1, \cdots, h-1$

$$\left| \int_{a+(Np^v)} (y_p - a_p)^t d\tilde{\mu} \right|_p \tag{4.2}$$

$$= \left| \sum_{j=0}^{t} \binom{t}{j} (-a_p)^{t-j} \mu_j(a + (Np^v)) \right|_p = o(p^{v(h-t)}) \text{ for } v \to \infty.$$

We construct $\{\mu_j\}$ out of the algebraic special values $L^*_{f_{k'}}(1+j, \chi)$ in such a way that the equality (3.1) of the Main Theorem 3.1 is satisfied:

$$ev_{k'} \left(\int_Y \chi(y)\, y_p^j \, d\mu_\alpha(y; f) \right) = i_p \left(R_{k'} \cdot \frac{p^{vj} G(\chi)}{\alpha_p(k')^v} L^*_{f_{k'}}(1+j, \bar{\chi}), \right)$$

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We construct the sequence μ_j as follows:

$$\mu_j = \ell_\alpha(\pi_\alpha(\Phi_j)), \quad (j = 0, 1, \cdots, k-2)$$

- Φ_j is a sequence of modular distributions on Y with values in an $\mathcal{A}$-module $\mathcal{M} = \mathcal{M}_N(\psi; \mathcal{A})$ of of modular forms with coefficients in $\mathcal{A}$ (it has INFINITE RANK):

$$\mathcal{M}_N(\psi; \mathcal{A}) := \bigcup_{v \geq 0} \mathcal{M}(Np^v, \psi; \mathcal{A}),$$

(*the modular forms* $\Phi_j(\chi)$ *are products of certain classical Eisenstein series in* $\mathcal{A}[\![q]\!]$)

- π_α is the canonical projector onto the characteristic $\mathcal{A}$-submodule $\mathcal{M}^\alpha = \mathcal{M}^\alpha(\mathcal{A})$ of Atkin's operator $U\left(\sum_{n\geq 0} b_n q^n\right) = \sum_{n \geq 0} b_{pn} q^n$

(KEY POINT: THE $\mathcal{A}$-MODULE $\mathcal{M}^\alpha(\mathcal{A})$ IS LOCALLY FREE OF FINITE RANK)

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- $\ell_\alpha \in \mathrm{Hom}_{\mathcal{A}}(\mathcal{M}^\alpha, \mathcal{A})$ is a $\mathcal{A}$-linear form (given by the Petersson scalar product with $h \in \mathcal{M}^\alpha$, as in Section 3: $h \longmapsto \dfrac{\langle f^0, h\rangle}{\langle f^0, f_0\rangle}$, normalized by the equality $\ell_\alpha(g) = 1$ for Coleman's eigenfunction $g = f_0 \in \mathcal{M}^\alpha$.

5 Criterion of admissibility

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THEOREM 5.1 *Let* $0 < |\alpha|_p < 1$ *and Suppose that there exists a positive integer* $\varkappa$ *such that the following conditions are satisfied: for all* $r = 0, 1, \cdots, h-1$ *with* $h = [\varkappa \mathrm{ord}_p \alpha] + 1$*, and* $v \geq 1$*,*

$$\Phi_r(a + (Np^v)) \in \mathcal{M}(Np^{\varkappa v}) \text{(the level condition)} \tag{5.1}$$

and the following p*-adic congruence holds: for all* $w \geq \max(\varkappa v, 1)$ *and for all* $t = 0, 1, \cdots, \varkappa h - 1$

$$U^w \sum_{r=0}^{t} \binom{t}{r} (-a_p)^{t-r} \Phi_r(a + (Np^v)) \equiv 0 \bmod p^{-vt} \tag{5.2}$$

(the divisibility condition)

Then the linear form given by $\tilde{\Phi}^\alpha(\delta_{a+(Np^v)} y_p^r) := \pi_\alpha(\Phi_r(a + (Np^v))$ *on local monomials (for all* $j = 0, 1, \cdots, h-1$*), is an* h*-admissible measure:* $\tilde{\Phi}^\alpha : \mathcal{P}^h(Y, \overline{\mathbb{Q}}) \to \mathcal{M}^\alpha \subset \mathcal{M}$

