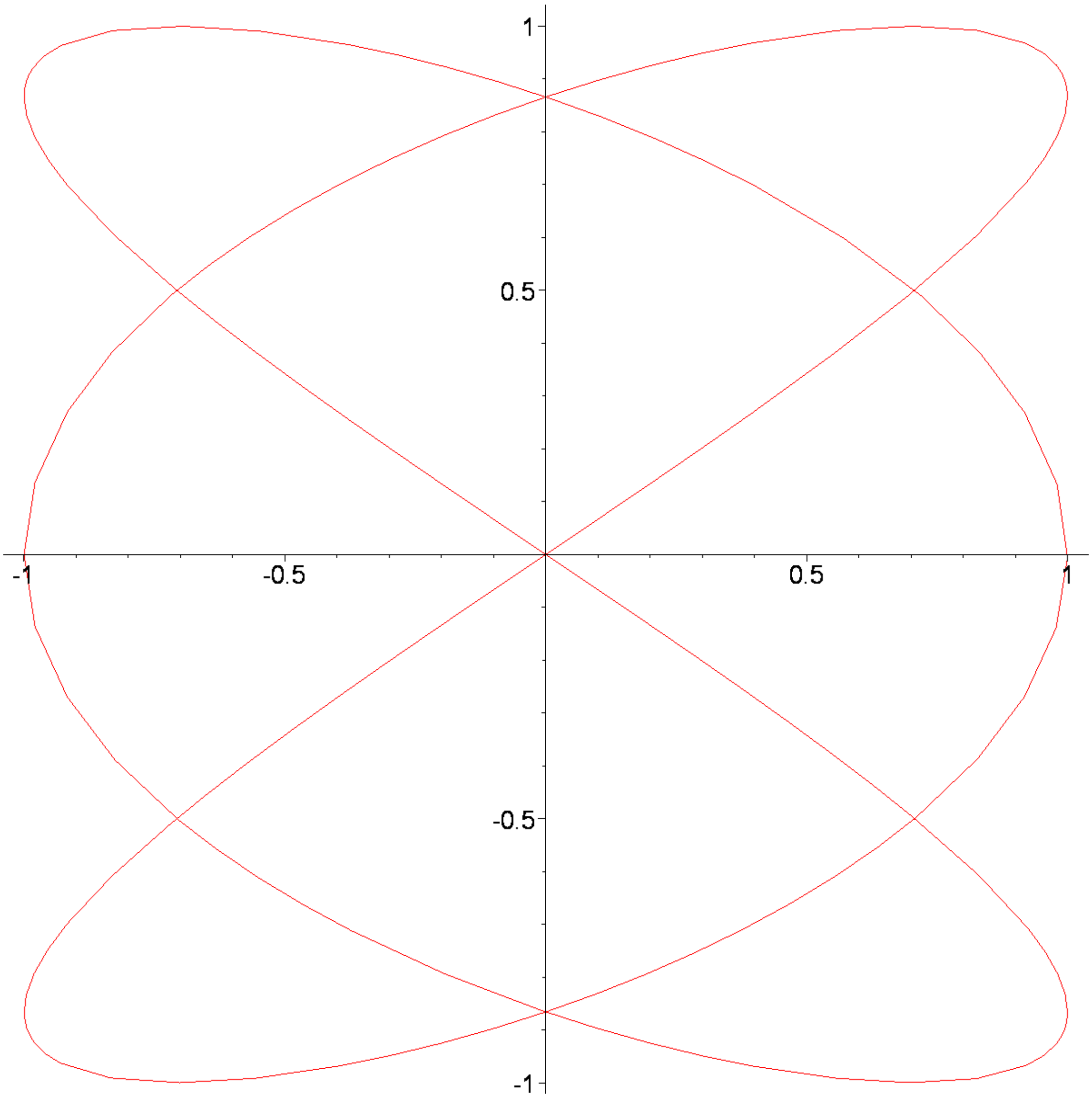
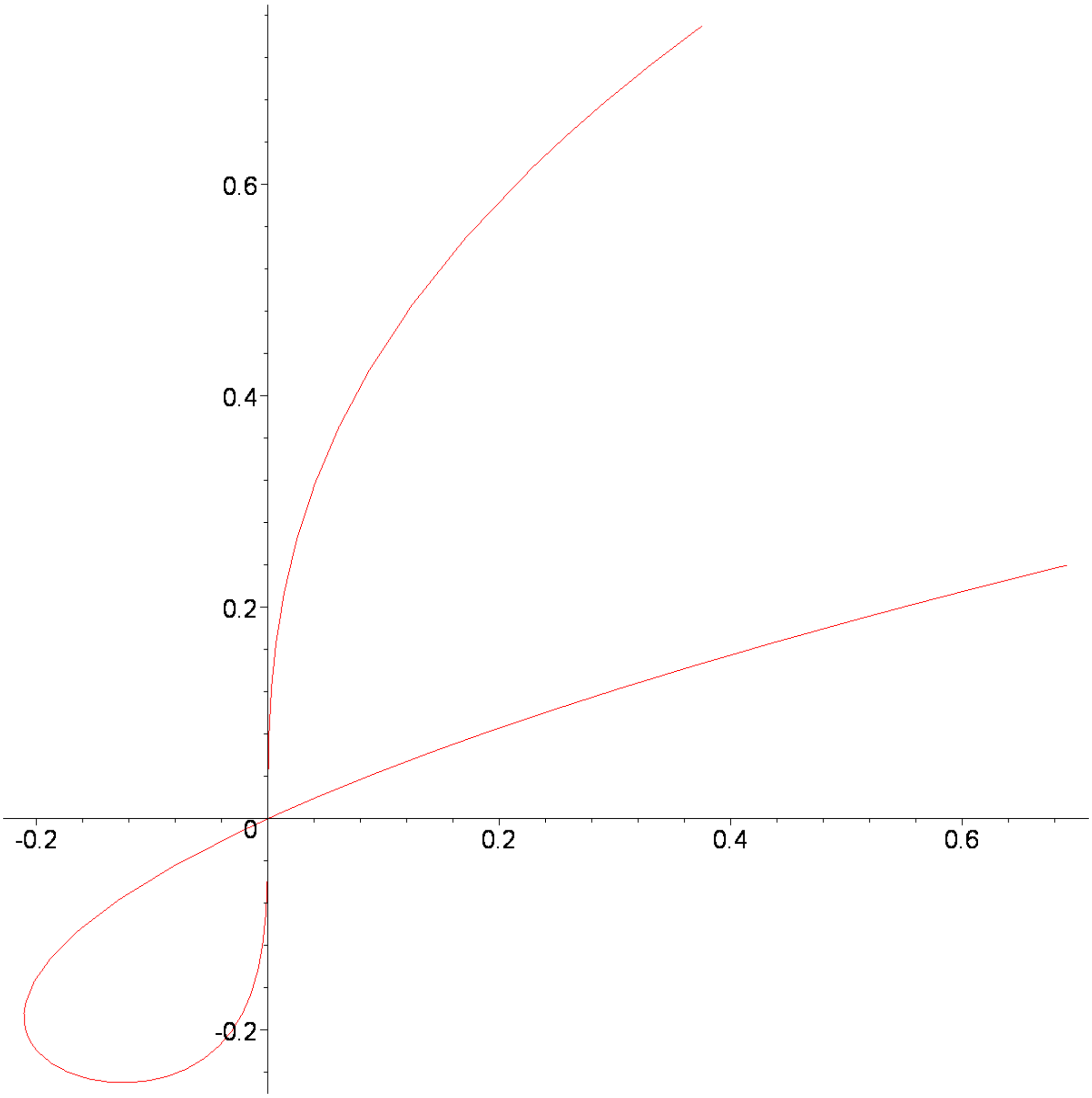
```
> #####################     FEUILLE TP MAT237
  ######################################    Exercice 1, F3
  plot([cos(3*t), sin(2*t), t=-Pi..Pi]);
```

