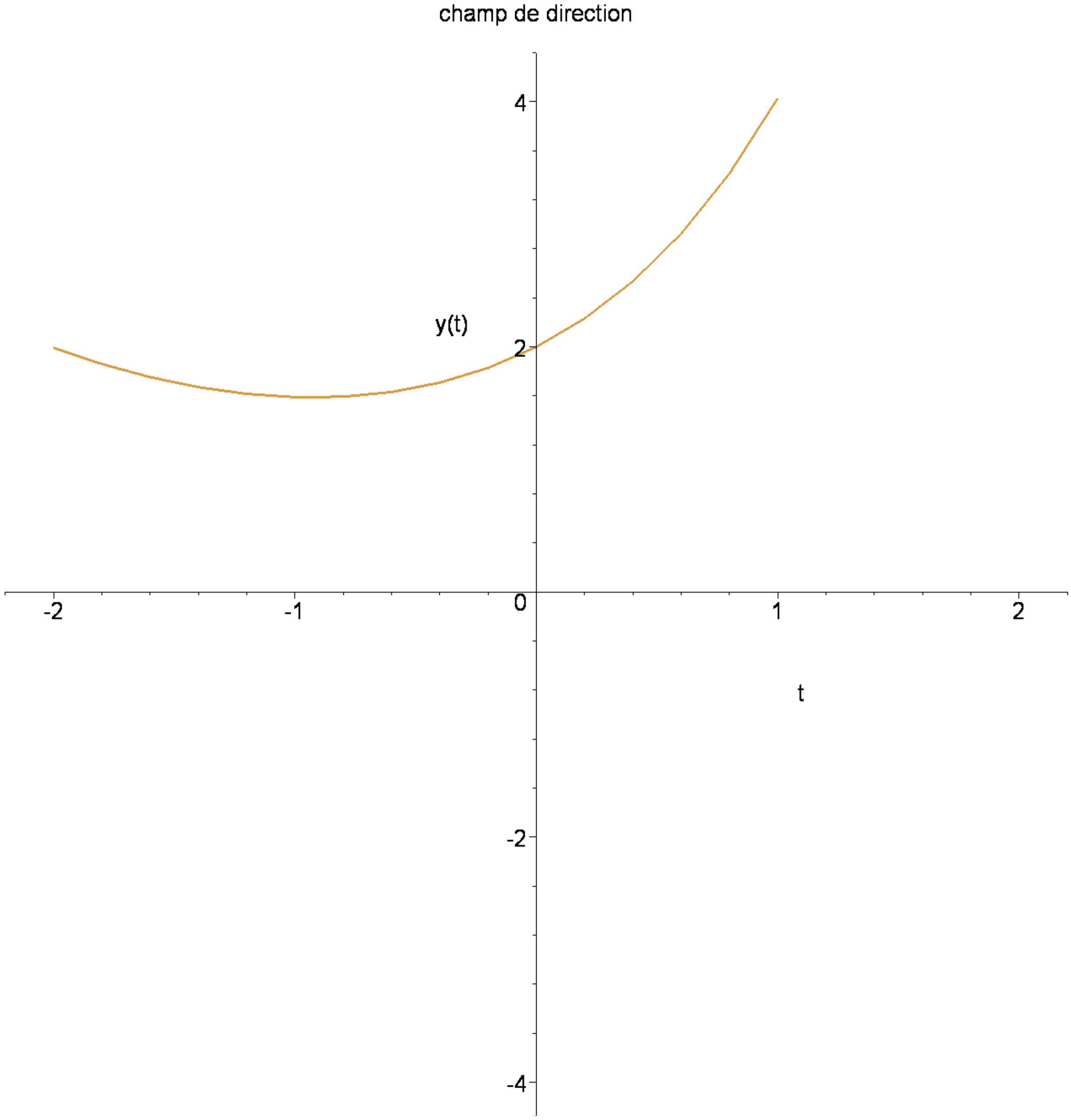
```
> ##############################   TD 28/11/06   Mat233b
> ############################### Ex1(2)
```

Exercice 1. Soient a \in R^*., et b, c \in R. Suivant les valeurs de a, b et c, résoudre l'équation différentielle ay" + by'+ cy = 0.

```
> restart;assume(a::real, b::real, c::real);
  a:=2:b:=-1:c:=-1:
  with(DEtools):
  ode := a*D(D(y))(t)+b*D(y)(t)+c*y(t);
  dsolve({ode, y(0)=2, D(y)(0)=1});
> DEplot(ode,y(t),t=-2..2,
  y=-4..4,[[y(0)=2,D(y)(0)=1]],
  linecolor=[ gold],title=`champ de direction`,
  color=y-1);
```

$$ode := 2\,(\mathrm{D}^{(2)})(y)(t) - \mathrm{D}(y)(t) - \mathrm{y}(t)$$

$$\mathrm{y}(t) = \frac{2}{3}\,\mathbf{e}^{\left(-\frac{t}{2}\right)} + \frac{4}{3}\,\mathbf{e}^{t}$$




```
>
> ############################    TD 5/12/06   Mat233b
```