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Proof uses the commutative diagram:

$$\begin{array}{ccccc} \mathcal{M}^\dagger(Np^{v+1}, \psi; \mathcal{A}) & \xrightarrow{\pi_{\alpha,v}} & \mathcal{M}^{\dagger\alpha}(Np^{v+1}, \psi; \mathcal{A}) & & \\ U^v \big\downarrow & & \big\downarrow \wr\, U^v & & \\ \mathcal{M}^\dagger(Np, \psi; \mathcal{A}) & \xrightarrow[\pi_{\alpha,0}]{} & \mathcal{M}^{\dagger\alpha}(Np, \psi; \mathcal{A}) & = \mathcal{M}^{\dagger\alpha}(Np^{v+1}, \psi; \mathcal{A}). \end{array}$$

The existence of the projectors $\pi_{\alpha,v}$ comes from Coleman's Theorem A.4.3 [CoPB].

On the right: U acts on the locally free $\mathcal{A}$-module $\mathcal{M}^\alpha(Np^{v+1}, \mathcal{A})$

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via the matrix:

$$\begin{pmatrix} \alpha & \cdots & \cdots & * \\ 0 & \alpha & \cdots & * \\ 0 & 0 & \ddots & \cdots \\ 0 & 0 & \cdots & \alpha \end{pmatrix} \text{ where } \alpha \in \mathcal{A}^\times$$

$$\Longrightarrow U^v \text{ is an isomorphism over Frac}(\mathcal{A}),$$

and one controls the denominators of the modular forms of all levels v by the relation:

$$\pi_{\alpha,v}(h) = U^{-v}\pi_{\alpha,0}(U^v h) =: \pi_\alpha(h) \tag{5.3}$$

The equality (5.3) can be used as the definition of π_α. The growth condition (1.1) for $\pi_\alpha(\Phi_r)$ is deduced from the congruences (5.2) between modular forms, using the relation (5.3).

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6 Modular Eisenstein distributions Φ_j

Let us fix an auxiliary Dirichlet character $\xi \mod p$, $\xi(-1) = \pm 1$, and use the method of Rankin-Selberg for the convolution

$$D(s,f,g) = L_N(2s+2-k-l, \psi\overline{\xi\chi}) \sum_{n=1}^{\infty} a_n b_n n^{-s}, \text{ where} \tag{6.1}$$

$$b_n = \sigma_{l-1,\bar\chi,\bar\xi}(n) = \sum_{d|n, d>0} \bar\chi(d)\bar\xi(n/d)d^{l-1},$$

are the Fourier coefficients of an Eisenstein series $g = \sum_{n=0}^{\infty} b_n q^n$ of weight l (and of Dirichlet character $\bar\chi\bar\xi$) if $\chi\xi(-1) = (-1)^l$, so that

$$L_g(s) = \sum_{n=1}^{\infty} b_n n^{-s} = L(s-l+1, \bar\chi)L(s, \bar\xi).$$

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The Rankin lemma (cf. [Ra52]) expresses $D(s,f,g)$ through the function

$$L_f(s-l+1,\bar\chi)L_f(s,\bar\xi). \tag{6.2}$$

Let us define the modular distributions Φ_j on a profinite group $Y=\varprojlim_v(\mathbb{Z}/Mp^v\mathbb{Z})^\times$ (for some M, divisible by N) in such a way that the modular form $\Phi_j(\chi)\in\mathcal{A}[\![q]\!]$ is a product of two Eisenstein series with coefficients in $\mathcal{A}$:
$ev_{k'}(\Phi_j(\chi))=(-1)^jE_{k'-1-j}(\xi,\chi)E_{1+j}(\psi\overline{\xi\chi})=:\Phi_{j,k'}(\chi)$.