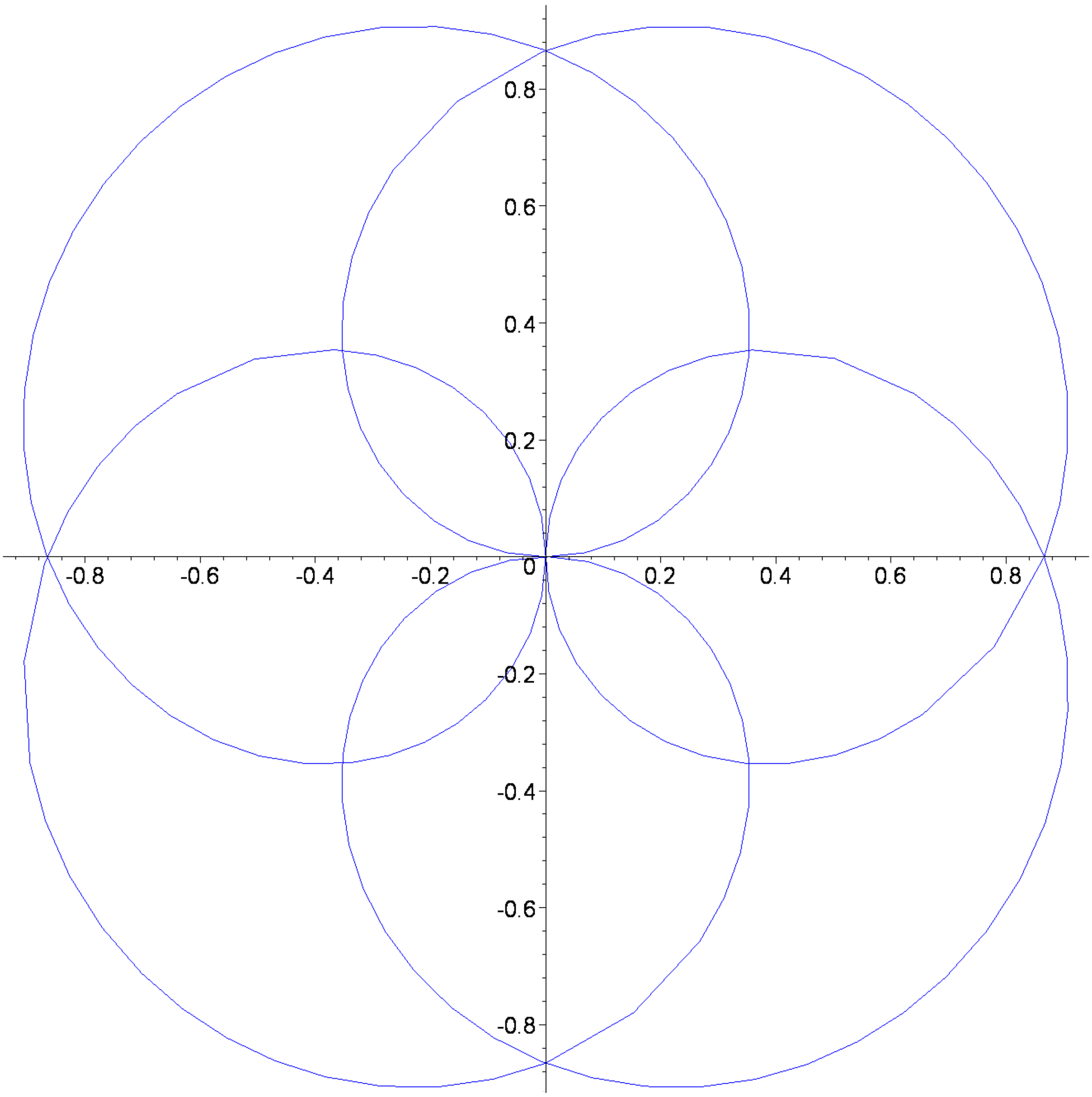



```
> ######################################    Exercice 2, F3
  restart:plot([2*t^4-2*t^3,t^2-t, t=-0.5...1.2]);
```




```
>
> ######################################     Exercice 3, F3
  with(plots):
  polarplot([sin(2*t/3),t,t=0..6*Pi],color=blue);
```

```
Warning, the name changecoords has been redefined
```




```
> 
> ########################################     Exercice 8.
  #Hyperbole
> restart;F:=x^2-y^2-1;Fx:=diff(F, x);Fy:=diff(F, y);
  y0:=1; x0:=sqrt(y0^2+1);
> 
```