Exercice 2. Sur l'intervalle I =]0,2[, on considére l'équation difféerentielle
(E): y''(t) + 3 tan t y'(t) -2y(t) = 0.
1) Quelle est la structure de l'éspace des solutions ?
2) Vérifier que la fonction f(t) = sin t est solution de (E).
3) Soit y une solution de (E). On pose w(t) =
det(y(t) ,f(t), y'(t) f'(t))
. Montrer que w est
solution d'une équation différentielle, que l'on résoudra.
4) Vérifier que.. Résoudre (E).

```
> ode := D(D(y))(t)+3*tan(t)*D(y)(t)-2*y(t);
```

$$ode := (D^{(2)})(y)(t) + 3\tan(t)\, D(y)(t) - 2\, y(t)$$

```
> with(DEtools):
  ode := D(D(y))(t)+3*tan(t)*D(y)(t)-2*y(t);
```

```
dsolve({ode, y(0)=a, D(y)(0)=b});
> #DEplot(ode,y(t),t=-2..2,
#y=-4..4,[[y(0)=2,D(y)(0)=1]],
#linecolor=[ gold],title=`champ de direction`,
#color=y-1);
```

$$ode := (D^{(2)})(y)(t) + 3\tan(t)\,D(y)(t) - 2\,y(t)$$

$$y(t) = b\sin(t) + a\,(2 - \cos(t)^2)$$

Réponse: $y(t) = b\sin(t) + a\,(2 - \cos(t)^2)$

```
> w:= D(y)*sin-y*D(sin);D(y/sin)(t);D(w/sin^2)(t);
>
```

$$w := D(y)\sin - y\cos$$

$$\frac{D(y)(t)}{\sin(t)} - \frac{y(t)\cos(t)}{\sin(t)^2}$$

$$\frac{(D^{(2)})(y)(t)\sin(t) + y(t)\sin(t)}{\sin(t)^2} - \frac{2\,(D(y)(t)\sin(t) - y(t)\cos(t))\cos(t)}{\sin(t)^3}$$

```
> ################################################################
##############
with(DEtools):
ode := D(D(y))(t)+3*tan(t)*D(y)(t)-2*y(t);
dsolve({ode, y(0)=2, D(y)(0)=1});
> DEplot(ode,y(t),t=-2..2,
y=-4..4,[[y(0)=2,D(y)(0)=1]],
linecolor=[ gold],title=`champ de direction`,
color=y-1);
> ################################################################
###################
```