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Explicitely, the Fourier coefficients of Φ_j (for $j=0,\dots,k'-2$) are given by

$$\Phi_j(a+Mp^v) \tag{6.3}$$
$$=\sum_{b\in Y_{Mp^v}}\psi\bar\xi(b)\sum_{n\ge 0}^{\infty}\sum_{n_1+n_2=n}A_j(n_1,ab)_vB_j(n_2,b)_vq^n\in\mathcal{A}[\![q]\!],\ \text{where}$$

$$A_j(n_1,ab)_v(k')=\sum_{\substack{d_1|n_1\\(n_1/d_1)\equiv ab \bmod Mp^v}}\xi(d_1)\mathrm{sgn}\,(d_1)d_1^{k'-2-j} \tag{6.4}$$
$$B_j(n_2,b)_v(k')=\sum_{\substack{d_2|n_2\\d_2\equiv b \bmod Mp^v}}\mathrm{sgn}\,(d_2)(n_2/d_2)^j\quad\text{for}\quad n_2>0.$$

(Note that the last series has constant coefficients). One verifies coefficient-by-coefficient that the distributions Φ_j satisfy the level condition with $\varkappa=1$, and the divisibility condition (5.1), (5.2):

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MAIN CONGRUENCE: For all $w \geq w_0(v)$ one has

$$U^w \sum_{j=0}^{t} \binom{t}{j} (-a_p)^{t-j} \Phi_j(a + (Mp^v)) \tag{6.5}$$

$$= \sum_{j=0}^{t} \binom{t}{j} (-a_p)^{t-j} \sum_{n\geq 0}^{\infty} \sum_{n_1+n_2=p^w n} (-1)^j A_j(n_1, ab)_v B_j(n_2, b)_v q^n$$

$$\stackrel{?}{\equiv} 0 (\mathrm{mod} p^{tv}).$$

Let us fix n_1 et n_2 with $n_1 + n_2 = p^w n$, $d_1|n_1$ and $d_2|n_2$ with $(n_1/d_1) \equiv ab \bmod Mp^v$ et $d_2 \equiv b \bmod Mp^v$, and write only the

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terms which depend on j:

$$\sum_{j=0}^{t} \binom{t}{j} (-a)^{t-j} (-1)^j d_1^{k'-2-j} \left(\frac{n_2}{d_2}\right)^j = d_1^{k'-2} \left(-a - \left(\frac{n_2}{d_1 d_2}\right)\right)^t \tag{6.6}$$

$$\equiv d_1^{k-2} d_2^{-t} \left(-a d_2 + \left(\frac{n_1}{d_1}\right)\right)^t \equiv 0 \bmod p^{vt}.$$

The congruence (6.6) is then satisfied for all $w \geq v(k'-1) > tv$ because $p \nmid d_2$ and

$$d_1^{k'-2-j} d_2^j \left(-\frac{n_2}{d_1 d_2}\right)^j \equiv d_1^{k'-2} \left(\frac{n_1}{d_1}\right)^j \bmod p^{tv}.$$

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7 Algebraic $\mathcal{A}$-linear form $\ell_\alpha : \mathcal{M}_N(\psi;\mathcal{A})^\alpha \to \mathcal{A}$

Let us describe a linear form ℓ_α on the locally free module $\mathcal{M}_N(\psi;\mathcal{A})^\alpha = \pi_\alpha(\mathcal{M}_N(\psi;\mathcal{A}))$ of finite rank.

Let us use a basis $\{g^i\}$ of $\mathcal{M}_N(\psi)^\alpha$ over the field of fractions $\mathrm{Frac}(\mathcal{A})$, such that $g^1 = g$ is fixed Coleman's eigenvector as above, and g^i are eigenfunctions of all Hecke operators T_l, $(l \nmid Np)$.

Define $\ell_\alpha(h) = x_1,$ where $h = \sum_i x_i g^i, x \in \mathcal{A}$ (the first coordinate of $h \in \mathcal{M}(\mathcal{A}; N, \psi)^\alpha$). An explicit evaluation in terms of the Peterson product shows:

$$ev_{k'}(\ell_\alpha(h)) = \ell_{\alpha(k')}(h_{k'}), \text{ where } h_{k'} = ev_{k'}(h) \in \mathcal{M}_{k'}(N,\psi).$$

The R.H.S can be computed for classical modular forms $h_{k'}$ through the (normalized) Petersson scalar product, moreover, $\ell_\alpha(g) = 1$.