$$F := x^2 - y^2 - 1$$

$$Fx := 2\,x$$

$$Fy := -2\,y$$

$$y0 := 1$$

$$x0 := \sqrt{2}$$

```
> 
```

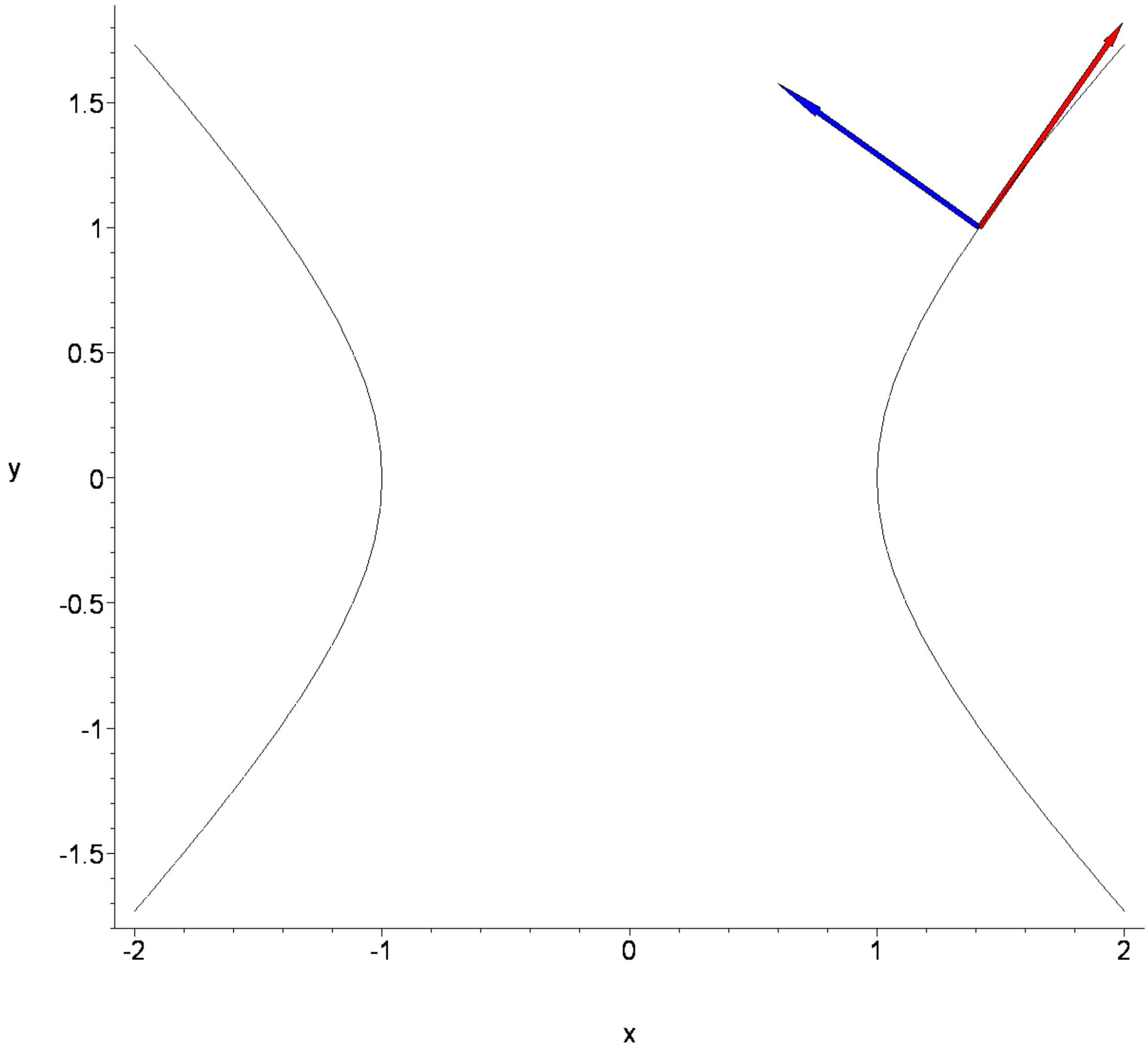
```
with(plots):b0:=implicitplot(F=0, x=-2..2,y=-2..2,color=black ,
numpoints=1000):
#y0:=1; x0:=sqrt(y0^2+1);
Fx0:=eval( Fx,[x=x0, y=y0]);
Fy0:=eval( Fy,[x=x0, y=y0]);
l0:=(Fx0^2+Fy0^2)^(1/2);
b1 := arrow(<x0,y0>, <-Fy0/l0,Fx0/l0>, width=[0.02, relative],
head_length=[0.1, relative],  color=red):
b2 := arrow( <x0,y0>, <-Fx0/l0,-Fy0/l0>,width=[0.02,relative],
color=blue):

display(b0, b1, b2, scaling=CONSTRAINED, axes=FRAMED);
```