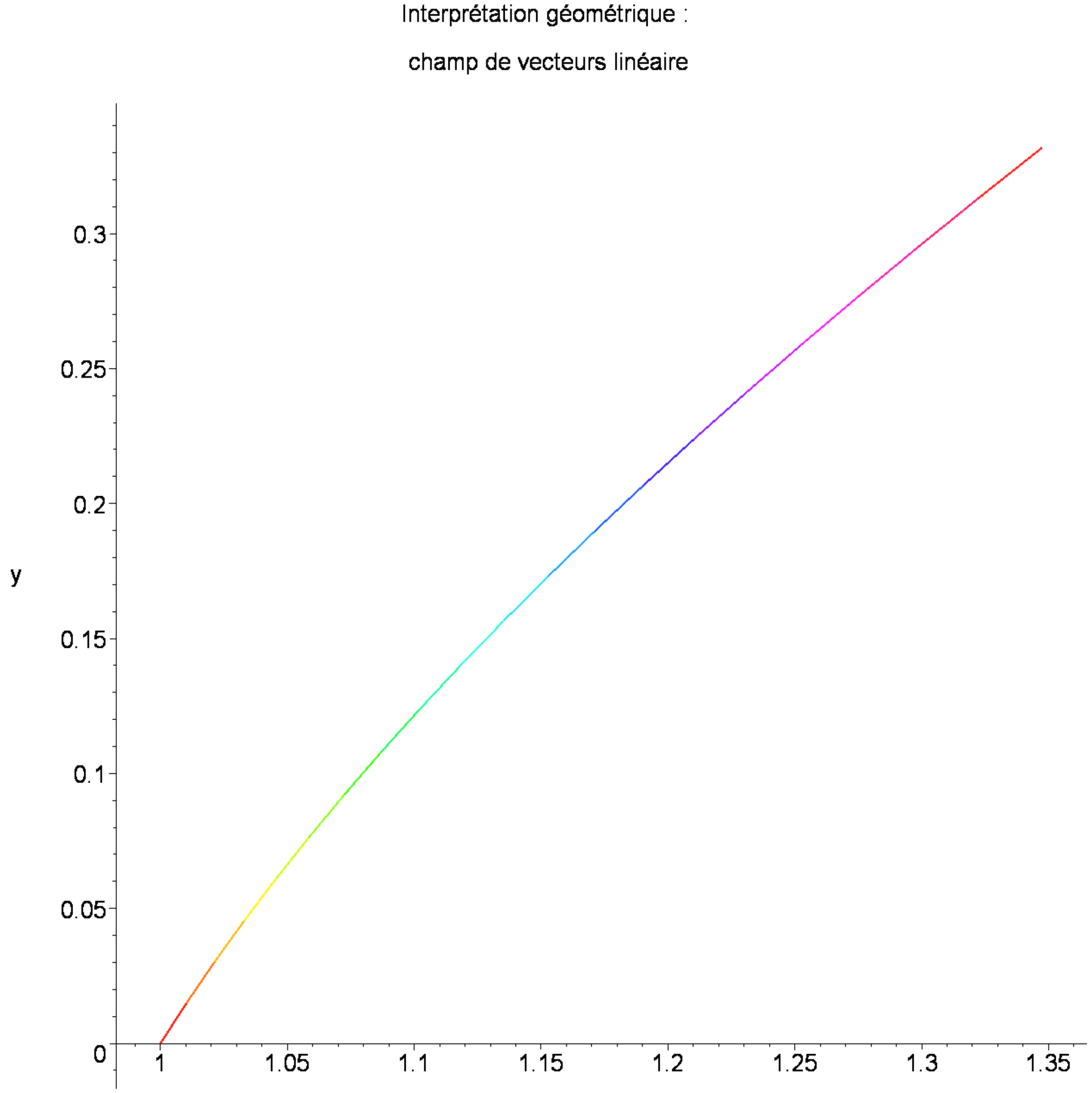
Exercice 4. Interprétation géométrique : champ de vecteurs linéaire ([YCV], p.20)
Résoudre le système différentiel
x'(t) = x(t) + 8y(t) + e^t
y'(t)= 2x(t) + y(t) + e^(-3t).

```
> restart;with(DEtools):
F2:=[diff(x(t),t)=x(t)+8*y(t)+exp(t),
diff(y(t),t)=2*x(t)+y(t)+exp(-3*t)];
>
```

$$F2 := \left[\frac{d}{dt}x(t) = x(t) + 8\,y(t) + e^t,\ \frac{d}{dt}y(t) = 2\,x(t) + y(t) + e^{(-3\,t)}\right]$$

```
>
> DEplot(F2,[x(t),y(t)],t=0..0.1,number=2,[
[x(0)=1,y(0)=0]],
#[x(0)=-0.1,y(0)=0.1]],
#[x(0)=5,y(0)=1]],
#[x(0)=-.5,y(0)=-.1]],
```

```
#stepsize=.2,
title=`Interprétation géométrique :
champ de vecteurs linéaire`,
color=[x(t), y(t), .1],
linecolor=t/2,
arrows=MEDIUM);
```




```
> restart;with(DEtools):dsys := {
diff(x(t),t)=x(t)+8*y(t)+exp(t),diff(y(t),t)=2*x(t)+y(t)+exp(-3*t),
        x(0)=1, y(0)=0};
```

$$dsys := \{\frac{d}{dt}\mathrm{x}(t) = \mathrm{x}(t) + 8\,\mathrm{y}(t) + \mathbf{e}^t, \frac{d}{dt}\mathrm{y}(t) = 2\,\mathrm{x}(t) + \mathrm{y}(t) + \mathbf{e}^{(-3\,t)}, \mathrm{x}(0) = 1, \mathrm{y}(0) = 0\}$$

```
> dsol := dsolve(dsys,'maxfun'=0);
```