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8 Proof of Main Theorem

Take the admissible measure $\tilde{\mu}_\alpha := \ell_{\alpha,f}(\tilde{\Phi}^\alpha),$ with $\tilde{\Phi}^\alpha$ constructed by the admissibility criterium of Theorem 5.1 out of products of Eisenstein series Φ_j and the linear form $\ell_{\alpha,f}$ (the Petersson product over $\mathcal{A}$). Let us compute the integrals

$$ev_{k'}\left(\int_Y \chi y_p^j \, d\tilde{\mu}_{\alpha,f}\right) = ev_{k'}(\ell_\alpha(\pi_\alpha(\Phi_j(\chi))) = \tag{8.1}$$
$$ev_{k'}(\ell_\alpha(U^{-v}\pi_{\alpha,0}U^v\Phi_j(\chi)))$$
$$= \ell_{\alpha(k')}(\pi_{\alpha(k')}\Phi_{j,k'}(\chi)) = \alpha(k')^{-v}\frac{\langle f^0_{k'}, U^v\Phi_{j,k'}(\chi)\rangle}{\langle f^0_{k'}, (f_{k'})_0\rangle}$$

for primitive Dirichlet characters χ mod p^v, using the relation (5.3): $\pi_\alpha(h) = U^{-v}\pi_{\alpha,0}(U^v h)$, where $\Phi_{j,k'} = ev_{k'}(\Phi_j) = (-1)^j E_{k'-1-j}(\xi,\chi)E_{1+j}(\psi\overline{\xi\chi})$. The value (8.1)

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can be computed using the Rankin–Selberg convolution:

$$L_{f_{k'}}(s-l+1,\bar\chi)L_{f_{k'}}(s,\bar\xi) = L_N(2s+2-k'-l,\psi\overline{\xi\chi})\sum_{n=1}^{\infty} a_n(k')b_n n^{-s}, \tag{8.2}$$

where $b_n = \sigma_{l-1,\bar\chi,\bar\xi}(n) = \sum_{d|n,d>0}\bar\chi(d)\bar\xi(n/d)d^{l-1}$, are the Fourier coefficients of an Eisenstein series $g = \sum_{n=0}^{\infty} b_n q^n$ of weight l with character $\bar\chi\bar\xi$) (if $\chi\xi(-1) = (-1)^l$).

Put $s = k'-1$, $l = k'-1-j$, $j = 0,\cdots k'-2$ with $k' > 2+j$, into (8.2):

$$L_{f_{k'}}(1+j,\bar\chi)L_{f_{k'}}(k'-1,\bar\xi) = L_N(1+j,\psi\overline{\xi\chi})\sum_{n=1}^{\infty} a_n(k')b_n n^{-k'+1}.$$

Using this equality, the R.H.S. of (8.1): is then computed using the

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Rankin–Selberg integral in the form:

$$ev_{k'}(\ell_\alpha(\pi_\alpha(\Phi_j(\chi))) = t_{k'}\cdot\frac{p^{\nu j}G(\chi)}{\alpha(k')^\nu}L^*_{f_{k'}}(1+j,\bar\chi),$$

$$\text{where } c^{\pm}(f_{k'}) = \frac{(-2i\pi)^{k'-1}\langle f_{k'},f_{k'}\rangle}{\Gamma(k-1)L_{f_{k'}}(k-1,\bar\xi)},$$

$G(\chi)$ denotes the Gauss sum of the character χ mod p^v, and $t_{k'}\in\mathbb{Q}^\times$ is an explicit elementary constant. Then one applies directly theorem 5.1 (the admissibility criterion) with $\varkappa = 1$, and the congruences (6.5) in order to obtain the required h-admissibles measures $\tilde\mu = \mu_{f,\alpha}$ in the form $\mu_{f,\alpha} = \ell_{f,\alpha}(\tilde\Phi^\alpha)$ (given by the sequence of the distributions $\Phi_j^\alpha = \pi_\alpha(\Phi_j)$).

After having an admissible measure $\tilde\Phi^\alpha$ with values in modular forms over the algebra $\mathcal{A}$, we then construct the required h-admissibles measures $\tilde\mu = \tilde\mu_{f,\alpha}$ in the form $\tilde\mu_{f,\alpha} = \ell_\alpha(\tilde\Phi^\alpha)$, as explained above.

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