Warning, the name changecoords has been redefined

$$Fx0 := 2\sqrt{2}$$

$$Fy0 := -2$$

$$l0 := \sqrt{12}$$




```
> ########################################     Exercice 9
```

```
#Ellipse
> restart;F:=2*x^2+3*y^2-1;Fx:=diff(F, x);Fy:=diff(F, y);
y0:=.1; solve(F,x)=0;x0:=max(eval(solve(F,x), y=y0));

>
```

$$F := 2\,x^2 + 3\,y^2 - 1$$

$$Fx := 4\,x$$

$$Fy := 6\,y$$

$$y0 := 0.1$$

$$\left(\frac{\sqrt{-6\,y^2+2}}{2}, -\frac{\sqrt{-6\,y^2+2}}{2}\right) = 0$$

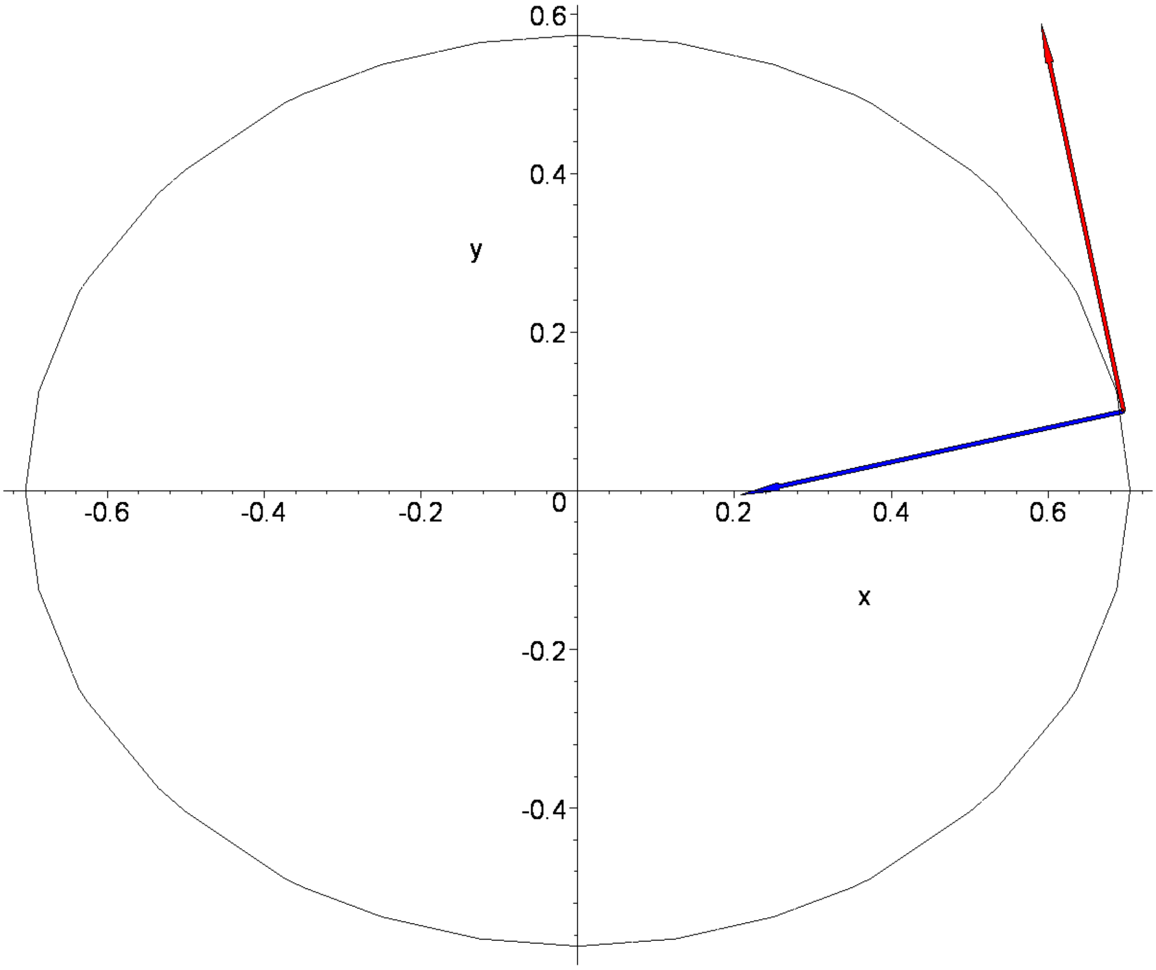
$$x0 := 0.6964194140$$

```
>
 with(plots):b0:=implicitplot(F=0, x=-2..2,y=-2..2,color=black ,
numpoints=1000):
#y0:=1; x0:=sqrt(y0^2+1);
Fx0:=eval( Fx,[x=x0, y=y0]);
Fy0:=eval( Fy,[x=x0, y=y0]);
l0:=(Fx0^2+Fy0^2)^(1/2);
b1 := arrow(<x0,y0>, <-Fy0/l0,Fx0/l0>, length=[0.5],
width=[0.01, relative], head_length=[0.1, relative],
color=red):
b2 := arrow( <x0,y0>, <-Fx0/l0,-Fy0/l0>,
length=[0.5],width=[0.01, relative], head_length=[0.1,
relative], color=blue):
display(b0, b1, b2, scaling=CONSTRAINED, axes=NORMAL);
```