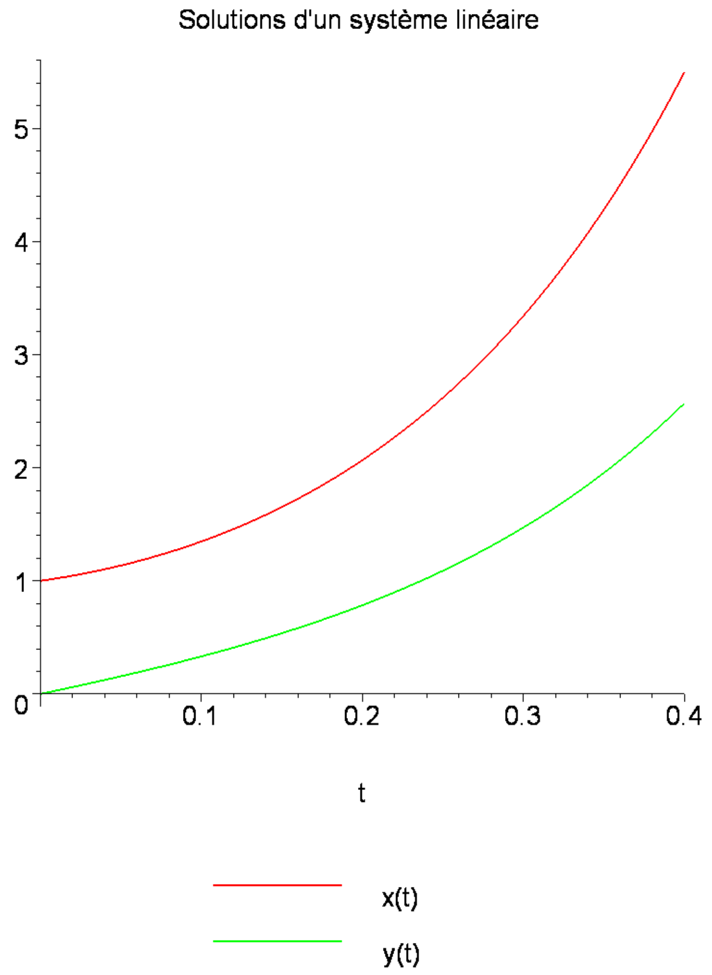
$$dsol := \{\mathrm{x}(t) = \frac{3}{4}\mathbf{e}^{(5\,t)} + \frac{1}{4}\mathbf{e}^{(-3\,t)} - t\,\mathbf{e}^{(-3\,t)}, \mathrm{y}(t) = -\frac{1}{4}\mathbf{e}^{(-3\,t)} + \frac{3}{8}\mathbf{e}^{(5\,t)} + \frac{1}{2}t\,\mathbf{e}^{(-3\,t)} - \frac{1}{8}\mathbf{e}^t\}$$

```
> X:=solve(dsol[1],x(t));
  Y:=solve(dsol[2],y(t));
```

$$X := \frac{3}{4}\,\mathbf{e}^{(5\,t)} + \frac{1}{4}\,\mathbf{e}^{(-3\,t)} - t\,\mathbf{e}^{(-3\,t)}$$

$$Y := -\frac{1}{4}\,\mathbf{e}^{(-3\,t)} + \frac{3}{8}\,\mathbf{e}^{(5\,t)} + \frac{1}{2}\,t\,\mathbf{e}^{(-3\,t)} - \frac{1}{8}\,\mathbf{e}^{t}$$

```
> with(plots):
  RX:=plot(X(t),t=0..0.4,
  color=red,
  #labels=[t, y],
  thickness=2,
  title="Solutions d'un système linéaire",
  legend="x(t)"
  ):
  RY:=plot(Y(t),t=0..0.4,
  color=green,
  #labels=[t, y],
  thickness=2,
  title="Solutions d'un système linéaire",
  legend="y(t)"
  ):
```



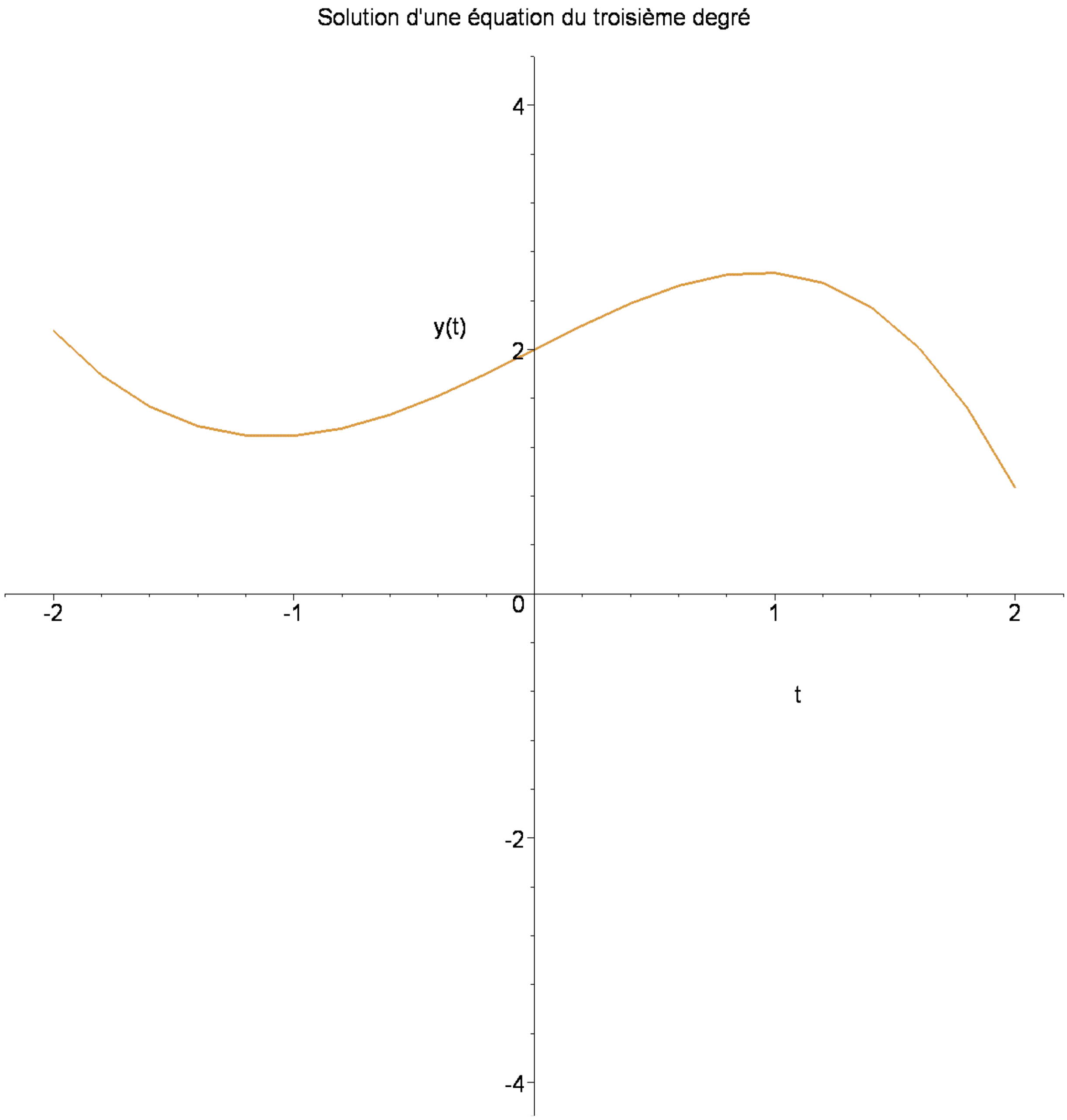
Exercice 5. (Complexification).
Résoudre l'équation différentielle y''' + y = 0.

```
> ode2 := D(D(D(y)))(t)+y(t);
> with(DEtools):
  dsolve({ode2, y(0)=a, D(y)(0)=b, D(D(y))(0)=c});
>
```

$$ode2 := (D^{(3)})(y)(t) + y(t)$$

$$y(t) = \left(\frac{a}{3} - \frac{b}{3} + \frac{c}{3}\right) e^{(-t)} + \frac{1}{3}\sqrt{3}\,(c+b)\, e^{\left(\frac{t}{2}\right)} \sin\left(\frac{\sqrt{3}\,t}{2}\right) + \left(\frac{b}{3} + \frac{2\,a}{3} - \frac{c}{3}\right) e^{\left(\frac{t}{2}\right)} \cos\left(\frac{\sqrt{3}\,t}{2}\right)$$

```
> DEplot(ode2,y(t),t=-2..2,
  y=-4..4,[[y(0)=2,D(y)(0)=1, D(D(y))(0)=0]],
  linecolor=[ gold],title=`Solution d'une équation du troisième
  degré`,
  color=y-1);
>
```