$$Fx0 := 2.785677656$$

$$Fy0 := 0.6$$

$$l0 := 2.849561370$$




```
>
> ########################################    Exercice 10
>
  #Parabole
> restart;F:=y^2-6*x;Fx:=diff(F, x);Fy:=diff(F, y);
  y0:=1; solve(F,x)=0;x0:=max(eval(solve(F,x), y=y0));

>
```

$$F := y^2 - 6\,x$$

$$Fx := -6$$

$$Fy := 2\,y$$

$$y0 := 1$$

$$\frac{y^2}{6} = 0$$

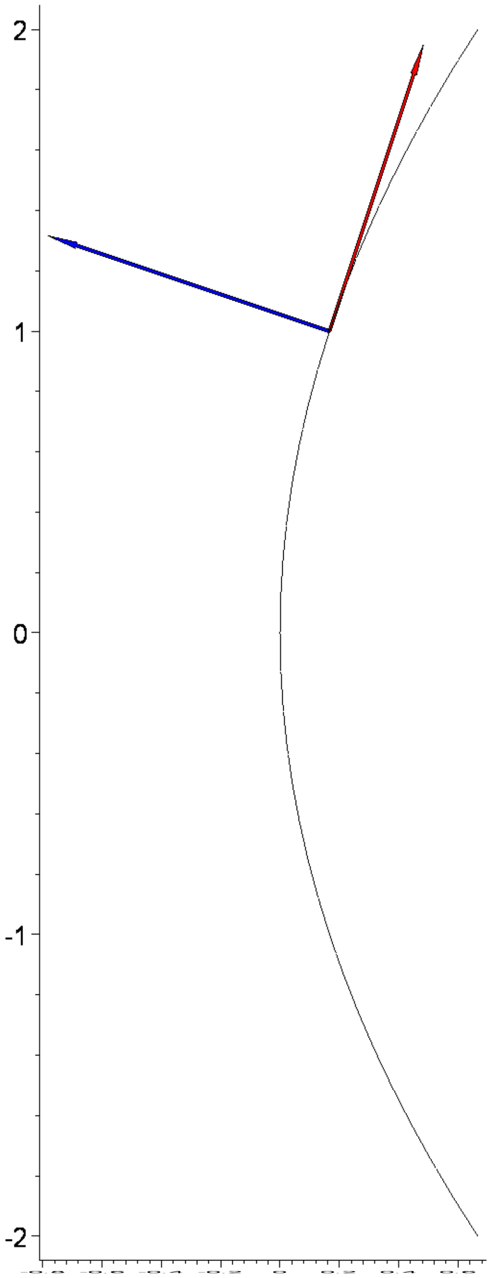
$$x0 := \frac{1}{6}$$

```
>
  with(plots):
 b0:=implicitplot(F=0, x=-2..2,y=-2..2,color=black ,
 numpoints=1000,coords=cartesian):
 #y0:=1; x0:=sqrt(y0^2+1);
 Fx0:=eval( Fx,[x=x0, y=y0]);
 Fy0:=eval( Fy,[x=x0, y=y0]);
 l0:=(Fx0^2+Fy0^2)^(1/2);
 b1 := arrow(<x0,y0>, <Fy0/l0,-Fx0/l0>, width=[0.01, relative],
 head_length=[0.1, relative], color=red):
 b2 := arrow( <x0,y0>, <Fx0/l0,Fy0/l0>,width=[0.01, relative],
 head_length=[0.1, relative], color=blue):

 display(b0, b1, b2, scaling=CONSTRAINED, axes=FRAMED);
>
Warning, the name changecoords has been redefined
```

$$Fx0 := -6$$

$$Fy0 := 2$$

$$l0 := \sqrt{40}$$




```
> 
> ########################################     Exercice 17
> restart;P:=exp(x)*cos(y)+x*y^2;Q:=(-exp(x)*sin(y)+x^2*y);
```

$$P := e^x \cos(y) + x\,y^2$$

$$Q := -e^x \sin(y) + x^2\,y$$

```
> diff(P,y);
  diff(Q,x);
```

$$-e^x \sin(y) + 2\,x\,y$$

$$-e^x \sin(y) + 2\,x\,y$$

```
> F:=exp(x)*cos(y)+x^2*y^2/2;'P'=diff(F,x);'Q'=diff(F,y);
```

$$F := e^x \cos(y) + \frac{x^2 y^2}{2}$$

$$P = e^x \cos(y) + x y^2$$

$$Q = -e^x \sin(y) + x^2 y$$

```
> int(P,x);
```

$$e^x \cos(y) + \frac{x^2 y^2}{2}$$

```
> #Soit a;b > 0. Calculer int( x^2*dy + y^2*dx, o u  a pour
  equation cart esienne l'une des
  # equations suivantes: x^2 +y^2-a*x=0; (x/a)^2 +(y/b)^2=1;
  #  (x/a)^2 +(y/b)^2-2*(x/a)-2*(y/b)=0;
> assume(a>0);assume(b>0);
> x:=(a/2)*(cos(t)+1);y:=(a/2)*(sin(t));
```

$$x := \frac{1}{2} a\sim (\cos(t) + 1)$$

$$y := \frac{1}{2} a\sim \sin(t)$$

```
> xp:=diff(x,t);yp:=diff(y,t);
```

$$xp := -\frac{1}{2} a\sim \sin(t)$$

$$yp := \frac{1}{2} a\sim \cos(t)$$

```
> x^2*yp+y^2*xp;
```

$$a^2 (\sqrt{2} \cos(t) + 1)^2 b \sqrt{2} \cos(t) - b^2 (\sqrt{2} \sin(t) + 1)^2 a \sqrt{2} \sin(t)$$

```
> simplify(int(x^2*yp+y^2*xp,t));
>
```

$$\frac{1}{24} a\sim^3 (\cos(t)^2 \sin(t) + 3 \sin(t) \cos(t) + 5 \sin(t) + 3 t - \cos(t)^3 + 3 \cos(t))$$

```
> x^2*yp+y^2*xp;
```

$$\frac{1}{8} a\sim^3 (\cos(t) + 1)^2 \cos(t) - \frac{1}{8} a\sim^3 \sin(t)^3$$

```
> simplify(int(x^2*yp+y^2*xp,t=0..2*Pi));
```

$$\frac{a\sim^3 \pi}{4}$$

```
> ################################################
  x:=a*(cos(t));y:=(b)*(sin(t));
```

$$x := a \cos(t)$$

$$y := b \sin(t)$$

```
> xp:=diff(x,t);yp:=diff(y,t);
```

$$xp := -a \sin(t)$$

$$yp := b \cos(t)$$

```
> simplify(int(x^2*yp+y^2*xp,t));
>
```

$$\frac{1}{3}\,a\,b\,(a\cos(t)^2\sin(t)+2\,a\sin(t)+3\,b\cos(t)-b\cos(t)^3)$$

```
> x^2*yp+y^2*xp;
```

$$a^2\cos(t)^3\,b-b^2\sin(t)^3\,a$$

```
> simplify(int(x^2*yp+y^2*xp,t=0..2*Pi));
```

$$0$$

```
> ##########################################
  restart;P:=y^2;Q:=x^2;
```

$$P := y^2$$

$$Q := x^2$$

```
> diff(P,y);
  diff(Q,x);
```

$$2\,y$$

$$2\,x$$

```
> ##############################################################
> restart;((x/a)-1)^2+((y/b)-1)^2=2;
```

$$\left(\frac{x}{a}-1\right)^2+\left(\frac{y}{b}-1\right)^2=2$$

```
> x:=a*(sqrt(2)*cos(t)+1);y:=(b)*(sqrt(2)*sin(t)+1);
```

$$x := a\,(\sqrt{2}\,\cos(t)+1)$$

$$y := b\,(\sqrt{2}\,\sin(t)+1)$$

```
> xp:=diff(x,t);yp:=diff(y,t);
```

$$xp := -a\,\sqrt{2}\,\sin(t)$$

$$yp := b\,\sqrt{2}\,\cos(t)$$

```
> simplify(int(x^2*yp+y^2*xp,t));
>
```

$$\frac{1}{3}\,a\,b\,(2\,a\cos(t)^2\sin(t)\sqrt{2}+6\,a\cos(t)\sin(t)+7\,a\sqrt{2}\,\sin(t)+6\,b\sin(t)\cos(t)-6\,b\,t$$
$$+9\,b\sqrt{2}\,\cos(t)+6\,a\,t-2\,b\sqrt{2}\,\cos(t)^3)$$

```
> x^2*yp+y^2*xp;
```