Exercice 6. (Changement de variable).
En effectuant un changement de variable x = g(t) conduisant `a une équation différentielle `a coefficients constants, résoudre l'équation (1+t^2)y'' +ty'+k^2y = 0.

```
> ode3 := (1+t^2)*D(D(y))(t)+t*D(y)(t)+k^2*y(t);
> with(DEtools):
  dsolve({ode3, y(0)=a, D(y)(0)=b});
```

$$ode3 := (1+t^2)\,(D^{(2)})(y)(t) + t\,D(y)(t) + k^2\,y(t)$$

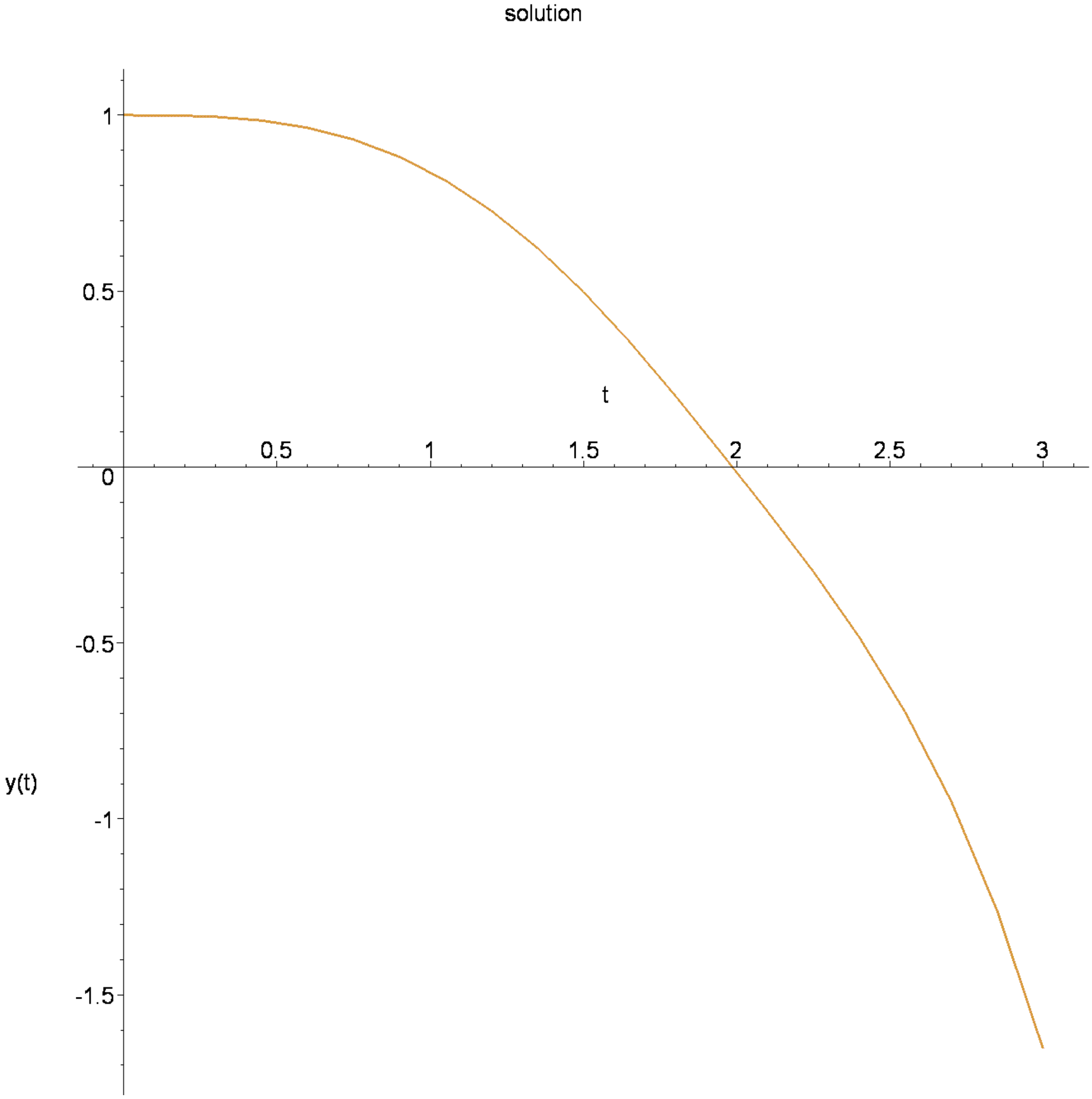
$$y(t) = \frac{b\sin(k\,\mathrm{arcsinh}(t))}{k} + a\cos(k\,\mathrm{arcsinh}(t))$$

Exercice 8. Soit f : R_+->R, de classe C^2, solution de l'équation différentielle y''(x) = -x|y(x)|, et telle que f(0) = 1, f'(0) = 0. Montrer que lim_{x\to\infty}f(x) =-\infty

```
>
  #assume(a::real, b::real, c::real);
  #a:=2:b:=-1:c:=-1:
```

```
with(DEtools):
ode4 := D(D(y))(t)+t*abs(y(t));
dsolve({ode4, y(0)=1, D(y)(0)=0});
> DEplot(ode4,y(t),t=0..3,
#y=-4..4,
number=2,[[y(0)=1,D(y)(0)=0]],
linecolor=[ gold],title=`solution`,
color=y-1);
```

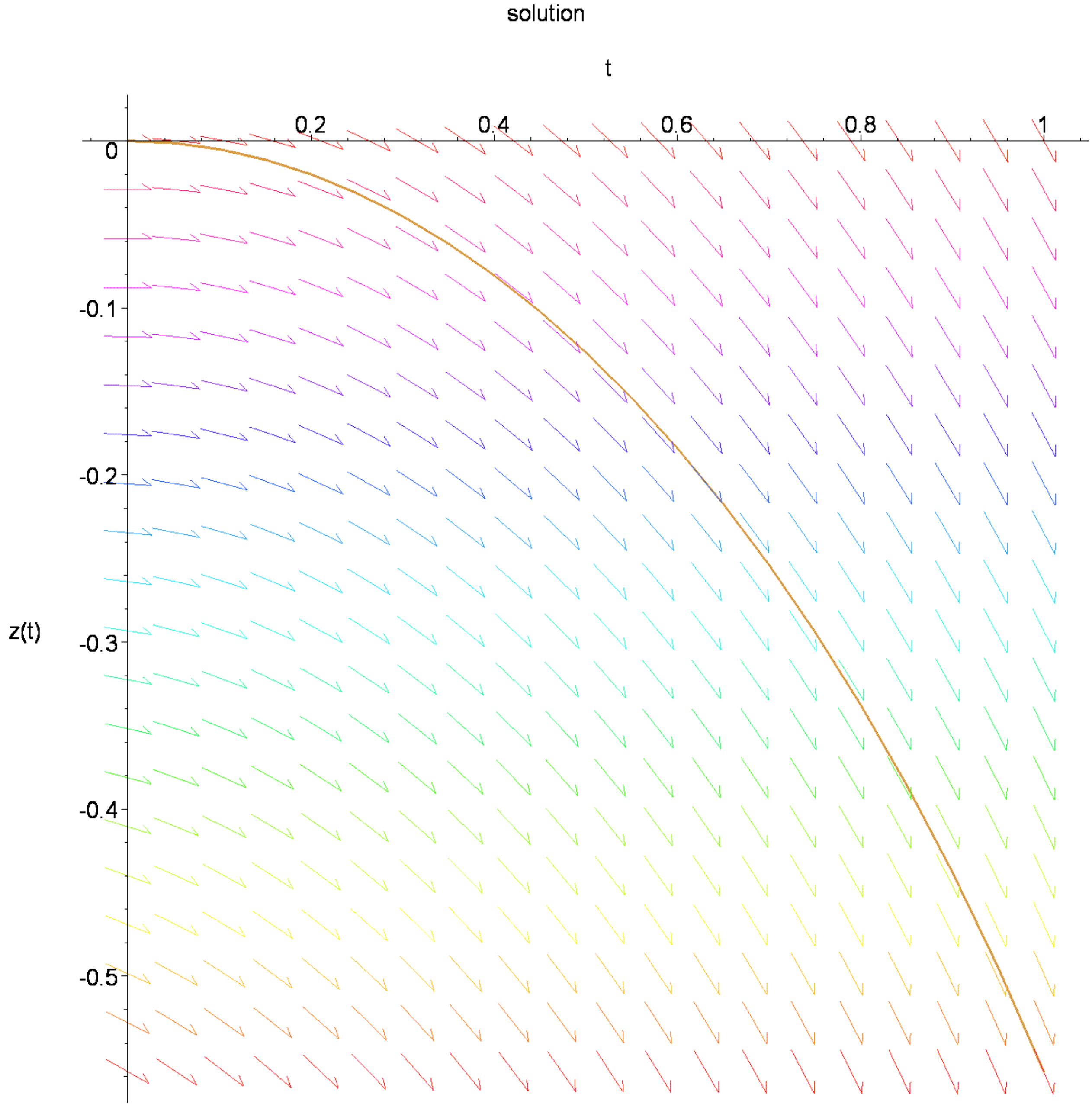
$$ode4 := (D^{(2)})(y)(t) + t\,|y(t)|$$




```
> with(DEtools):
ode5 := D(z)(t)+t+z(t)^2;
#dsolve({ode5, z(0)=0});
> DEplot(ode5,z(t),t=0..1,
#y=-4..4,
number=1,[[z(0)=0]],
```