$$a^2\,(\sqrt{2}\,\cos(t)+1)^2\,b\sqrt{2}\,\cos(t)-b^2\,(\sqrt{2}\,\sin(t)+1)^2\,a\sqrt{2}\,\sin(t)$$

```
> simplify(int(x^2*yp+y^2*xp,t));
```

$$\frac{1}{3}\,a\,b\,(2\,a\cos(t)^2\sin(t)\sqrt{2}+6\,a\cos(t)\sin(t)+7\,a\sqrt{2}\,\sin(t)+6\,b\sin(t)\cos(t)-6\,b\,t$$
$$+9\,b\sqrt{2}\,\cos(t)+6\,a\,t-2\,b\sqrt{2}\,\cos(t)^3)$$

```
> simplify(int(x^2*yp+y^2*xp,t=0..2*Pi));
```

$$4\,a^2\,b\,\pi-4\,a\,b^2\,\pi$$

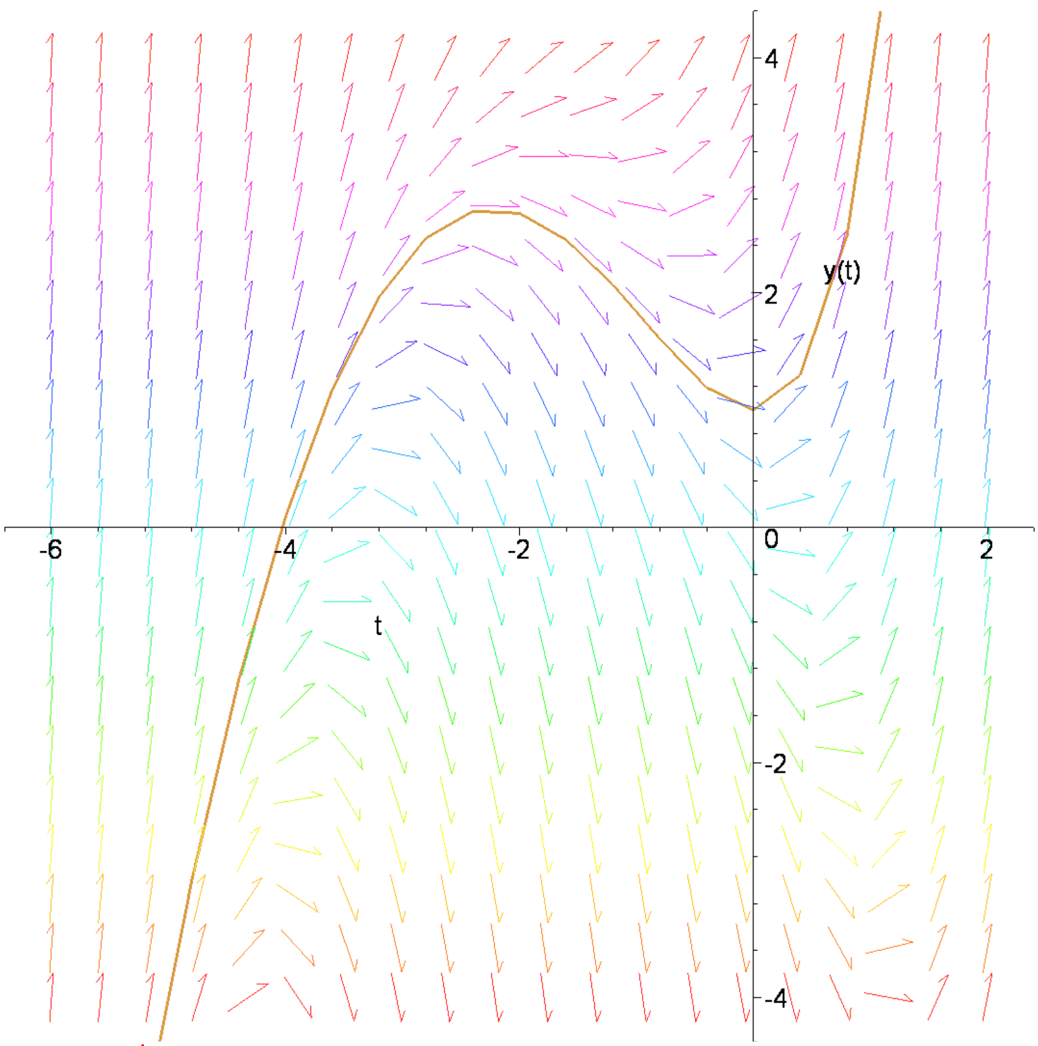
```
> ################################################################
```

```
###############
################################################################
###############
############################## Ex1(2), F4
 restart;with(DEtools):
ode := diff(y(t),t)=y(t)+t^2+3*t-1;
dsolve({ode, y(0)=1});
> DEplot(ode,y(t),t=-6..2,
y=-4..4,[[y(0)=1]],
linecolor=[ gold],title=`champ de direction`,
color=y-1);
```

$$ode := \frac{d}{dt}\mathrm{y}(t) = \mathrm{y}(t) + t^2 + 3\,t - 1$$

$$\mathrm{y}(t) = -4 - 5\,t - t^2 + 5\,\mathrm{e}^t$$

champ de direction




```
> y:=-4-5*t-t^2+5*exp(t);
```

$$y := -4 - 5\,t - t^2 + 5\,\mathrm{e}^t$$

```
> fsolve(y=0, t=-5..1);
```

$$-4.029355723$$

```
> yp:=diff(-4-5*t-t^2+5*exp(t),t);
```

$$yp := -5 - 2\,t + 5\,e^t$$

```
> evalf(solve(yp=0));
```

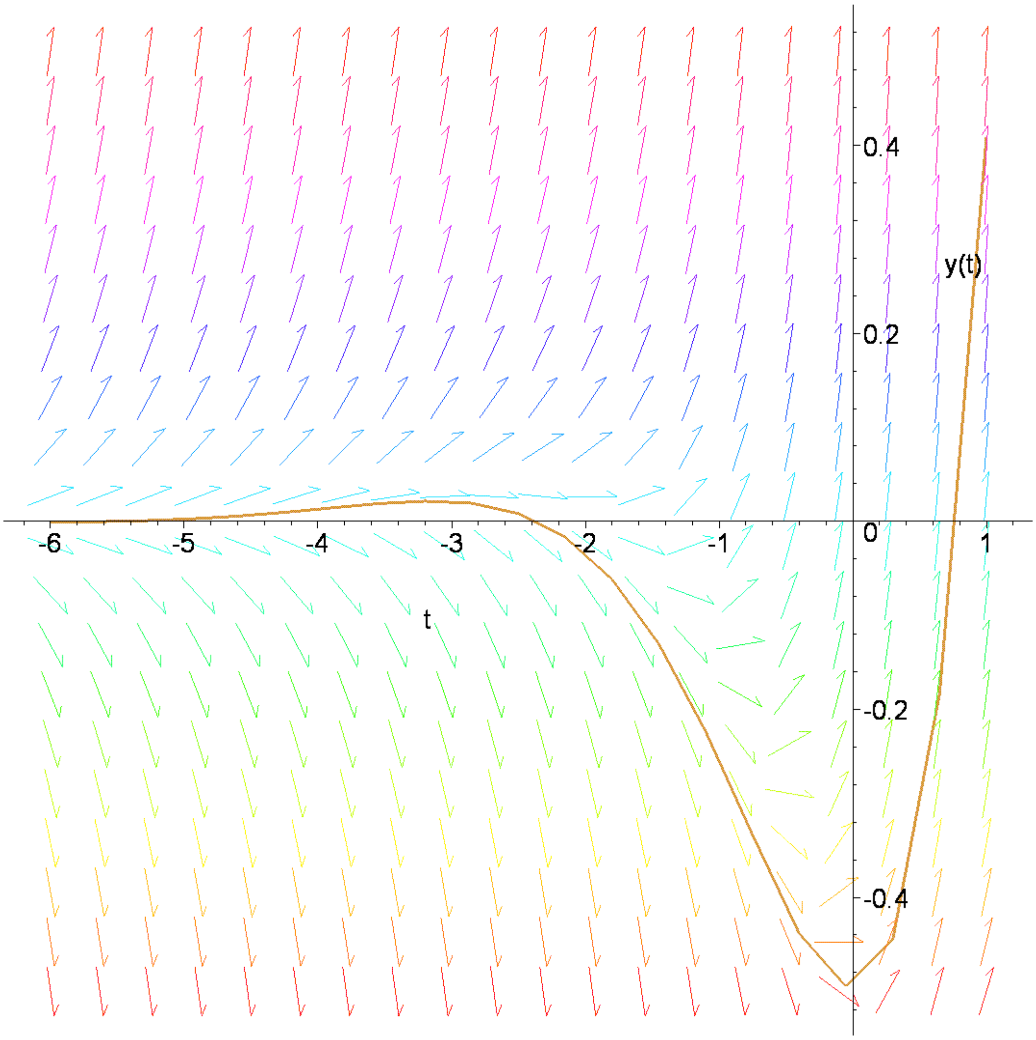
-2.231611884, 0.

```
> ################################ Ex1(3)

  restart;with(DEtools):
  ode := diff(y(t),t)=2*y(t)+exp(t)*cos(t);
  dsolve({ode, y(Pi/4)=0});
> DEplot(ode,y(t),t=-6..1,
  y=-0.5..0.5,[[y(Pi/4)=0]],
  linecolor=[gold],title=`champ de direction`,
  color=y-1);
```

$$ode := \frac{d}{dt}\,y(t) = 2\,y(t) + e^t \cos(t)$$

$$y(t) = -\frac{1}{2}\,e^t \cos(t) + \frac{1}{2}\sin(t)\,e^t$$

champ de direction




```
>
```