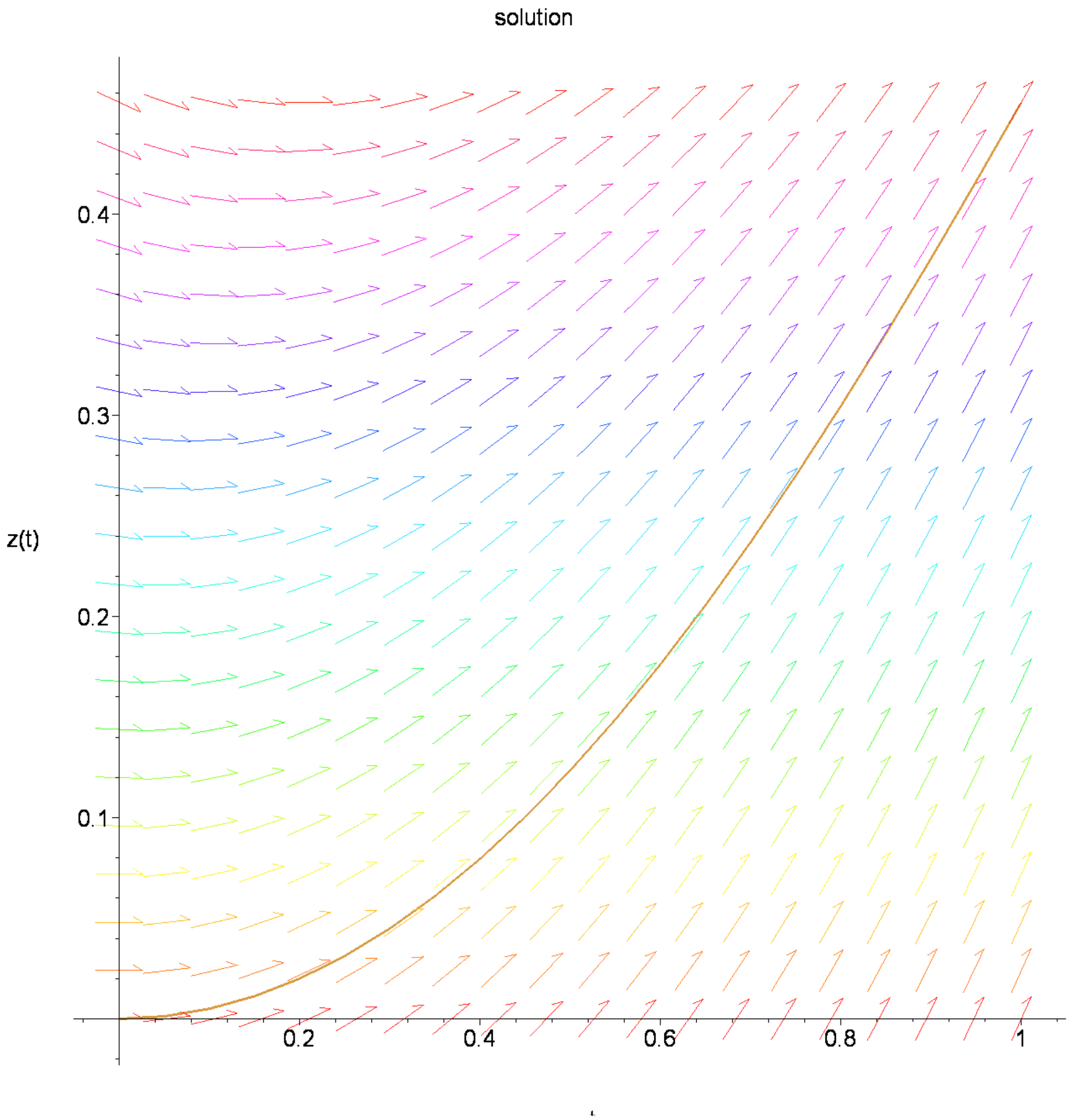
```
linecolor=[ gold],title=`solution`,
color=z-1);
```

$$ode5 := \mathrm{D}(z)(t)+t+z(t)^2$$




```
> with(DEtools):
  ode6 := D(z)(t)-t+z(t)^2;
  #dsolve({ode6, z(0)=0});
> DEplot(ode6,z(t),t=0..1,
  #y=-4..4,
  number=1,[[z(0)=0]],
  linecolor=[ gold],title=`solution`,
  color=z-1);
```

$$ode6 := \mathrm{D}(z)(t)-t+z(t)^2$$


solution
z(t)
0.1
0.2
0.3
0.4
0.2
0.4
0.6
0.8
1

>